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Constructing Smoothing Information Potential Fields with Partial Differential Equations

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Abstract: Sensornets is an important activity in the fields of computer science. Every sensor node collects the information and cooperates with others for certain purpose. A smooth version of the information distribution on the nodes is expected for some applications. Partial Differential Equations (PDEs) are introduced to determine such information potential fields and they can be solved by some numerical methods. The solutions preserve major features of original distributions which are advantageous to most applications such as navigation in parking lots, environment monitoring, etc. The simulation results show that such methods are convenient to be applied in sensornets.

Key words: Sensornets, potential field, partial differential equation, finite element method, meshless method

INTRODUCTION

The rapid development of sensornets brings new problems to many applicable scenarios (Akyildiz *et al*., 2007; Akyildiz *et al*., 2002). Such application scenarios on distributed data gathering systems have already distinguished the merits of cheap sensor networks beyond traditional centralized sensing systems. That sensornets will move beyond military deployments and the monitoring of animal or other natural habitats to the places where humans work and live: homes, cars, buildings, roads, cities, etc. Note that in these human spaces sensornets serve users embedded in the same physical space as the network. The information collected by the nodes composes a discrete distribution and most applications will be processing on it, such as navigation in parking lots, environment monitoring and so on (Atluri and Zhu, 1998; Belytschko *et al*., 1996).

Those application scenarios can be different from traditional scientific monitoring applications by several characteristics. The information is updated dynamically such as parking spaces are freed up or occupied over time, road conditions are changing at different periods of the day. Thus the navigation system needs to accommodate these environmental changes. The information and its change can be associated with a potential field. How to obtain this potential field efficiently is crucial to related application in sensornets.

With the traditional methods, one or more related optimization problem will be introduced for a certain purpose. It is sure that these methods are efficient and convenient if the amount of nodes involved in the problem is not very large. When the amount increases rapidly, the computation of solving the related optimization problem will increase more rapidly. So, it is necessary to seek novel methods with more adaptivity to the great development of sensornets.

Construct a smoothing information potential field and then intend to the purpose is a new idea. Many mathematical theories and numerical algorithms can be applied for it. Partial differential equations have been applied in various fields (Zhou and Mu, 2010; Zhou *et al*., 2009, 2010; Qiao *et al*., 2009, 2010). As a special Partial Differential Equation (PDE), Laplace equation is introduced to determine a smoothing information field (Lin *et al*., 2008). After the building of the gradient field, some certain objects such as navigation can be achieved by simple algorithms. Most of these gradient-based approaches (Chu *et al*., 2002; Faruque and Helmy, 2004; Faruque *et al*., 2005; Liu *et al*., 2005) use the natural gradients of physical phenomena, since the spatial distribution of many physical quantities, e.g., temperature measurements for heat, follows a natural diffusion law. However, it is still complex and cost much to solve a PDE by many methods such as finite element method, meshless method and so on (Zhang, 1996; Zhang *et al*., 2001). What method and algorithm should be selected is the next problem. The finite differential method and iterative algorithm are common selections.

In this study, several PDEs are introduced to construct the information potential fields in sensornets. They are proposed to determine a smooth version of the

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original potential field which keep the main features. These features are useful to most applications. Then, some numerical methods are applied to solve above PDEs. Several simulation examples are presented to verify the effectivity.

INFORMATION DISTRIBUTIONS AND INFORMATION FIELDS

As mentioned before, potential fields are introduced to describe the nodes' information distributions and be advantageous to various applications. Most of those applications benefit from the potential fields by some gradient algorithms. It is important for these algorithms to establish a proper field suitable. More exactly, a smooth field with most original features is expected. The smoothness will be beneficial to the gradient calculation and algorithm implemented, while the reservation of features is a necessary guarantee for purpose accomplished. As shown in Fig. 1a and b, a classical discrete information distribution and the related smooth information field are presented.

