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Performance of Alamouti-Based HARQ for Slow Fading MIMO Channel

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Abstract: Traditional hybrid automatic repeat request (HARQ) protocols extract time diversity in a scalar fading channel. The performance depends critically on the channel variation over retransmissions. The slow fading channel model considered allows no time diversity for the HARQ protocols to exploit. But traditional protocols can not effectively exploit spatial diversity provided by a Multiple Input Multiple Output (MIMO) system. A new Alamouti-based HARQ transmission scheme for a MIMO system in a slowly varying channel is proposed. This new scheme is a combination of the packet combining HARQ and the Alamouti Space-Time Coding (STC). This technique increases the efficiency of HARQ packet transmission by exploiting both the spatial and time diversity of the MIMO channel. It uses the full diversity of Alamouti STC and the added gain of the packet combining scheme to provide reliable communication. The Packet Error Rate (PER) analysis of space-time coded MIMO-HARQ is presented. An n-dimension Pair Wise Error Probability (PWEP) analysis of the optimal Alamouti-based HARQ protocol is derived. Simulation results show that this new scheme outperforms traditional Chase Combining (CC) and reveal the gain of ARQ feedback in space-time coded MIMO systems.

Key words: Alamouti code, hybrid ARQ, multiple-antenna, chase combining

INTRODUCTION

Hybrid ARQ techniques use Forward Error Correction (FEC) with the automatic repeat request (ARQ) protocol to recover erroneous packets caused by the channel noise and interferences. The HARQ schemes are usually considered to exploit both the high coding gain of FEC and the rate flexibility of ARQ protocol such that data can be transmitted with a minimum error. In a pure ARQ protocol, a received packet containing error is discarded and a retransmission of the packet is requested. In HARQ, earlier received erroneous packets are combined in an intelligent way with the subsequent received packets to improve the decoding reliability. The MIMO systems are known to increase the spectral efficiency and the capacity of a communication system. Combined with HARQ, a MIMO system can potentially provide higher throughput packet data services with higher reliability. Furthermore, through proper arrangement of the retransmitted packets, one can improve the performance of a MIMO system. However, most of the study of hybrid ARQ techniques is focused on scalar channels. Kim and Skoglund (2007) and Chuang *et al.* (2008) focused on the jointly design of MIMO transmission and ARQ feedback about MIMO-HARQ design. Oh *et al.* (2004) studied receiver processing of ARQ retransmissions. Koike *et al.* (2004), Carvalho and Popovski (2008) have studied HARQ schemes jointly considering STC and packet retransmission. For the sake of exploiting the additional spatial degrees of freedom, the bits or symbols rearrangement for retransmissions is studied by Carvalho and Popovski (2008) and the linear precoder design is considered in (Zheng *et al.*, 2007). Despite the many efforts in studying MIMO-HARQ design, there are still some unsolved problems. The DMT-based approach focuses only on the high Signal-to-Noise Ratio (SNR) asymptotics and gives the tradeoff between multiplexing gain, diversity gain and ARQ delay. It requires a some of Space-Time Codes (STC) those rates ought to be proportional to the SNR. However, in practice, one is also interested in designing a fixed-rate STC that operates well within a range of finite SNRs. For the practical HARQ retransmission protocol design, the known works provide separate and ad hoc designs.

There are mainly two types of HARQ combining scheme: the packet combining (Chase, 1985) and Incremental Redundancy (IR) (Sesia *et al.*, 2004). In IR-type ARQ, retransmissions only carry portions of the

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data packet. It presents an efficient technique for increasing the system throughput while keeping the error performance acceptable. Present main objective is to reduce the number of ARQ rounds required to correctly decode a data packet while keeping the receiver affordable complexity of computational load and memory requirements. We focus on space-time Bit-Interleaved Coded Modulation (BICM) transmitter schemes with Chase-type ARQ, in which the data packet is entirely retransmitted. The choice of BICM is due to the simplicity of this coding scheme and the efficiency of its iterative decoding receiver in achieving high diversity and coding gains over blockfading MIMO channels. In this study, we restrict present work to chase-type ARQ. A conventional Log-Likelihood Ratio (LLR)-level combining are used, where extrinsic LLRs corresponding to multiple transmissions are simply added together before SISO decoding. In the packet combining, the receiver combines noisy packets to obtain a packet with a code rate which is low enough such that reliable communication is possible even for low quality channels. we provide numerical simulations for some MIMO configurations demonstrating the superior performance of the proposed the Alamouti-based HARQ protocol compared with CC protocol.

Throughout the study the following notations will be used. Matrices and vectors are denoted with inclined bold capital and lowercase letters, respectively. $\mathbf{A}(i, j)$ is the (i, j)-th element of the matrix $\mathbf{A}$ and $\mathbf{a}(i)$ is the i-th element of the vector $\mathbf{a}$. $\mathbb{E}[\cdot]$ denotes the expectation of a random variable. a^* denotes the conjugate of the complex number a, Re{a} is the real part of a and $\mathbf{a}^T$ is the transpose of vector $\mathbf{a}$. tr($\mathbf{A}$) and $\|\mathbf{A}\|$ denotes the trace and Frobenius norm of matrix $\mathbf{A}$, respectively. $\mathbf{A}^H$ denotes the Hermitian of the complex matrix $\mathbf{A}$. vec(•) is the vectorization operation. $|C|$ denotes the cardinality of the set C. $\langle \mathbf{A}, \mathbf{B} \rangle = \mathrm{tr}(\mathbf{A}\mathbf{B}^H)$ denotes the Frobenius inner product of matrices $\mathbf{A}$ and $\mathbf{B}$ of the same dimension.

