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Precoding for Non-Coordinative Multi-Cell Multi-Antenna Networks

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Abstract: In multi-cell multi-antennas networks with uplink training, the channel estimate at the base station in one cell jammed by users from other cells, the performance of systems suffers tremendous losses due to the use of corrupted pilots. In addition, as the MMSE precoding matrix exists singularity problem when coordinative method is used, The goal is to investigate precoding techniques for downlink transmission to mitigate this corruption and thus increase achievable rates. A multi-cell MMSE precoding depending on the pilots assigned to the users is proposed, which does not need coordination. This precoding is the optimal solution of an optimization problem, which consists of the mean-square error of signals and the mean-square interference. Simulation results show that this precoding method can effectively reduce intra/inter-cell interference.

Key words: Multi-cell MIMO, orthogonality, optimization, precoding matrix

INTRODUCTION

In multi-cell multi-antennas wireless networks with uplink training, uplink pilot symbols is a crucial problem, which results in channel estimate and consequently determines system performance. When the channel estimate at the base station in one cell is jammed by users from other cells, orthogonality of pilots will be lost, the use of non-orthogonal pilots causes corruption, Hence, the corrupted training must be adequately mitigated and utilized. The goal of this study is to exploit techniques to mitigate this corruption and thus increase achievable rates.

In the multi-cell scenario, when Channel State Information (CSI) at the base stations is available, the most of reports focus on the gain obtained via coordination of the base stations (Zhang and Dai, 2004; Venkatesan *et al*., 2007; Jing *et al*., 2008). Zhang and Dai (2004) presented jointly both beamforming and pre-coding approaches. The sum capacity of the multiantenna Gaussian Broadcast Channel (BC) based upon Dirty Paper Coding (DPC) was reported by Weingarten *et al*. (2006). The capacity of DPC was compared to that of TDMA MIMO Gaussian BC by Jindal and Goldsmith (2005). For the non-fading scenario with random phases, The high SNR performance gap between the DPC and Zero-Forcing (ZF) precoders was characterized in Jing *et al.* (2008), which indicates a singularity problem in certain network settings and demonstrate that the MMSE precoder does not completely resolve the singularity problem.

For the problem of deficiency of channel CSI, Time-Division Duplex (TDD) systems operation and linear ZF precoder were considered and a lower bound on sum capacity was given by Marzetta (2006), results show that it is always beneficial to increase the number of antennas at the base station.

Motivated by the singularity problem of the MMSE precoding matrix while using coordination as claimed by Jing *et al.* (2008), a multi-cell MMSE precoding depending on the pilots assigned to the users is proposed in this study, which does not need coordination between base stations required by the jointly precoding techniques.

SYSTEM MODEL

In a cellular system with L cells, $l \in \{1, 2, ..., L\}$, each cell provided with one base station with M antennas and K single-antenna users, $K \le M$. The signal vector received at the m-th antenna of the base station of the l-th cell is:

$$\mathbf{y}_{lm} = \sqrt{p_r}\sum_{j=1}^{L}\sum_{k=1}^{K}(\sqrt{\rho_{jlk}}\,h_{jlkm})(\sqrt{\xi}\omega_{jk}) + \mathbf{n}_{lm} \quad (1)$$

where, p_r is the average power at each user. ξ is the length of pilot symbols transmitted by all users of all cells in every coherence interval, the pilot vector $\sqrt{\xi}\omega_{jk}$

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transmitted by the k-th user in the j-th cell satisfying $\omega_{jk}^H \omega_{jk} = 1$ orthogonality condition is optimal (Dong and Tong, 2002). $\sqrt{\rho_{jlk}}\, h_{jlkm}$ denote the propagation factor between the m-th base station antenna of the l-th cell and the k-th user of the j-th cell, here, $\{\rho_{jlk}\}$ are given nonnegative constants and $\{h_{jlkm}\}$ are unknown i.i.d. zero-mean, circularly-symmetric complex Gaussian $\mathcal{CN}(0,1)$ random variables. In every OFDM sub-band, channel reciprocity for the forward and reverse links, i.e., the propagation factor is same for both forward and reverse links and block fading, i.e., $\{h_{jlkm}\}$ remains unchanged for a duration of T symbols. The additive noises $\mathbf{n}_{lm}$ at all terminals are i.i.d. $\mathcal{CN}(0,1)$ random variables.

Let $\mathbf{Y}_l = [y_{l1},\ldots,y_{lM}]$, $\Lambda_{jl} = \mathrm{diag}\{[\rho_{jl1},\ldots,\rho_{jlK}]\}$, $\Omega_j = [\omega_{j1},\ldots,\omega_{jK}]$, $\mathbf{N}_l = [\mathbf{n}_{l1},\ldots,\mathbf{n}_{lM}]$ and $\mathbf{H}_{jl} = \begin{bmatrix} h_{jl11} & \cdots & h_{jl1M} \\ \vdots & \ddots & \vdots \\ h_{jlK1} & \cdots & h_{jlKM} \end{bmatrix}$.