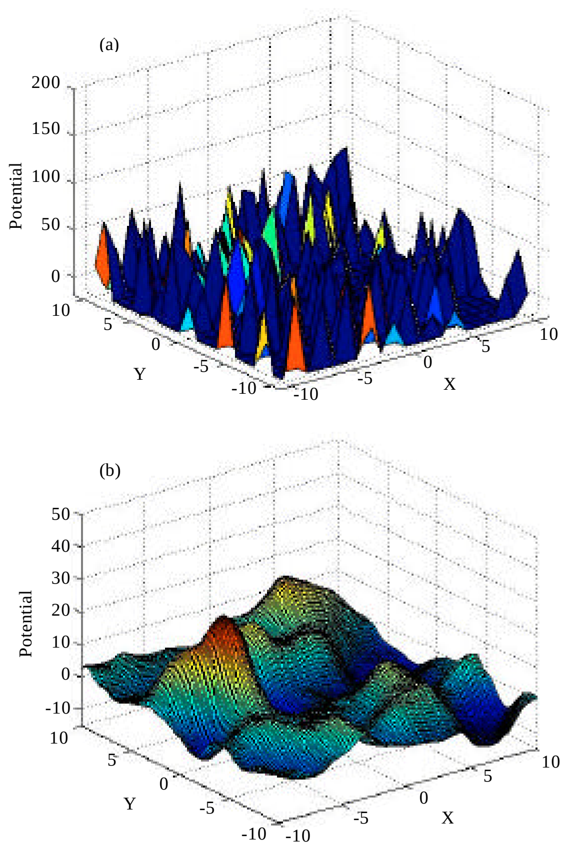

Fig. 1: (a, b) Distribution and smooth field

Many mathematical methods and techniques can be introduced to construct information potential fields such as polynomial fitting, nonlinear fitting, polynomial interpolation, least-squares fitting, Redial Basis Function (RBF) interpolation and so on. They have been applied in various fields. With the development of those methods, the error of the surface becomes smaller gradually and the smoothness is improved steadily.

PDE-BASED MODELS

In some scenarios, the local information level is advantageous to the applications. The local information level is a special average of the information in the local region. Some filters/kernels can be useful to get it by the convolution formula. The Gaussian kernel is a most common selection as known to all in many applications. The action of Gaussian kernel is equal to an evolution process of the original distribution governed by a diffusion equation. It is similar with some physical phenomenon such as a material diffusing in another material. Considering with the requirement of smoothness, it is reasonable to adopt the diffusion equation as denoted by following:

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} = F \cdot \nabla^2 u(x,t), & (x,y) \in \Omega, \\ u(x,y,0) = u_0(x,y), & (x,y) \in \Omega, \\ u(x,y) = f_D(x,y), & (x,y) \in \Gamma_D, \\ \nabla u(x,y) \cdot \vec{N} + b(x,y) \cdot u(x,y) = f_N(x,y), & (x,y) \in \Gamma_N \end{cases} \quad (1)$$

where, $\Gamma = \partial\Omega$, u(x, y, t) denotes the information field which we expected and $u_0(x, y,)$ denotes the original information distribution. With the proper setting of diffusion coefficient F, we can obtained a smooth information field by solving the Eq. 1.

In some other scenarios, the local extreme will be used to achieve the task. Then Laplace Eq. 2 and Poisson Eq. 3 can be introduced.

$$\begin{cases} \nabla^2 u(x,y) = 0, & (x,y) \in \Omega, \\ u(x,y) = f_D(x,y), & (x,y) \in \Gamma_D, \\ \nabla u(x,y) \cdot \vec{N} + b(x,y) \cdot u(x,y) = f_N(x,y), & (x,y) \in \Gamma_N \end{cases} \quad (2)$$

where, $\Gamma = \partial\Omega$. In fact, u(x, y) is assumed as a harmonic in domain Ω (Lin *et al*., 2008).

It is acknowledged that the value of any internal point can be determined uniquely if the values of the information field function on the boundaries are given. What's more, based on the maximum principle, the certain extreme points must lie the boundary. Namely, with the Dirichlet condition and Nuemman condition, the field can be obtained by solve the Laplace problem.

$$\begin{cases} div[-c\cdot\nabla u]+a\cdot u=f, & (x,y)\in\Omega, \\ u(x,y)=f_D(x,y), & (x,y)\in\Gamma_D, \\ \nabla u(x,y)\cdot\overline{N}+b(x,y)\cdot u(x,y)=f_N(x,y), & (x,y)\in\Gamma_N \end{cases} \quad (3)$$

where, $\Gamma = \partial\Omega$. If the information potential field is constructed by Poisson equation, u(x, y) is not need to be a harmonic and it will be more consistent with the real world.