MIMO CHANNEL MODEL WITH ARQ FEEDBACK

In Fig. 1, we focuses on a single-user multiple-antenna slow fading wireless system with $M_T \times M_R$ transmit-receive antennas. The receiver is assumed to have perfect knowledge of H is due to the slowly varying nature of the channel helps receiver channel estimation. The transmitter is assumed to have no knowledge of $\mathbf{H}$ before the transmission. The MIMO channel $\mathbf{H} \in \mathbb{C}^{M_R \times M_T}$ is assumed to be a random matrix with independent and identically distributed (i.i.d.) complex circularly symmetric Gaussian entries with unit variance. The channel matrix $\mathbf{H}$ is assumed to be fixed within the channel coherence time (τ_c symbols) and varieties independently with a different value in the next coherence block.

As soon as the transmission of current message including possible retransmissions is finished, the next message is encoded and transmitted immediately. It is assumed that there is an error-free and delay-free ARQ feedback link and there is a very large buffer of information messages available at the transmitter. N denotes the maximum allowable ARQ rounds, after ARQ round N retransmissions, if the receiver still cannot decode the message successfully, no further attempt will be tried and a decoding failure is declared. The receiver sends messages to inform the transmitter the successful decoding by ACK and failed decoding by NACK at each ARQ round, respectively. For the transmission of each message, the input information message $c \in \mathcal{C}$ is sent to a MIMO-HARQ encoder, here, $\mathcal{C}$ denotes a set of uniformly distributed messages. encoder outputs N matrix subcodewords $\{\mathbf{X}_n(c)\}_{n=1}^{N}$, each corresponds to one ARQ round. If the decoding fails at the (n-1)-th ARQ round, subcodeword $\mathbf{X}_n(c)$ will be transmitted in the n-th ARQ round. The overall codeword after the n-th ARQ round is denoted by $\mathbf{X}^{(n)}(c) \triangleq [\mathbf{X}_1(c), \cdots, \mathbf{X}_n(c)],\ \forall n = 1, \cdots, N.$ The decoder performs optimum decoding for all the received packets in ARQ round 1,...,n. Each subcodeword satisfies