The received signal in Eq. 1 can be expressed as:

$$\mathbf{Y}_l = \sqrt{p_r \xi} \sum_{j=1}^{L} \Omega_j \Lambda_{jl}^{\frac{1}{2}} \mathbf{H}_{jl} + \mathbf{N}_l \tag{2}$$

The MMSE estimate $\hat{\mathbf{H}}_{jl}$ is:

$$\hat{\mathbf{H}}_{jl} = \sqrt{p_r \xi} \Lambda_{jl}^{\frac{1}{2}} \Omega_j^H \left(\mathbf{I} + p_r \xi \sum_{i=1}^{L} \Omega_i \Lambda_{il} \Omega_i^H \right)^{-1} \mathbf{Y}_l \tag{3}$$

Let the base station of the l-th cell transmit symbols $\mathbf{s}_l = [s_{l1}, s_{l2}, \cdots, s_{lK}]^T$ to users and the linear precoding matrix $\mathbf{B}_l = f(\hat{\mathbf{H}}_l) \in \mathbb{C}^{M\times K}$, where $\hat{\mathbf{H}}_l = [\hat{\mathbf{H}}_{1l}, \hat{\mathbf{H}}_{2l}, \cdots, \hat{\mathbf{H}}_{Ll}]$ denote the MMSE estimate of the channel between this base station and all users. The function f(•) corresponds to the specific precoding method operated at the base station. Assume that the average power constraint at the base station satisfy conditions $\mathbb{E}[\mathbf{s}_l] = \mathbf{0}$, $\mathbb{E}[\mathbf{s}_l \mathbf{s}_l^H] = \mathbf{I}$ and $\mathrm{tr}\{\mathbf{B}_l^H \mathbf{B}_l\} = 1$.

The signal received by the users in the j-th cell is:

$$\mathbf{x}_j = \sqrt{p_s} \sum_{l=1}^{L} \Lambda_{jl}^{\frac{1}{2}} \mathbf{H}_{jl} (\mathbf{B}_l \mathbf{s}_l) + \mathbf{z}_j \tag{4}$$

where, p_s is the average power at the base station. $\mathbf{B}_l\mathbf{s}_l$ is the signal vector transmitted by this base station. $\mathbf{z}_j$ is the additive noise. The signal received by the k-th user can be expressed as:

$$x_{jk} = \sqrt{p_s \sum_{l=1}^{L} \sum_{i=1}^{K} \rho_{jlk}} \, [h_{jlk1}, h_{jlk2}, \cdots, h_{jlkM}] \mathbf{b}_{li} s_{li} + z_{jk} \tag{5}$$

where, $\mathbf{b}_{li}$ is the i-th column of the precoding matrix $\mathbf{B}_l$ and z_{jk} is the k-th element of $\mathbf{z}_j$ The signal received by the k-th user in the j-th cell can be rewritten as:

$$\begin{aligned} x_{jk} &= \sum_{l=1}^{M} \sum_{i=1}^{K} g_{li}^{jk} s_{li} + z_{jk} \\ &= \mathbb{E}[g_{jk}^{jk}] s_{jk} + \underbrace{(g_{jk}^{jk} - \mathbb{E}[g_{jk}^{jk}]) s_{jk} + \sum_{l\neq j} \sum_{i\neq k} g_{li}^{jk} s_{li} + z_{jk}}_{\text{Effective noise } z_{jk}^{+}} \end{aligned} \tag{6}$$

where, $g_{li}^{jk} = \sqrt{p_s \rho_{jlk}} [h_{jlk1}, \cdots, h_{jlkM}] \mathbf{b}_{li}$, z_{jk}^{+} and s_{jk} are uncorrelated, $\mathbb{E}[|z_{jk}^{+}|^2] = 1 + \mathrm{var}\{|g_{jk}^{jk}|\} + \sum_{l\neq j} \sum_{i\neq k} \mathbb{E}[|g_{li}^{jk}|^2]$, the achievable rate of the k-th user in the j-th cell is:

$$R_{jk} = C\left(\frac{|\mathbb{E}[g_{jk}^{jk}]|^2}{1 + \mathrm{var}\{|g_{jk}^{jk}|\} + \sum_{l\neq j} \sum_{i\neq k} \mathbb{E}[|g_{li}^{jk}|^2]} \right) \tag{7}$$

where, $C(x) = \log_2(1+x)$, x is a variable.

