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Controller Design and Stabilization and for a Class of Bilinear System

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Abstract: This study presents a bilinear control scheme for a class of bilinear system. A controller is proposed to globally stabilize the bilinear system. Then, the stabilization condition is derived to guarantee the stabilizability of the control system in terms of Linear Matrix Inequalities (LMIs). Finally, a numerical example is utilized to demonstrate the feasibility and effectiveness of the proposed control scheme.

Key words: Stabilization, bilinear system, linear matrix inequality, Lyapunov function, parallel distributed compensation

INTRODUCTION

Bilinear systems and controls have been successfully applied to a wide variety of fields in recent years (Mohler, 1973, 1991; Elliott, 1999; Chen and Chen, 2008; Kim *et al*., 2005; Li *et al*., 2008). Two distinguished merits should be pointed here for selecting bilinear models to describe nonlinear models. One is that bilinear systems are an adequate approximation than linear models for some real-world systems, including engineering applications in nuclear, thermal and chemical processes and many other non-engineering applications in biology, socioeconomics and immunology (Mohler, 1991, 1973). The other is that many real physic processes can be appropriately modeled as bilinear systems when linear models are inadequate, for example, the population of biological specie (Mohler, 1991). For these two reasons, bilinear system is therefore very essential to design its controller, to explore the stability (Xiang-Shun and Hua-Jing, 2009) and to improve performance by applying various control techniques (Li *et al*., 2008; Zhang and Li, 2010; Tang *et al*., 2005; Sun, 2007).

The main contributions of this study are (1) designing a bilinear controller for the bilinear system and (2) describing the stabilization conditions for the bilinear system via LMI.

SYSTEM DESCRIPTION AND CONTROLLER DESIGN

Here, we will introduce the bilinear system and then develop the controller. Firstly, consider a class of bilinear system is described as below:

$$\dot{x}(t) = Ax(t) + Bu(t) + Nx(t)u(t) \tag{1}$$

where, $x \in R^{n\times 1}$ is state, $u \in R$ is control input, $A \in R^{n\times n}$, $B \in R^{n\times 1}$ and $N \in R^{n\times n}$.

The controller for the bilinear system Eq. 1 is formulated as follows:

$$u(t) = \frac{\rho Kx(t)}{\sqrt{1 + x^T K^T Kx}}$$

$$= \rho \sin\theta$$

$$= \rho Kx \cos\theta \tag{2}$$

where,

$$\sin\theta = \frac{Kx(t)}{\sqrt{1 + x^T(t) K^T Kx(t)}}$$

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$$\cos\theta = \frac{1}{\sqrt{1 + x^T(t)K^TKx(t)}}$$

$$\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}],\ K \in R^{1\times n}$$

is a vector to be determined and $K \in R^{1\times n}$ is a scalar to be assigned.

The control objective is to design a controller Eq. 2 to stabilize the bilinear system Eq. 1 Substituting Eq. 2 into Eq. 1, one can get the closed-loop system:

$$\dot{x}(t) = (A + \rho N \sin\theta + \rho BK\cos\theta)x(t) \tag{3}$$

The main result on the asymptotic stability of the bilinear controlled system is propounded in the next section. Before discussing the proof, we first give the following results which will be used in the proof of our main results.

Lemma 1: Lo and Lin (2006), Chen *et al.* (2008) and Xu and Lam (2005): Given any matrices A, B and ε with appropriate dimensions such that ε>0 we have:

$$A^TB + AB^T \le \varepsilon A^TA + \frac{1}{\varepsilon}B^TB \tag{4}$$

RESULTS

Theorem 1: If there exist a symmetric and positive definite matrix P, a scalar ρ, some vectors K and some scalars ε_i = 1,2,3, such that the following LMI (Eq. 4) is satisfied, then the bilinear system (Eq. 1) is globally asymptotically stable via the feedback controller (Eq. 3):

$$\begin{bmatrix} QA^T + AQ & * & * & * \\ \varepsilon_1 NQ & -\varepsilon_1^2 I & * & * \\ \varepsilon_2 (BK)Q & 0 & -\varepsilon_2^2 I & * \\ \varepsilon_3 \rho I & 0 & 0 & -\varepsilon_3^2 I \end{bmatrix} < 0 \tag{5}$$

where, $Q = P^{-1}$ and * denotes the transposed elements in the symmetric positions.