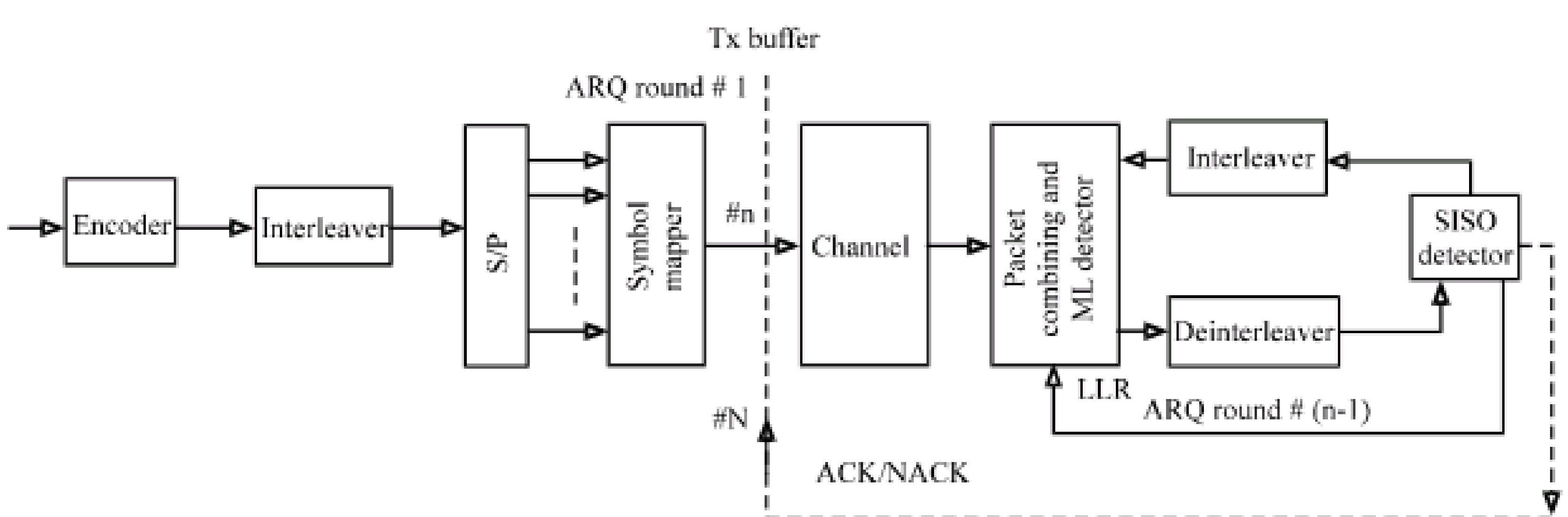

Fig. 1: BICM with MIMO-HARQ system model

$\mathbf{X}_n(c) \in \mathbb{C}^{M_T \times (l_n/r_n)}$, where, l_n/r_n denotes the length of the subcodeword, r_n is determined by the coding rate of the space--time structure as $r_n \triangleq K_n/\tau_n$, where, K_n and τ_n are the number of symbols and the time slots per space-time codeword, respectively and l_n is the capacity-achieving channel code length. According to the relationship between τ_c and $\{l_n/r_n\}_{n=1}^N$. A slow fading MIMO channel model is considered, where, $\tau_c \gg \sum_{n=1}^N (l_n/r_n)$. This means the MIMO channel keeps fixed throughout all possible retransmissions. There is no time diversity to exploit by retransmissions in this models. The overall transmission rate at the n-th ARQ round is:

$$R^{(n)} = \frac{\log_2 |\mathcal{C}|}{\sum_{i=1}^n (l_i/r_i)}, \quad \forall\, n = 1, \cdots, N \tag{1}$$

The received signal corresponding to the n-th transmission can be expressed as:

$$\mathbf{Y}_n = \sqrt{\frac{\gamma}{M_T}} \mathbf{H}\mathbf{X}_n + \mathbf{N}_n$$

and the overall received signal after the *n*-th ARQ round is:

$$\mathbf{Y}^{(n)} = \sqrt{\frac{\gamma}{M_T}} \mathbf{H}\mathbf{X}^{(n)} + \mathbf{N}^{(n)}$$

where, $\mathbf{Y}^{(n)} \triangleq [\mathbf{Y}_1, \cdots, \mathbf{Y}_n]$, $\mathbf{X}^{(n)} \triangleq [\mathbf{X}_1, \cdots, \mathbf{X}_n]$, and $\mathbf{N}^{(n)} \triangleq [\mathbf{N}_1, \cdots, \mathbf{N}_n]$. The additive noise $\mathbf{N}^{(n)}$ has i.i.d. entries $\mathbf{N}^{(n)}(i,j) \sim \mathcal{CN}(0,1)$. The overall transmit codewords $\mathbf{X}^{(N)}$c are normalized to satisfy the average power constraint,

$$\mathbb{E}\left[\left\|\mathbf{X}^{(N)}(c)\right\|^2\right] \le M_T \sum_{n=1}^N (l_n/r_n)$$

where, $\mathbb{E}[\bullet]$ is the expectation for all possible codewords. The average SNR per receive antenna is γ.

DEGREES OF FREEDOM IN THE MIMO-HARQ PROTOCOL

In this study, a traditional stop-and-wait Chase Combining (CC) (Chase, 1985) HARQ protocol is considered, which belongs to the category of diversity combining. Upon each retransmission request, the transmitter simply repeats the same packet. Different diversity combining schemes can be used at the receiver, among which the Chase combining, which essentially is a maximum ratio combining of all the received packets in the scalar channel, gives the best performance.

Due to the randomness of the channel matrix **H** and the maximum ARQ round N, the successful communication rate R is a random variable. we define two events for ARQ round n, n = 1...,N $\{\mathcal{D}_n$ = successful decoding at the end of ARQ round n$\}$ and $\{\mathcal{M}_n$ = ARQ round n is enabled$\}$. because a decoding failure is equivalent to a channel outage, thus:

$$\mathbb{P}\{\mathcal{D}_n\} = \mathbb{P}\left\{C^{(n)}(\mathbf{H}_{eq}^{(n)}) \ge R^{(n)}\right\} \tag{2}$$

where, $C^{(n)}(\mathbf{H}_{eq}^{(n)})$ is the equivalent channel capacity at ARQ round n, $R^{(n)}$ given by Eq. 1 is the overall communication rate at round n.

Equation 2 leads to $\mathcal{D}_{n-1} \subseteq \mathcal{D}_n$ and hence $\mathbb{P}\{\mathcal{M}_n\} = \mathbb{P}\left\{\overline{\mathcal{D}_1}, \cdots, \overline{\mathcal{D}_{n-1}}\right\} = \mathbb{P}\left\{\overline{\mathcal{D}_{n-1}}\right\}$. According to definitions of event $\mathcal{D}_n$ and $\mathcal{M}_n$ and using $\mathbb{P}\{\mathcal{D}_0\} = 0$, the successful communication rate R can be written as:

$$R = \begin{cases} R^{(n)}, & \text{if } \{\mathcal{M}_n, \mathcal{D}_n\}, \forall n = 1, \cdots, N \\ 0, & \text{if } \overline{\mathcal{D}_N} \end{cases} \tag{3}$$

$$= \begin{cases} R^{(n)}, & \text{if } \left\{\overline{\mathcal{D}_{n-1}}, \mathcal{D}_n\right\}, \forall n = 1, \cdots, N \\ 0, & \text{if } \overline{\mathcal{D}_N} \end{cases} \tag{4}$$

the average rate is:

$$\bar{R} = \sum_{n=1}^N R^{(n)} \mathbb{P}\{\overline{\mathcal{D}_{n-1}}, \mathcal{D}_n\} \tag{5}$$

$$= \sum_{n=1}^N R^{(n)} \mathbb{P}\{C^{(n-1)}(\mathbf{H}_{eq}^{(n-1)}) < R^{(n-1)}, C^{(n)}(\mathbf{H}_{eq}^{(n)}) \ge R^{(n)}\} \tag{6}$$