ACHIEVABLE RATE

To make problem simple and manifest the primary effect of pilot corruption which is correlative between the precoding matrix at the base station in a cell and users in other cells. Setting that one user per cell, all users use the same pilot $\omega_{j1} = \omega$. For ZF precoding, the precoding vector used at the base station in the l-th cell is:

$$\mathbf{b}_l = \frac{\hat{\mathbf{h}}_{ll}^H}{\sqrt{\hat{\mathbf{h}}_{ll} \hat{\mathbf{h}}_{ll}^H}}$$

From Eq. 4, the received signal by the user in the j-th cell is:

$$x_{jk} = \underbrace{\sqrt{p_s \rho_{jj}} \mathbf{h}_{jj} \mathbf{b}_j}_{\text{Effective channel gain}} s_j + \sum_{l\neq j} \underbrace{\sqrt{p_s \rho_{jl}} \mathbf{h}_{jl} \mathbf{b}_l s_l}_{\text{Signal and interference}} + z_j \tag{8}$$

Using matrix inversion lemma (Zhang, 2004). Simplify the MMSE estimate $\mathbf{h}_{jl}$ in Eq. 3.

$$\begin{aligned} \hat{\mathbf{h}}_{jl} &= \sqrt{p_r \xi \rho_{jl}} \omega^H \left(\mathbf{I} + \omega \left(p_r \xi \sum\nolimits_{i=1}^{L} \rho_{il} \right) \omega^H \right)^{-1} \mathbf{Y}_l, \\ &= \frac{\sqrt{p_r \xi \rho_{jl}}}{1 + p_r \xi \sum\nolimits_{i=1}^{L} \rho_{il}} \omega^H \mathbf{Y}_l \end{aligned}$$

and obtain

$$\frac{\hat{\mathbf{h}}_{jl}^H}{\| \hat{\mathbf{h}}_{jl} \|_2} = \frac{\mathbf{Y}_l^H \omega}{\| \omega^H \mathbf{Y}_l \|_2} \tag{9}$$

Let $\mathbf{h}_{jl} = \hat{\mathbf{h}}_{jl} + \tilde{\mathbf{h}}_{jl}$, obtain:

$$\mathbf{h}_{jl} \mathbf{b}_l = \mathbf{h}_{jl} \frac{\hat{\mathbf{h}}_{jl}^H}{\| \hat{\mathbf{h}}_{jl}^H \|_2} = \| \hat{\mathbf{h}}_{jl}^H \|_2 + \tilde{\mathbf{h}}_{jl} \frac{\hat{\mathbf{h}}_{jl}^H}{\| \hat{\mathbf{h}}_{jl}^H \|_2} \tag{10}$$

where, $\hat{\mathbf{h}}_{jl}$ is independent of $\hat{\mathbf{h}}_{jl}$

$$\hat{\mathbf{h}}_{jl} \sim \mathcal{CN}\left(\mathbf{0}, \frac{p_r\xi\rho_{jl}}{1+p_r\xi\sum_{i=1}^{L}\rho_{il}}\mathbf{I}\right)$$

and

$$\tilde{\mathbf{h}}_{jl} \sim \mathcal{CN}\left(\mathbf{0}, \frac{1+p_r\xi\sum_{i\neq j}\rho_{il}}{1+p_r\xi\sum_{i=1}^{L}\rho_{il}}\mathbf{I}\right)$$

The first and second order moments of $\{\mathbf{h}_{jl}\mathbf{b}_l\}$ are respectively:

$$\mathbb{E}[\mathbf{h}_{jl}\mathbf{b}_l] = \mathbb{E}[\|\hat{\mathbf{h}}_{jl}^H\|_2] = \sqrt{\frac{p_r\xi\rho_{jl}}{1+p_r\xi\sum_{i=1}^{L}\rho_{il}}}\mathbb{E}[\Theta] \tag{11}$$

where, $\Theta = \sqrt{\sum_{m=1}^{M}|x_m|^2}$, i.i.d. $\{x_m\} \sim \mathcal{CN}(0,1)$.

$$\begin{aligned}\mathbb{E}\left[\|\mathbf{h}_{jl}\mathbf{b}_l\|_2^2\right] &= \mathbb{E}\left[\|\hat{\mathbf{h}}_{jl}^H\|_2^2\right] + \mathbb{E}\left[\frac{\hat{\mathbf{h}}_{jl}}{\|\hat{\mathbf{h}}_{jl}^H\|_2}\tilde{\mathbf{h}}_{jl}^H\tilde{\mathbf{h}}_{jl}\frac{\hat{\mathbf{h}}_{jl}^H}{\|\hat{\mathbf{h}}_{jl}^H\|_2}\right] \\ &= \frac{p_r\xi\rho_{jl}}{1+p_r\xi\sum_{i=1}^{L}\rho_{il}}\mathbb{E}[\Theta^2] + \frac{1+p_r\xi\sum_{i\neq j}\rho_{il}}{1+p_r\xi\sum_{i=1}^{L}\rho_{il}}\end{aligned} \tag{12}$$