Proof: Consider the Lyapunov function candidate:

$$V(x(t)) = x(t)^T P x(t) \tag{6}$$

where, P is a constant, symmetric and positive definite matrix. Clearly, V(x(t)) is positive definite and radially unbounded. The time derivative of V(x(t)) becomes:

$$\dot{V}(x(t)) = \dot{x}(t)^T P x(t) + x^T(t) P \dot{x}(t) \tag{7}$$

Substituting Eq. 3 into 4, we get:

$$\dot{V}(x) = (A + \rho N\sin\theta + \rho BK\cos\theta)^T P + P(A + \rho N\sin\theta + \rho BD\cos\theta))x(t)$$

$$= x^T(t)(A^TP + PA_1 + \rho K^TB^TP\cos\theta$$

$$+\rho N^TP\sin\theta + \rho PN\sin\theta + \rho PBK\cos\theta)x(t) \tag{8}$$

Consider the equation from the Eq. 8:

$$A^TP + PA + \rho N^TP\sin\theta + \rho K^TB^TP\cos\theta + \rho PN\sin\theta + \rho PBK\cos\theta \tag{9}$$

Multiplying the above equation on the left and right by P^{-1} and defining a new variable $Q = P^{-1}$, we can rewrite Eq. 9 as:

$$Q(A + \rho N\sin\theta + \rho BK\cos\theta)^T + (A + \rho N\sin\theta + \rho BK\cos\theta)Q \tag{10}$$

According to Lemma 1, the above equation can be rewritten as:

$$QA^T + AQ + QN^TNQ + QK^TB^TBKQ + \rho^2 I \tag{11}$$

Assuming that Eq. 11 is negative, then applying the Schur complement to Eq. 11 results in:

$$\begin{bmatrix} QA^T + AQ & * & * & * \\ \varepsilon_1 NQ & -\varepsilon_1^2 I & * & * \\ \varepsilon_2 (BK)Q & 0 & -\varepsilon_2^2 I & * \\ \varepsilon_3 \rho I & 0 & 0 & -\varepsilon_3^2 I \end{bmatrix} < 0 \tag{12}$$

which is the LMI (Eq. 5). Thus $\dot{V}(x(t)) < 0$, if Eq. 5 is satisfied. This completes the proof of the theorem.

Example: Here, the proposed method is used to design a controller for a class of bilinear system. The bilinear system is described as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) + Nx(t)u(t) \tag{13}$$

where,

$$A = \begin{bmatrix} -8.22 & -4.13 \\ 7.1 & -4 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } N = \begin{bmatrix} 7 & 0 \\ 0 & 10 \end{bmatrix}$$

Let, ρ = 0.3 and choose the controller gain matrix as K = [-0.5, -1]. Applying ρ and all these matrices to

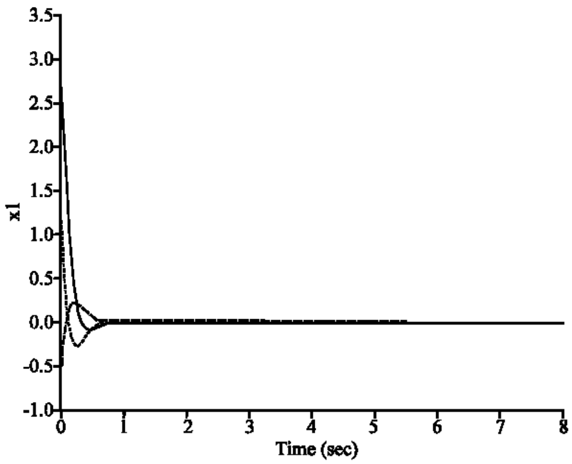

Fig. 1: State responses of x_1 under three different initial conditions $(x(0)) = [3.1 \quad -2]^T$: solid line, $x(0) = [-0.8 \quad -1.3]^T$: dash line, $x(0) = [1.5 \quad 1.5]^T$: dot line