From Eq. 1, obtain $R^{(0)} = \infty$ and $C^{(0)}(\mathbf{H}_{eq}) = 0$ due to $\mathcal{D}_{n-1} \subseteq \mathcal{D}_n$ get $\mathbb{P}\{\overline{\mathcal{D}_{n-1}}, \mathcal{D}_n\} = \mathbb{P}\{\mathcal{D}_n\} - \mathbb{P}\{\mathcal{D}_{n-1}\}$. Hence, Eq. 5 can be rewritten as:

$$\bar{R} = \sum_{n=1}^N R^{(n)} \left(\mathbb{P}\{\mathcal{D}_n\} - \mathbb{P}\{\mathcal{D}_{n-1}\}\right)$$

$$= \sum_{n=1}^N \left(R^{(n)} - R^{(n+1)}\right) \mathbb{P}\{\mathcal{D}_n\} \tag{7}$$

$$= \sum_{n=1}^N \left(R^{(n)} - R^{(n+1)}\right) \mathbb{P}\left\{C^{(n)}(\mathbf{H}_{eq}^{(n)}) \ge R^{(n)}\right\} \tag{8}$$

clearly, $R^{(N+1)} = 0$.

The equivalent channel capacity $C^{(n)}(\mathbf{H}_{eq}^{(n)})$ at ARQ round n is determined by the HARQ protocol and the instantaneous channel matrix H, Thus, the transmit covariance matrices could be variational over retransmissions. Thus, these degrees of freedom shall be exploited in MIMO-HARQ protocol.

DETECTION ERROR PROBABILITY ANALYSIS

Here, the Pair Wise Error Probability (PWEP) (Tarokh *et al*., 1998) is studied, an error probability analysis of the optimal Alamouti-based HARQ protocol is performed.

Assume that the signal vector $\mathbf{s} = [s_1,...,s_K]^T$ is chosen from a uniformly distributed set $\mathcal{M}$, here, $\mathbf{s}_j \in \mathcal{M}$, its cardinality is denoted by $|\mathcal{M}|$. The optimum Maximum-Likelihood (ML) decoding rule for deciding between two possible codewords for a given channel realization and the receiver observation is:

$$\left\| \mathbf{Y}^{(n)} - \sqrt{\frac{\gamma}{M_T}}\mathbf{H}\mathbf{X}^{(n)}(\mathbf{s}_i) \right\|^2 \underset{\mathbf{s}_j}{\overset{\mathbf{s}_i}{\lessgtr}} \left\| \mathbf{Y}^{(n)} - \sqrt{\frac{\gamma}{M_T}}\mathbf{H}\mathbf{X}^{(n)}(\mathbf{s}_j) \right\|^2$$

Based upon ML metric $\| \mathbf{Y}^{(n)} - \sqrt{\frac{\gamma}{M_T}}\mathbf{H}\mathbf{X}^{(n)}(\mathbf{s}_i) \|^2$, the pairwise decision metric in the n-th ARQ round is:

$$\Delta_{i,j}^{(n)} = \left\| \mathbf{Y}^{(n)} - \sqrt{\frac{\gamma}{M_T}}\mathbf{H}\mathbf{X}^{(n)}(\mathbf{s}_i) \right\|^2 - \left\| \mathbf{Y}^{(n)} - \sqrt{\frac{\gamma}{M_T}}\mathbf{H}\mathbf{X}^{(n)}(\mathbf{s}_j) \right\|^2$$

The probability of a decoding error after ARQ round n is:

$$P_e^{(n)} = \frac{1}{|\mathcal{M}|}\sum_{j=1}^{|\mathcal{M}|} \mathbb{P}\left\{ \mathbf{s}^{(1)} \neq \mathbf{s}_j, \cdots, \mathbf{s}^{(n)} \neq \mathbf{s}_j \mid \mathbf{s}_j \right\} \quad (9)$$

$$= \frac{1}{|\mathcal{M}|}\mathbb{E}\Big[\sum_{j=1}^{|\mathcal{M}|}\mathbb{P}\Big\{\bigcup_{\substack{i_1=1\\ i_1\neq j}}^{|\mathcal{M}|}\cdots\bigcup_{\substack{i_n=1\\ i_n\neq j}}^{|\mathcal{M}|}\{\Delta_{i_1,j}^{(1)} < 0, \cdots, \Delta_{i_n,j}^{(n)} < 0\} \mid \mathbf{H},\mathbf{s}_j\Big\}\Big]$$

$$\leq \frac{1}{|\mathcal{M}|}\sum_{j=1}^{|\mathcal{M}|}\sum_{\substack{i_1=1\\ i_1\neq j}}^{|\mathcal{M}|}\cdots\sum_{\substack{i_n=1\\ i_n\neq j}}^{|\mathcal{M}|}\mathbb{E}\left[\mathbb{P}\{\Delta_{i_1,j}^{(1)} < 0, \cdots, \Delta_{i_n,j}^{(n)} < 0 \mid \mathbf{H},\mathbf{s}_j\}\right] \quad (10)$$

where, $\mathbf{s}_j$ is the transmitted vector, $\mathbf{s}^{(n)}$ is the detected vector after ARQ round n.