The expectation and the variance of effective channel gain in Eq. 8 are respectively:

$$\mathbb{E}[\sqrt{p_s\rho_{jj}}\mathbf{h}_{jj}\mathbf{b}_j] = \left(p_s\rho_{jj}\frac{p_r\xi\rho_{jj}}{1+p_r\xi\sum_{i=1}^{L}\rho_{ij}}\right)^{1/2}\mathbb{E}[\Theta] \tag{13}$$

$$\mathrm{var}\{\|\sqrt{p_s\rho_{jj}}\mathbf{h}_{jj}\mathbf{b}_j\|_2\} = p_s\rho_{jj}\left(\frac{p_r\xi\rho_{jj}\mathrm{var}\{\Theta\}}{1+p_r\xi\sum_{i=1}^{L}\rho_{ij}} + \frac{1+p_r\xi\sum_{i\neq j}\rho_{ij}}{1+p_r\xi\sum_{i=1}^{L}\rho_{ij}}\right) \tag{14}$$

The first and second order moments of the signal and interference terms in Eq. 8 are respectively:

$$\mathbb{E}[\sqrt{p_s\rho_{jl}}\mathbf{h}_{jl}\mathbf{b}_l s_l] = 0 \tag{15}$$

$$\mathbb{E}[\|\sqrt{p_s\rho_{jl}}\mathbf{h}_{jl}\mathbf{b}_l s_l\|_2^2] = p_s\rho_{jl}\left(\frac{p_r\xi\rho_{jl}\mathbb{E}[\Theta^2]}{1+p_r\xi\sum_{i=1}^{L}\rho_{il}} + \frac{1+p_r\xi\sum_{i\neq j}\rho_{il}}{1+p_r\xi\sum_{i=1}^{L}\rho_{il}}\right) \tag{16}$$

By substituting Eq. 13-16 for Eq. 7. Θ has a scaled χ^2 distribution with 2M degrees of freedom, scalar factor is $1/\sqrt{2}$. Eq. 7 can be simplified as:

$$R_j = C\left(\frac{\dfrac{p_s p_r\xi\rho_{jj}^2}{1+p_r\xi\sum_{i=1}^{L}\rho_{ij}}\mathbb{E}^2[\Theta]}{1+\dfrac{p_s p_r\xi\rho_{jj}^2\mathrm{var}\{\Theta\}}{1+p_r\xi\sum_{i=1}^{L}\rho_{ij}} + \sum_{l\neq j}\dfrac{p_s p_r\xi\rho_{jl}^2\mathbb{E}[\Theta^2]}{1+p_r\xi\sum_{i=1}^{L}\rho_{il}} + \sum_{l=1}^{L}p_s\rho_{jl}\dfrac{1+p_r\xi\sum_{i\neq j}^{L}\rho_{il}}{1+p_r\xi\sum_{i=1}^{L}\rho_{il}}}\right) \tag{17}$$

where, $\mathbb{E}[\Theta] = \frac{\Gamma(M+\frac{1}{2})}{\Gamma(M)}$, here, $\Gamma(\bullet)$ is the Gamma function, $\mathbb{E}[\Theta^2] = M$ and $\mathrm{var}\{\Theta\} = M - \mathbb{E}^2[\Theta]$.

UPPER BOUND

Assuming that all channel matrices $\hat{\mathbf{F}}_{jl}$ are available to all base stations, the effective channel matrices $\mathbf{F}_{jl}\mathbf{B}_l$ are known to all users and all base stations transmits data only to the users situated in their cells. Using precoding matrices $\mathbf{B}_1$ and $\mathbf{B}_2$, the total sum rate is:

$$\begin{aligned}&R(\mathbf{B}_1, \mathbf{B}_2, \mathbf{F}_{11}, \mathbf{F}_{12}, \mathbf{F}_{21}, \mathbf{F}_{22}) \\ &= \sum_{j=1}^{K}C\left(\frac{|\mathbf{f}_{11j}^H\mathbf{b}_{1j}|^2}{\sigma^2\mathrm{tr}(\mathbf{B}_1^H\mathbf{B}_1) + \sum_{r\neq j}|\mathbf{f}_{11j}^H\mathbf{b}_{1r}|^2 + \frac{\mathrm{tr}(\mathbf{B}_1^H\mathbf{B}_1)}{\mathrm{tr}(\mathbf{B}_2^H\mathbf{B}_2)}\sum_r|\mathbf{f}_{21j}^H\mathbf{b}_{2r}|^2}\right) \\ &+ \sum_{j=1}^{K}C\left(\frac{\frac{\mathrm{tr}(\mathbf{B}_1^H\mathbf{B}_1)}{\mathrm{tr}(\mathbf{B}_2^H\mathbf{B}_2)}|\mathbf{f}_{22j}^H\mathbf{b}_{2j}|^2}{\sigma^2\mathrm{tr}(\mathbf{B}_1^H\mathbf{B}_1) + \frac{\mathrm{tr}(\mathbf{B}_1^H\mathbf{B}_1)}{\mathrm{tr}(\mathbf{B}_2^H\mathbf{B}_2)}\sum_{r\neq j}|\mathbf{f}_{22j}^H\mathbf{b}_{2r}|^2 + \sum_r|\mathbf{f}_{12j}^H\mathbf{b}_{1r}|^2}\right)\end{aligned}$$

where, $\mathbf{f}_{itj}^H$, (i =1, 2) is the j-th row of $\mathbf{F}_{it}$ and $\mathbf{b}_{ir}$ is the r-th column of $\mathbf{B}_i$.