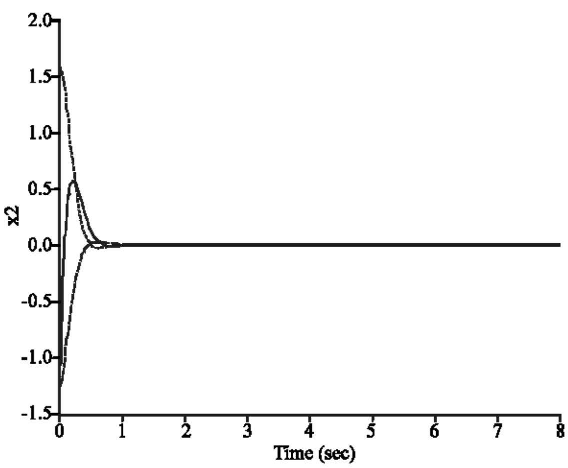

Fig. 2: State responses of x_2 under three different initial conditions $(x(0)) = [3.1 \quad -2]^T$: solid line, $x(0) = [-0.8 \quad -1.3]^T$: dash line, $x(0) = [1.5 \quad 1.5]^T$: dot line)

inequalities Eq. 4 in Theorem 1 and utilizing the LMI tool box (Tanaka and Wang, 2001), one can figure out the common positive-definite matrices,

$$Q = \begin{bmatrix} 0.2872 & -0.0223 \\ -0.0223 & 0.5529 \end{bmatrix} \text{ and } P = \begin{bmatrix} 3.4931 & 0.1409 \\ 0.1409 & 1.8142 \end{bmatrix}$$

The simulations are performed under three different initial conditions $x(0) = [3.1 \quad -2]T$, $x(0) = [-0.8 \quad -1.3]^T$, and $x(0) = [1.5 \quad 1.5]^T$, respectively. Figure 1 shows state responses of x_1 and Fig. 2 shows state responses of x_2. From these two Fig. 1 and 2, one can find that the states

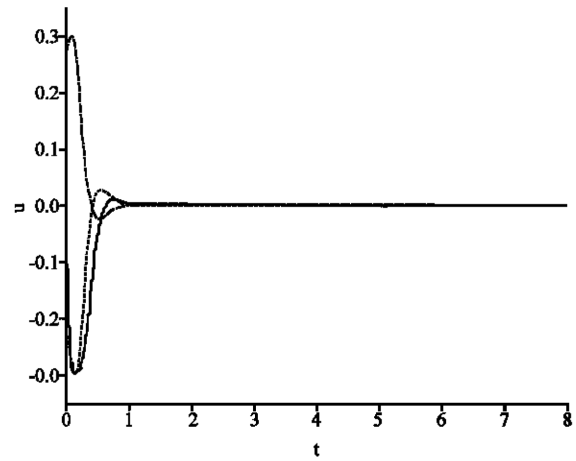

Fig. 3: Control input u(t) for three different initial conditions $(x(0)) = [3.1 \quad -2]^T$: solid line, $x(0) = [-0.8 \quad -1.3]^T$: dash line, $x(0) = [1.5 \quad 1.5]^T$: dot line)

x_1 and x_2 converge to the equilibrium state within 1 sec. Control input u(t) for three different initial conditions is shown in Fig. 3. From these simulation results, the proposed control scheme for bilinear system is effective and feasible.

CONCLUSIONS

In this study, we have developed a novel controller for a class of bilinear system. Considering to stabilize this kind of problem and to guarantee the stability of the controlled system, the sufficient condition has been derived via LMI in detail. Finally, a numerical simulation has been adopted to demonstrate the feasibility and effectiveness of the proposed schemes.

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