However, the performance analysis of MIMO-HARQ is intractable, which comes from Eq. 10. Analyzing MIMO-HARQ requires the n-th pairwise error probability $\mathbb{P}\{\Delta_{i_1,j}^{(1)} < 0, \cdots, \Delta_{i_n,j}^{(n)} < 0 \mid \mathbf{H},\mathbf{s}_j\}$, where, the decision metrics $\{\Delta_{i_l,j}^{(l)}\}_{l=1}^{n}$ are random variables and correlated.

To simplify the derivation of n-PWEP, define:

$$\mathbf{Y}_{i,j}^{(n)} \triangleq \sqrt{\frac{\gamma}{M_T}}\mathbf{H}(\mathbf{X}^{(n)}(\mathbf{s}_i) - \mathbf{X}^{(n)}(\mathbf{s}_j))$$
$$\mathbf{y}_i^{(n)} \triangleq \mathrm{vec}(\mathbf{Y}_i^{(n)})$$
$$\mathbf{d}_{i,j}^{(n)} \triangleq \mathrm{vec}(\mathbf{Y}_{i,j}^{(n)}) = \mathbf{y}_i^{(n)} - \mathbf{y}_j^{(n)}$$
$$\mathbf{n}^{(n)} \triangleq \mathrm{vec}(\mathbf{N}^{(n)}).$$

Conditioning on $\mathbf{H}$ and $\mathbf{s}_j$ is transmitted, obtain:

$$\begin{aligned}\Delta_{i,j}^{(n)} &= \| \mathbf{Y}_{i,j}^{(n)} + \mathbf{N}^{(n)} \|^2 - \| \mathbf{N}^{(n)} \|^2 \\ &= \| \mathbf{Y}_{i,j}^{(n)} \|^2 + 2\mathrm{Re}\{\mathrm{tr}(\mathbf{Y}_{i,j}^{(n)}(\mathbf{N}^{(n)})^H)\} \\ &= (d_{i,j}^{(n)})^2 + G_{i,j}^{(n)}\end{aligned}$$

where, $d_{i,j}^{(n)} = \| \mathbf{Y}_{i,j}^{(n)} \|$ is the Euclidean distance and $G_{i,j}^{(n)} \triangleq 2\mathrm{Re}\{\mathrm{tr}(\mathbf{Y}_{i,j}^{(n)}(\mathbf{N}^{(n)})^H)\}$.

$$\begin{aligned}G_{i,j}^{(n)} &= \mathrm{tr}(\mathbf{Y}_{i,j}^{(n)}(\mathbf{N}^{(n)})^H + (\mathbf{Y}_{i,j}^{(n)})^H\mathbf{N}^{(n)}) \\ &= (\mathbf{n}^{(n)})^H\mathbf{d}_{i,j}^{(n)} + (\mathbf{d}_{i,j}^{(n)})^H\mathbf{n}^{(n)}\end{aligned}$$

With $\mathbb{E}[G_{i,j}^{(n)}] = 0$ and

$$\begin{aligned}\mathbb{E}[(G_{i,j}^{(n)})^2] &= \mathbb{E}\left[(\mathbf{n}^{(n)})^H\mathbf{d}_{i,j}^{(n)} + (\mathbf{d}_{i,j}^{(n)})^H\mathbf{n}^{(n)}\right] \\ &= \mathbb{E}\left[2(\mathbf{n}^{(n)})^H\mathbf{d}_{i,j}^{(n)}(\mathbf{d}_{i,j}^{(n)})^H\mathbf{n}^{(n)}\right] = 2(d_{i,j}^{(n)})^2\end{aligned}$$

where, $\mathbf{N}^{(n)}$ has Complex Circularly Symmetric Gaussian (CCSG) entries with unit variance. The n-PWEP becomes:

$$\mathbb{P}\{\Delta_{i_1,j}^{(1)} < 0, \cdots, \Delta_{i_n,j}^{(n)} < 0 \mid \mathbf{H},\mathbf{s}_j\} =$$
$$\mathbb{P}\{G_{i_1,j}^{(1)} < -(d_{i_1,j}^{(1)})^2, \cdots, G_{i_n,j}^{(n)} < -(d_{i_n,j}^{(n)})^2 \mid \mathbf{H},\mathbf{s}_j\}$$