Let $\mathbf{F}_{jl} = \sqrt{p_s}\Lambda_{jl}^{\frac{1}{2}}\mathbf{H}_{jl}$, $\hat{\mathbf{F}}_{jl} = \sqrt{p_s}\Lambda_{jl}^{\frac{1}{2}}\hat{\mathbf{H}}_{jl}$ and $\tilde{\mathbf{F}}_{jl} = \mathbf{F}_{jl} - \hat{\mathbf{F}}_{jl}$ for all j and *l*. Since, $\hat{\mathbf{H}}_{jl}$ are the MMSE estimates, Elements of $\tilde{\mathbf{F}}_{jl}$ are i.i.d. complex Gaussian random variables with known variances. Let $\tilde{\mathbf{F}}_{jl}^{(i)}$, (j, l = 1,2; i = 1,…,L) be random matrices following the corresponding distributions. Let $\mathbf{F}_{jl}^{(i)} = \hat{\mathbf{F}}_{jl} + \tilde{\mathbf{F}}_{jl}^{(i)}$,

$$\begin{aligned}&\mathbb{E}_{\tilde{\mathbf{F}}_{jl}, z_i, s_i}[R(\mathbf{B}_1, \mathbf{B}_2, \mathbf{F}_{11}, \mathbf{F}_{12}, \mathbf{F}_{21}, \mathbf{F}_{22})] \\ &= \lim_{L\to\infty}\frac{1}{L}\sum_{i=1}^{L}R(\mathbf{B}_1, \mathbf{B}_2, \mathbf{F}_{11}^{(i)}, \mathbf{F}_{12}^{(i)}, \mathbf{F}_{21}^{(i)}, \mathbf{F}_{22}^{(i)})\end{aligned} \tag{18}$$

For a finite L, after taking partial derivatives of Eq. 18 with respect to elements of $\mathbf{B}_1$ and $\mathbf{B}_2$ we can obtain the elements of $\mathbf{B}_1$ and $\mathbf{B}_2$ then reuse these in Eq. 18 to obtain an approximation in terms of the average sum rate.

OPTIMIZATION PROBLEM

Assuming that pilots in every cell is orthogonal and zero-forcing is performed on the users in every cell. The precoding matrix corresponding to this zero-forcing approach is:

$$\mathbf{B}_l = \frac{\hat{\Sigma}_{ll}^H\left(\hat{\Sigma}_{ll}\hat{\Sigma}_{ll}^H\right)^{-1}}{\sqrt{\mathrm{tr}\left[\left(\hat{\Sigma}_{ll}\hat{\Sigma}_{ll}^H\right)^{-1}\right]}} \tag{19}$$

where, $\hat{\Sigma}_{ll} = \sqrt{p_s}\Lambda_{ll}^{\frac{1}{2}}\hat{\mathbf{H}}_{ll}$.

In the j-th cell, the signal received by the users in this cell given by Eq. 4 is a function of all the precoding matrices. Consider the signal and interference terms corresponding to the base station in the *l*-th cell. Based on these terms, construct the following optimization problem to obtain the precoding matrix $\mathbf{B}_l$ Using the notation: $\mathbf{F}_{jl} = \sqrt{p_s}\Lambda_{jl}^{\frac{1}{2}}\mathbf{H}_{jl}$, $\hat{\mathbf{F}}_{jl} = \sqrt{p_s}\Lambda_{jl}^{\frac{1}{2}}\hat{\mathbf{H}}_{jl}$ and $\tilde{\mathbf{F}}_{jl} = \mathbf{F}_{jl} - \hat{\mathbf{F}}_{jl}$ for all j and *l*. The optimization problem is:

$$\underset{\mathbf{B}_l,\alpha}{\text{Minimize}} \quad \mathbb{E}_{\tilde{\mathbf{F}}_{jl},\mathbf{z}_l,\mathbf{s}_l}\left[\underbrace{\left\|\alpha(\mathbf{F}_{ll}\mathbf{B}_l\mathbf{s}_l + \mathbf{z}_l) - \mathbf{s}_l\right\|_2^2}_{\text{Sum of squares of errors}} + \underbrace{\sum_{j\neq l}\left\|\alpha\beta(\mathbf{F}_{jl}\mathbf{B}_l\mathbf{s}_l)\right\|_2^2}_{\text{Sum of squares of interference}}\right] \quad (20)$$

$$\text{Subject to} \quad \text{tr}\{\mathbf{B}_l^H\mathbf{B}_l\} = 1$$

where, β is the relative weights control parameter of the optimization problem. The optimal solution $\mathbf{B}_l^{opt}$ in Eq. 20 is the multi-cell MMSE precoding matrix.