In other scenarios, partial differential equations different with above mentioned can be applied.

NUMERICAL EXAMPLES AND SIMULATIONS

For solving the Eq. 1-3, it is necessary to adopt some numerical methods and algorithms such as Finite Differential Method (FDM), Finite Element Method (FEM), iterative algorithm and so on.

Finite differential method and iterative algorithm are simple and easy to be applied in solving Diffusion Eq. 1. The numerical formula is denoted as Eq. 4 and the evolution process of the information are shown in Fig. 2a-d.

$$u_{i,j}^{n+1} = u_{i,j}^{n} + \tau\cdot(\frac{u_{i+1,j}^{n} + u_{i-1,j}^{n} - 2\cdot u_{i,j}^{n}}{h_x^2} + \frac{u_{i+1,j}^{n} + u_{i-1,j}^{n} - 2\cdot u_{i,j}^{n}}{h_y^2}) \quad (4)$$

Finite element method or meshless method can be applied to solve the Laplace and Poisson equations.

A distribution of some information nodes in domain G = [−1, 1]×[−1, 1] is shown in Fig. 3a. The information quantities on nodes (−0.5, 0) and (0.5, 0) are set to be 2 and 1. Set $\Gamma_D = \partial\Omega$, $\Omega = G - \{(-0.5, 0), (0.5, 0)\}$ and give the corresponding boundary conditions, a smooth potential field is determined as shown in Fig. 3b for advanced application.

Figure 4a shows a random information distribution of sensornets. Similar boundary conditions are set for the Laplace problem. The expected potential field is