Due to $\mathbf{g}^{(n)} \triangleq (G_{i_1,j}^{(1)}, \cdots, G_{i_n,j}^{(1)})^T$ is an n-dimensional real Gaussian random vector. Thus, attend to the statistics of $\mathbf{g}^{(n)}$. The mean is $\mathbb{E}[\mathbf{g}^{(n)}] = \mathbf{0}$. As for the covariance matrix $\mathbb{E}[(\mathbf{g}^{(n)})^2]$, due to its symmetric property, the focus is on $\mathbf{R}_{\mathbf{g}^{(n)}}(k,l) \triangleq \mathbb{E}[G_{i_k,j}^{(k)}G_{i_l,j}^{(l)}]$, for all $l \geq k$.
Due to

$$\mathbf{n}^{(l)} = \mathrm{vec}(\mathbf{N}^{(l)}) = \mathrm{vec}([\mathbf{N}^{(k)}, \mathbf{N}_{l-k}]) = \begin{bmatrix}\mathbf{n}^{(k)} \\ \mathbf{n}_{l-k}\end{bmatrix}$$

and

$$\mathbf{d}_{i_l,j}^{(l)} = \mathbf{y}_{i_l}^{(l)} - \mathbf{y}_j^{(l)} = \begin{bmatrix}\mathbf{y}_{i_l}^{(k)} - \mathbf{y}_j^{(k)} \\ \mathbf{y}_{i_l,l-k} - \mathbf{y}_{j,l-k}\end{bmatrix} = \begin{bmatrix}\mathbf{d}_{i_l,j}^{(k)} \\ \mathbf{d}_{i_l,j,l-k}\end{bmatrix}$$

Hence,

$$\begin{aligned}\mathbf{R}_{\mathbf{g}^{(n)}}(k,l) &= \mathbb{E}\left[\left((\mathbf{n}^{(k)})^H\mathbf{d}_{i_k,j}^{(k)} + (\mathbf{d}_{i_k,j}^{(k)})^H\mathbf{n}^{(k)}\right)\left((\mathbf{n}^{(l)})^H\mathbf{d}_{i_l,j}^{(l)} + (\mathbf{d}_{i_l,j}^{(l)})^H\mathbf{n}^{(l)}\right)\right] \\ &= 2\mathrm{Re}\left\{\mathbb{E}\left[(\mathbf{d}_{i_k,j}^{(k)})^H\mathbf{n}^{(k)}\left((\mathbf{d}_{i_l,j}^{(l)})^H\mathbf{n}^{(l)}\right)^*\right]\right\}\end{aligned}$$

where, using the circularly symmetric property of the i.i.d. complex Gaussian random matrix $\mathbf{N}$, Hence,

$$\begin{aligned}&\mathbb{E}\left[(\mathbf{d}_{i_k,j}^{(k)})^H\mathbf{n}^{(k)}\left((\mathbf{d}_{i_l,j}^{(l)})^H\mathbf{n}^{(l)}\right)^*\right] \\ &= \mathbb{E}\left[(\mathbf{d}_{i_k,j}^{(k)})^H\mathbf{n}^{(k)}\left((\mathbf{d}_{i_l,j}^{(k)})^H\mathbf{n}^{(k)} + (\mathbf{d}_{i_l,j,l-k})^H\mathbf{n}_{l-k}\right)^*\right] \\ &= \left[(\mathbf{d}_{i_k,j}^{(k)})^H\mathbf{n}^{(k)}\left((\mathbf{d}_{i_l,j}^{(k)})^H\mathbf{n}^{(k)}\right)^*\right] \\ &= (\mathbf{d}_{i_k,j}^{(k)})^H\mathbb{E}\left[\mathbf{n}^{(k)}(\mathbf{n}^{(k)})^H\right]\mathbf{d}_{i_l,j}^{(k)} \\ &= (\mathbf{d}_{i_k,j}^{(k)})^H\mathbf{d}_{i_l,j}^{(k)} = \left\langle \mathbf{Y}_{i_k,j}^{(k)}, \mathbf{Y}_{i_l,j}^{(k)} \right\rangle\end{aligned} \quad (11)$$

where, $\mathbf{n}^{(k)}$ and n_{l-k} are independent.

$$\mathbf{R}_{g^{(n)}}(k,l) = 2\mathrm{Re}\left\{\left\langle \mathbf{Y}^{(k)}_{i_k,j}, \mathbf{Y}^{(k)}_{i_l,j}\right\rangle\right\}, \quad \forall l > k$$
$$\mathbf{R}_{g^{(n)}}(k,k) = \| \mathbf{Y}^{(k)}_{i_k,j} \|^2$$
$$\mathbf{R}_{g^{(n)}}(k,l) = \mathbf{R}_{g^{(n)}}(l,k), \quad \forall l < k,$$

and have $\mathbf{g}^{(n)} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{g^{(n)}})$. The n-dimensional joint Gaussian Probability Density Function (PDF) of $\mathbf{g}^{(n)}$ is:

$$f_{g^{(n)}}(\mathbf{g}) = \frac{1}{(2\pi)^{\frac{n}{2}}\sqrt{\det(\mathbf{R}_{g^{(n)}})}}\exp\left(-\frac{1}{2}\mathbf{g}^T\mathbf{R}_{g^{(n)}}{}^{-1}\mathbf{g}\right)$$

Without loss of generality, for MIMO-HARQ with N, assuming zero mean, the n-dimensional Q function for a real Gaussian random vector $\mathbf{x} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_x)$.