OPTIMAL SOLUTION

Let $\tilde{\mathbf{f}}_{jlm}$ is the m-th column of $\tilde{\mathbf{F}}_{jl}$. h_{jlm} and $\hat{\mathbf{h}}_{jlm}$ as well as, similarly. From Eq. 3, obtain,

$$\begin{aligned}\tilde{\mathbf{f}}_{jlm} &= \sqrt{p_s}\Lambda_{jl}^{\frac{1}{2}}(\mathbf{h}_{jlm} - \hat{\mathbf{h}}_{jlm}) \\ &= \sqrt{p_s}\Lambda_{jl}^{\frac{1}{2}}\left(\mathbf{h}_{jlm} - \sqrt{p_r\xi}\Lambda_{jl}^{\frac{1}{2}}\Omega_j^H\left(\mathbf{I} + p_r\xi\sum_{i=1}^{L}\Omega_i\Lambda_{il}\Omega_i^H\right)^{-1}\mathbf{y}_{lm}\right)\end{aligned}$$

For given j and *l*, $\{\tilde{\mathbf{f}}_{jlm}\}_{m=1}^M$ follows i.i.d. zero-mean $\mathcal{CN}$ distribution. Therefore, $\mathbb{E}[\tilde{\mathbf{F}}_{jl}^H\tilde{\mathbf{F}}_{jl}] = \delta_{jl}\mathbf{I}_M$, where,

$$\begin{aligned}\delta_{jl} &= \mathbb{E}[\tilde{\mathbf{f}}_{jlm}^H\tilde{\mathbf{f}}_{jlm}] \\ &= p_s\text{tr}\left\{\Lambda_{jl}^{\frac{1}{2}}\left(\mathbf{I}_K - \mathbb{E}[\hat{\mathbf{h}}_{jlm}\hat{\mathbf{h}}_{jlm}^H]\right)\Lambda_{jl}^{\frac{1}{2}}\right\} \\ &= p_s\text{tr}\left\{\Lambda_{jl}^{\frac{1}{2}}\left(\mathbf{I}_K - p_r\xi\Lambda_{jl}^{\frac{1}{2}}\Omega_j^H\left(\mathbf{I} + p_r\xi\sum_{i=1}^{L}\Omega_i\Lambda_{il}\Omega_i^H\right)^{-1}\Omega_j\Lambda_{jl}^{\frac{1}{2}}\right)\Lambda_{jl}^{\frac{1}{2}}\right\} \\ &= p_s\text{tr}\left\{\Lambda_{jl}^{\frac{1}{2}}\left(\mathbf{I}_K + p_r\xi\Lambda_{jl}^{\frac{1}{2}}\Omega_j^H\left(\mathbf{I} + p_r\xi\sum_{i\neq j}\Omega_i\Lambda_{il}\Omega_i^H\right)^{-1}\Omega_j\Lambda_{jl}^{\frac{1}{2}}\right)^{-1}\Lambda_{jl}^{\frac{1}{2}}\right\}\end{aligned}$$

The last step results from matrix inversion lemma (Zhang, 2004). Let

$$\Lambda_{jl} = \left(\mathbf{I} + p_r\xi\sum\nolimits_{i\neq j}\Omega_i\Lambda_{il}\Omega_i^H\right)^{-1}$$

obtain

$$\delta_{jl} = p_s\text{tr}\left\{\Lambda_{jl}\left(\mathbf{I}_K + p_r\xi\Lambda_{jl}^{\frac{1}{2}}\Omega_j^H\Lambda_{jl}\Omega_j\Lambda_{jl}^{\frac{1}{2}}\right)^{-1}\right\} \qquad (21)$$

Simplify the objective function of the problem in Eq. 20:

$$\begin{aligned}J(\mathbf{B}_l,\alpha) &= \mathbb{E}\left[\left\|\alpha(\mathbf{F}_{ll}\mathbf{B}_l\mathbf{s}_l + \mathbf{z}_l) - \mathbf{s}_l\right\|_2^2 + \sum_{j\neq l}\left\|\alpha\beta\mathbf{F}_{jl}\mathbf{B}_l\mathbf{s}_l\right\|_2^2\right] \\ &= \mathbb{E}\left[\left\|(\alpha\mathbf{F}_{ll}\mathbf{B}_l - \mathbf{I}_K)\mathbf{s}_l\right\|_2^2 + \sum_{j\neq l}\left\|\alpha\beta\mathbf{F}_{jl}\mathbf{B}_l\mathbf{s}_l\right\|_2^2\right] + \alpha^2K \\ &= \text{tr}\left\{\mathbb{E}\left[(\alpha\mathbf{F}_{ll}\mathbf{B}_l - \mathbf{I}_K)^H(\alpha\mathbf{F}_{ll}\mathbf{B}_l - \mathbf{I}_K) + \sum_{j\neq l}\alpha^2\beta^2\mathbf{B}_l^H\mathbf{F}_{jl}^H\mathbf{F}_{jl}\mathbf{B}_l\right]\right\} + \alpha^2K \\ &= \text{tr}\left\{\alpha^2\mathbf{B}_l^H\mathbb{E}\left[\mathbf{F}_{ll}^H\mathbf{F}_{ll}\right]\mathbf{B}_l + \sum_{j\neq l}\alpha^2\beta^2\mathbf{B}_l^H\mathbb{E}\left[\mathbf{F}_{jl}^H\mathbf{F}_{jl}\right]\mathbf{B}_l - \alpha\mathbf{B}_l^H\hat{\mathbf{F}}_{ll}^H - \alpha\hat{\mathbf{F}}_{ll}\mathbf{B}_l\right\} \\ &\quad + (\alpha^2+1)K \\ &= \text{tr}\left\{\alpha^2\mathbf{B}_l^H\left(\hat{\mathbf{F}}_{ll}^H\hat{\mathbf{F}}_{ll} + \beta^2\sum_{j\neq l}\hat{\mathbf{F}}_{jl}^H\hat{\mathbf{F}}_{jl} + \left(\delta_{ll} + \beta^2\sum_{j\neq l}\delta_{jl}\right)\mathbf{I}_M\right)\mathbf{B}_l - \alpha\mathbf{B}_l^H\hat{\mathbf{F}}_{ll}^H - \alpha\hat{\mathbf{F}}_{ll}\mathbf{B}_l\right\} \\ &\quad + (\alpha^2+1)K\end{aligned}$$

The optimization problem in Eq. 20 can be expressed as the Lagrangian formulation $L(\mathbf{B}_l, \alpha, \lambda) = J(B_l, \alpha)+\lambda(\text{tr}\{\mathbf{B}_l^H\mathbf{B}_l\} - 1)$.
Let,

$$\mathbf{R} = \hat{\mathbf{F}}_{ll}^H\hat{\mathbf{F}}_{ll} + \beta^2\sum\nolimits_{j\neq l}\hat{\mathbf{F}}_{jl}^H\hat{\mathbf{F}}_{jl} + \left(\delta_{ll} + \beta^2\sum\nolimits_{j\neq l}\delta_{jl} + \frac{\lambda}{\alpha^2}\right)\mathbf{I}_M$$

obtain

$$L(\mathbf{B}_l,\alpha,\lambda) = \|\alpha\mathbf{R}^{\frac{1}{2}}\mathbf{B}_l - \mathbf{R}^{-\frac{1}{2}}\hat{\mathbf{F}}_{ll}^H\|_2^2 - \text{tr}\left\{\hat{\mathbf{F}}_{ll}\mathbf{R}^{-1}\hat{\mathbf{F}}_{ll}^H\right\} + (\alpha^2+1)K - \lambda \qquad (22)$$

Let $\alpha\mathbf{R}^{\frac{1}{2}}\mathbf{B}_l = \mathbf{R}^{-\frac{1}{2}}\hat{\mathbf{F}}_{ll}^H$, $L(\mathbf{B}_l,\alpha,\lambda)$ is minimized, obtain $\mathbf{B}_l^{opt} = \alpha^{-1}\mathbf{R}^{-1}\hat{\mathbf{F}}_{ll}^H$. Let $L(\alpha,\lambda) = L(\mathbf{B}_l^{opt},\alpha,\lambda)$ and obtain:

$$L(\alpha,\lambda) = -\text{tr}\left\{\hat{\mathbf{F}}_{ll}\mathbf{R}^{-1}\hat{\mathbf{F}}_{ll}^H\right\} + (\alpha^2+1)K - \lambda \qquad (23)$$

Factorizing $\hat{\mathbf{F}}_{ll}^H\hat{\mathbf{F}}_{ll} + \beta^2\sum_{j\neq l}\hat{\mathbf{F}}_{jl}^H\hat{\mathbf{F}}_{jl}$ into the form $\mathbf{G}^H\text{diag}(d_1, d_2,...d_M)$, where, $\mathbf{G}^H\mathbf{G} = \mathbf{I}_M$. Let $\delta = \delta_{ll} + \beta^2\sum_{j\neq l}\delta_{jl}$,

$$\begin{aligned}\mathbf{R}^{-1} &= \left(\mathbf{G}^H\text{diag}\{[d_1,d_2,\cdots,d_M]\}\mathbf{G} + \left(\delta + \frac{\lambda}{\alpha^2}\right)\mathbf{I}_M\right)^{-1} \\ &= \left(\mathbf{G}^H\text{diag}\left\{\left[d_1 + \delta + \frac{\lambda}{\alpha^2}, d_2 + \delta + \frac{\lambda}{\alpha^2}, \cdots, d_M + \delta + \frac{\lambda}{\alpha^2}\right]\right\}\mathbf{G}\right)^{-1} \\ &= \mathbf{G}^H\text{diag}\left\{\left[\left(d_1 + \delta + \frac{\lambda}{\alpha^2}\right)^{-1}, \left(d_2 + \delta + \frac{\lambda}{\alpha^2}\right)^{-1}, \cdots, \left(d_M + \delta + \frac{\lambda}{\alpha^2}\right)^{-1}\right]\right\}\mathbf{G}\end{aligned} \qquad (24)$$