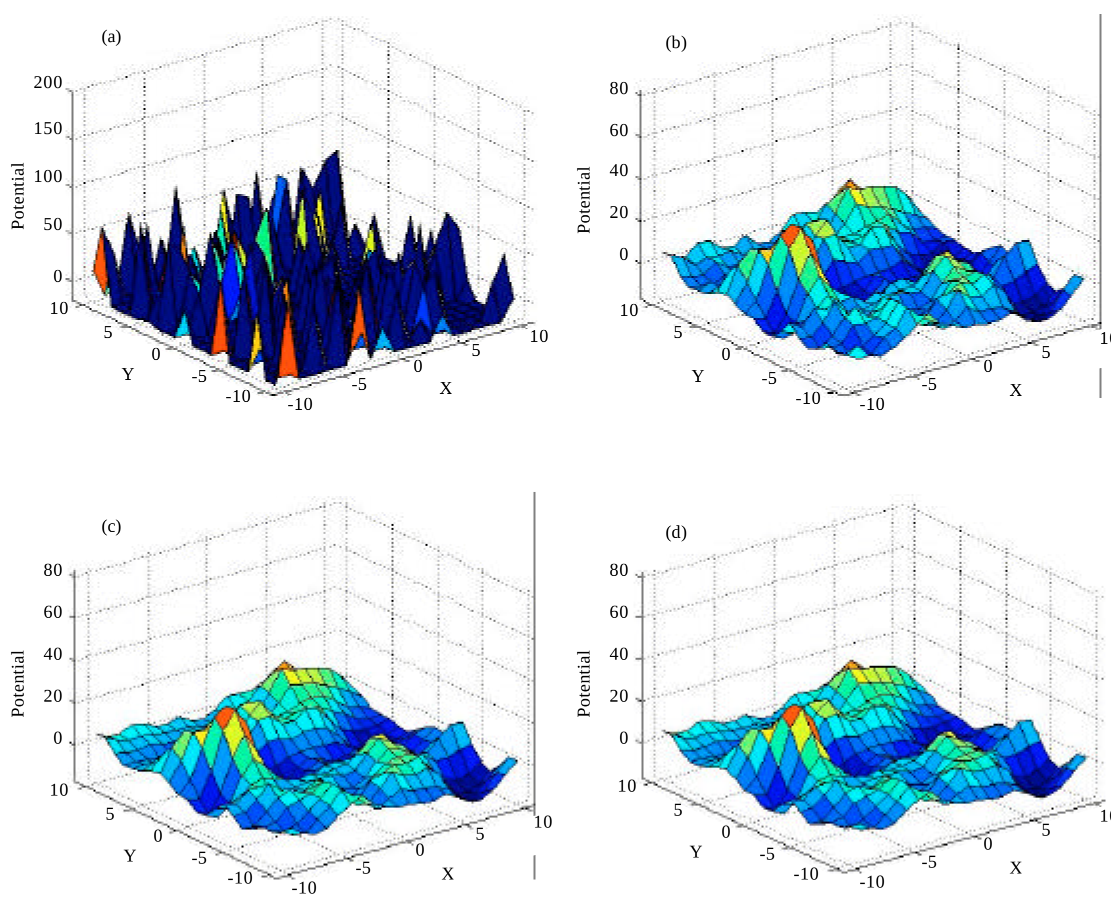

Fig. 2: (a-d) A potential field determined by Diffusion equation

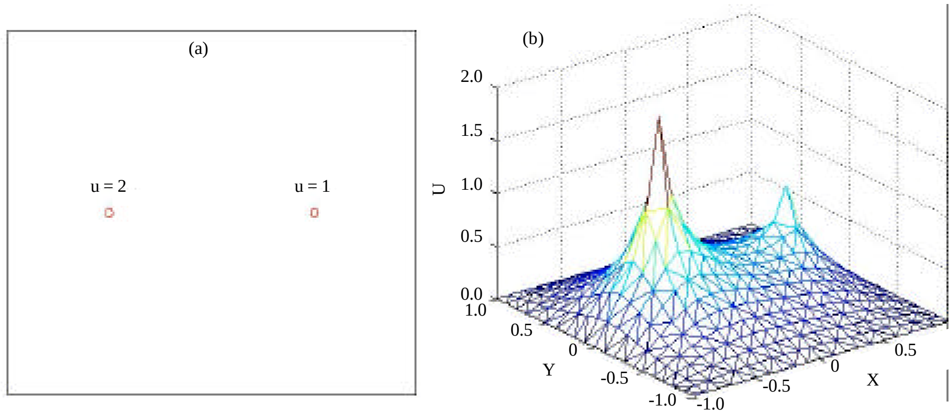

Fig. 3: A potential field determined by Laplace equation

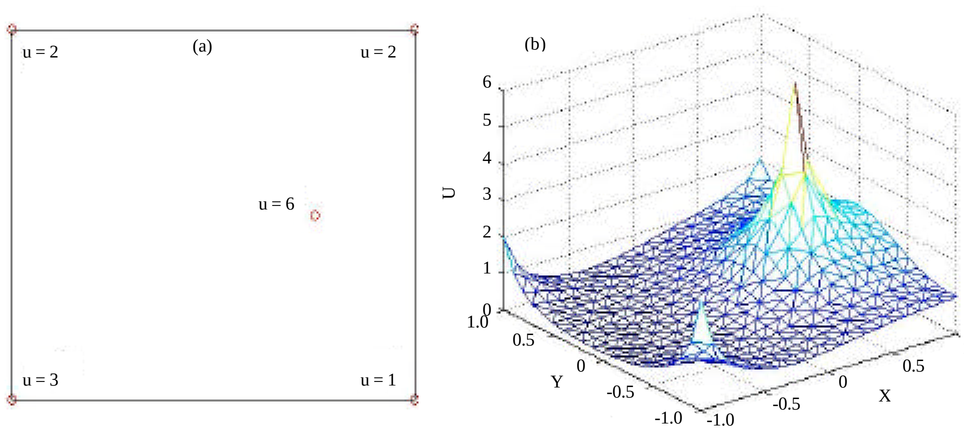

Fig. 4: A potential field determined by Poisson equation

approximated by the proposed method. The result is shown in Fig. 4b. The smoothness and reservation of main feathers can be advantageous to various applications.

CONCLUSIONS

In many application scenarios, the nodes' information generate a discrete distribution. A smooth version of the distribution is expected and it will benefit the later application. With this purpose, Diffusion, Laplace and Poisson equations are introduced to construct the information potential fields in sensornets. They are the smooth versions of some original information distributions and keep the main features. These features are useful to many applications. Then, some numerical methods are applied to solve above PDEs. Several simulation examples are presented to verify the effectivity.

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