$$\Delta_n(\mathbf{x}, \mathbf{R}_x) = \frac{1}{(2\pi)^{\frac{n}{2}}\sqrt{\det(\mathbf{R}_x)}}\int_{x(1)}^{\infty}\cdots\int_{x(n)}^{\infty}\exp\left(-\frac{1}{2}\mathbf{g}^T\mathbf{R}_x{}^{-1}\mathbf{g}\right)d\mathbf{g}$$

Hence, The union bound on $P_e^{(n)}$ in Eq. 10 is:

$$P_e^{(n)} \le \frac{1}{|\mathcal{M}|}\sum_{j=1}^{|\mathcal{M}|}\sum_{\substack{i_1=1\\ i_1\neq j}}^{|\mathcal{M}|}\cdots\sum_{\substack{i_n=1\\ i_n\neq j}}^{|\mathcal{M}|}\mathbb{E}_{\mathbf{H}}\left[\Delta_n\left((\mathbf{d}^{(n)})^2, \mathbf{R}_{g^{(n)}}\right)\right]$$

Where,

$$\mathbf{d}^{(n)} = (d^{(1)}_{i_1,j}, \cdots, d^{(n)}_{i_n,j})^T$$

RESULTS AND DISCUSSION

Alamouti-based HARQ protocol: Consider a Alamouti scheme with ARQ feedback for MIMO i.i.d. Gaussian slow fading channel with maximum ARQ rounds N = 2, which reveals the significance of ARQ feedback in different settings. Let $M_T = 2$, $M_R = 1$, N = 2, the following Alamouti-based HARQ protocol is studied.

The overall codeword of Alamouti code (Alamouti, 1998) after N = 2 transmissions is:

$$\mathbf{X} = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} = [\mathbf{x}_1, \mathbf{x}_2]$$

The first transmission is a spatial multiplexing, which is optimal. The traditional use of Alamouti code would transmit **X** in two channel uses. however, while using ARQ feedback, the first and second column can be separately transmitted. $\mathbf{x}_1$ is sent first to try exploiting the channel, the receiver gets $\mathbf{y}_1 = \mathbf{h}_1 s_1 + \mathbf{h}_2 s_2 + \mathbf{n}_1$ and jointly decodes ($\mathbf{s}_1$, $\mathbf{s}_2$) in a Maximum-likelihood manner. when the first decoding attempt fails, a NACK will be sent back to the transmitter asking for the transmission of the second column $\mathbf{x}_2$ With both columns transmitted, the receiver can perform the usual Alamouti decoding to recover $\mathbf{s}_1$ and $\mathbf{s}_2$ in the second decoding attempt.

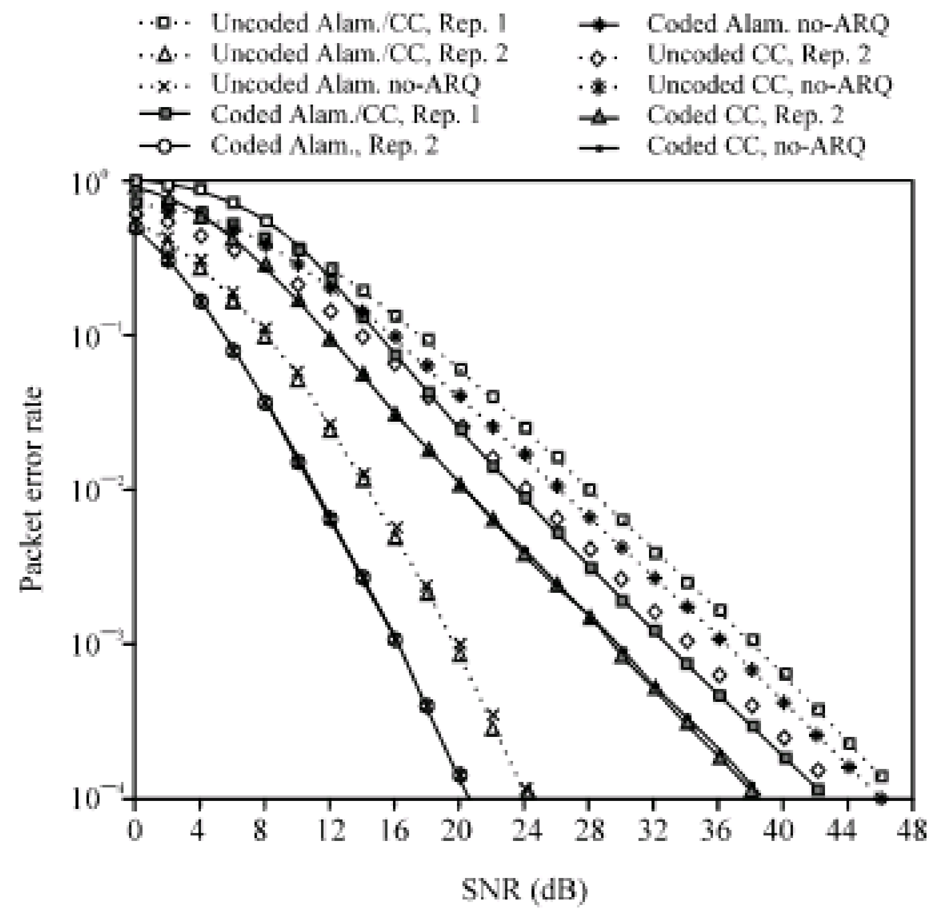

Fig. 2: Packet Error Rate (PER) vs. SNR for a 2×1 MISO Gaussian channel with ARQ round N = 2. Alamouti and CC are simulated with both uncoded and BICM coded

Simulation setting and results: Alamouti-based protocol and the traditional CC repetition scheme are simulated. Both uncoded and coded transmission are assembled with a Gray-coded QPSK constellation on each antenna. The ML decoding is performed for each ARQ rounds, which gives the optimal decoding performance. The encoder is a 1/2-rate binary convolutional code with generators $[133, 171]_8$. The encoder is followed by a block interleaver. The receiver detector performs ML soft-output with bit Log-Likelihood Ratio (LLR) computation concatenated with a bit deinterleaver and convolutional decoder implements a soft-input-soft-output Viterbi algorithm.

Figure 2 shows the performance comparison of Alamouti-based and CC HARQ protocols with ARQ round N = 2 and no-ARQ, with both uncoded and BICM coded transmissions in a 2×1 MISO Gaussian channel. The Packet Error Rate (PER) is a function of the average received SNR. The Alamouti-based protocol is predominant superior to the traditional CC ARQ approach in all settings. Clearly, there is a diversity loss for the CC protocol.

Analysis of gains of HARQ protocols: The gains of Alamouti-based HARQ are not only from packet error rate but also from the receiver soft-output ML detection complexity. Detector orthogonalize the two transmitted Alamouti symbols for independent bit LLR generation, whereas, the CC protocol is of higher complexity due to generated bit LLR is consists of an equivalent 2×2 caused by repetition in MIMO system.

The gains of HARQ protocols without ARQ feedback quite different with different system settings. For the uncoded case, The packet error rate gain of the two simulated HARQ protocols in no-ARQ transmission is distinct, For the coded case, these distinct is as much as negligible.

For example, the decoding error probability of at ARQ $N = 2$ rund is $P_{ARQ} = \mathbb{P}\{\overline{\mathcal{D}_1}, \overline{\mathcal{D}_2}\}$, comparing the case of no ARQ $P_{no\text{-}ARQ} = \mathbb{P}\{\overline{\mathcal{D}_2}\}$. The gain of HARQ is $P_{no\text{-}ARQ} - P_{ARQ} = \mathbb{P}\{\overline{\mathcal{D}_2}\} - \mathbb{P}\{\overline{\mathcal{D}_1}, \overline{\mathcal{D}_2}\} = \mathbb{P}\{\mathcal{D}_1, \overline{\mathcal{D}_2}\} \geq 0$. The event $\{\mathcal{D}_1, \overline{\mathcal{D}_2}\}$ implies that there is a correct decoding at the first ARQ round, but it makes an incorrect decoding as it performs decoding for these packets. this event is possible happened in the uncoded case, while the probability of noise realization at first round is very small, the decoding is correct, but if the probability is very large in the second round, it makes the decoding for these transmissions fail. Due to the channel matrix keeps fixed for all ARQ rounds, hence, the exclusive explain for happened fact is caused by the noise. However, for the channel coding case, the impact of noise will at last disappear as packet length increases. The noise will be completely averaged out and leads to $\mathcal{D}_{n-1} \subseteq \mathcal{D}_n$, have $\mathbb{P}\{\mathcal{D}_1, \overline{\mathcal{D}_2}\} = 0$.

CONCLUSIONS AND FUTURE WORK

Hybrid automatic repeat request (HARQ) is an important protocol used in packet transmission to provide reliable data communication. The MIMO systems are also well known to increase the spectral efficiency and the capacity of a communication system. In this study, a MIMO HARQ technique is proposed for slow fading MIMO Channel. The fundamental performance of Hybrid ARQ protocols are studied in a multiple-antenna channel. The new technique exploits both the space-time coding gain of Alamouti STC and the packet combining gain. It retransmits the HARQ packet using an orthogonal Alamouti STC and combined all the received packets. Alamouti-based HARQ design method based on the error probability analysis is also presented. Simulation shows that the Alamouti-based protocol is remarkably superior to the traditional CC ARQ. Note that the technique is valid only in a slow varying channel. Extension to MIMO channel with more than two transmit and receive antennas is under investigation. Note that for the Alamouti-based protocol, it is the assumption that the channel has to keep fixed over different ARQ rounds. For time-varying channels, protocols exploiting both time and spatial diversity should be considered and how to efficiently design the resulting protocol could be challenging problem. For the subjects of next work, there are several potential problems that have not been concerned herein. For MIMO ARQ schemes, the Alamouti STC is proved to be optimal performance, However, there are some other settings in existence unknown codes are close to the optimal performance. Numerical search of optimal codes is still a research topic. Incorporating the receiver lower decoding complexity into the STC HARQ design is under consideration. The Alamouti-based protocol profits from not only wonderful error performance, but also the receiver soft-output ML detection complexity at each ARQ rounds. It is ideal target that the STCs to not only satisfy the Alamouti-based HARQ design method presented herein, but also has the fast-decodable property for decoding in each ARQ round.

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