After substituting Eq. 24 in Eq. 23,

$$L(\alpha,\lambda) = -\sum_{m=1}^{M}\frac{f_m}{d_m + \delta + \dfrac{\lambda}{\alpha^2}} + (\alpha^2+1)K - \lambda \qquad (25)$$

where, f_m is the m-th diagonal element of $\mathbf{G}\hat{\mathbf{F}}_{ll}^H\hat{\mathbf{F}}_{ll}\mathbf{G}^H$. After differentiating Eq. 25 with respect to α and β and let resulting equations to 0, obtain $\lambda/\alpha^2 = K$ and

$$\mathbf{B}_l^{opt} = (\alpha^{opt})^{-1}\left(\hat{\mathbf{F}}_{ll}^H\hat{\mathbf{F}}_{ll} + \beta^2\sum\nolimits_{j\neq l}\hat{\mathbf{F}}_{jl}^H\hat{\mathbf{F}}_{jl} + \left(\delta_{ll} + \beta^2\sum\nolimits_{j\neq l}\delta_{jl} + K\right)\mathbf{I}_M\right)^{-1}\hat{\mathbf{F}}_{ll}^H$$

where, α^{opt} is such one that satisfying $\|\mathbf{B}_l^{opt}\|_2^2 = 1$.

Let $\zeta = \delta_{ll} + \beta^2 \sum_{j\neq l} \delta_{jl} + K$, get,

$$\mathbf{B}_l^{opt} = (\alpha^{opt})^{-1} \left(\hat{\mathbf{F}}_{ll}^H \hat{\mathbf{F}}_{ll} + \beta^2 \sum_{j\neq l} \hat{\mathbf{F}}_{jl}^H \hat{\mathbf{F}}_{jl} + \zeta \mathbf{I}_M \right)^{-1} \hat{\mathbf{F}}_{ll}^H$$

RESULTS AND DISCUSSION

Using sum rate as the performance metric and chiefly focusing on behavior of the inter-cell interference except for scheduling, power control and other techniques.

No. 1 setting for 2 cells scenario: Let K users in every cell and pilot length of $\xi = K$, the same orthogonal pilots are used in the two cells. Let the propagation factors ρ_{jlk} for all k is:

$$\rho_{jlk} = \begin{cases} 1, & \text{if } j = 1 \\ a, & \text{if } j \neq 1 \end{cases}$$

No. 2 setting for 4 non-adjacent cells scenario: Assume that ZF precoding and GPS precoding (Gomadam *et al*., 2008) uses only pilots from users situated in the same cell as the base station and the multi-cell MMSE precoding is always used with one Frame Relay (FR-1). Let K = 2 users in every cell and pilot length of $\xi = 4$. Orthogonal pilots are used in the No. 1 and No. 2 cells. The pilots used in the No. 1 cell are used in the No. 3, so do No. 2 cell and No. 4 cell. Let the sum rate per cell $R^{sum} = L^{-1}\Sigma_{j,k}R_{jk}$ and the minimum rate $R = \min_{jk} R_{jk}$ achieved by all users. Let the propagation factors for all k is:

$$\rho_{jlk} = \begin{cases} 1 & \text{if } j = l \\ a & \text{if } (j,l) \in \{(1,2),(2,1),(3,4),(4,3)\} \\ b & \text{for all other j and } l \end{cases}$$

Sum rates per cell for different precoding methods for No. 1 Setting are shown in Fig. 1. The improved sum rates are benefit from decreasing a, it shows that the reuse techniques is effective.

Sum rates per cell for different precoding methods for No. 2 setting are plotted in Fig. 2, here, L = 4, a = 0.8, b = 0.1a. It shows that the multi-cell MMSE precoding is effective and powerful, so do in Fig. 3 and 4. Nevertheless, the antenna potential can not be fully exploited, it imply that there are obstructing effect when pilot corruption occurs, this phenomenon also happen in Fig. 1 and 4.

In Fig. 3, Minimum rates for No. 2 setting for ZF and multi-cell MMSE precoding are plotted and show that improved performance of multi-cell MMSE precoding alters in large range resulting from a and b. It owes to MMSE precoding matrices at the base stations produce slightly interference to other cells.

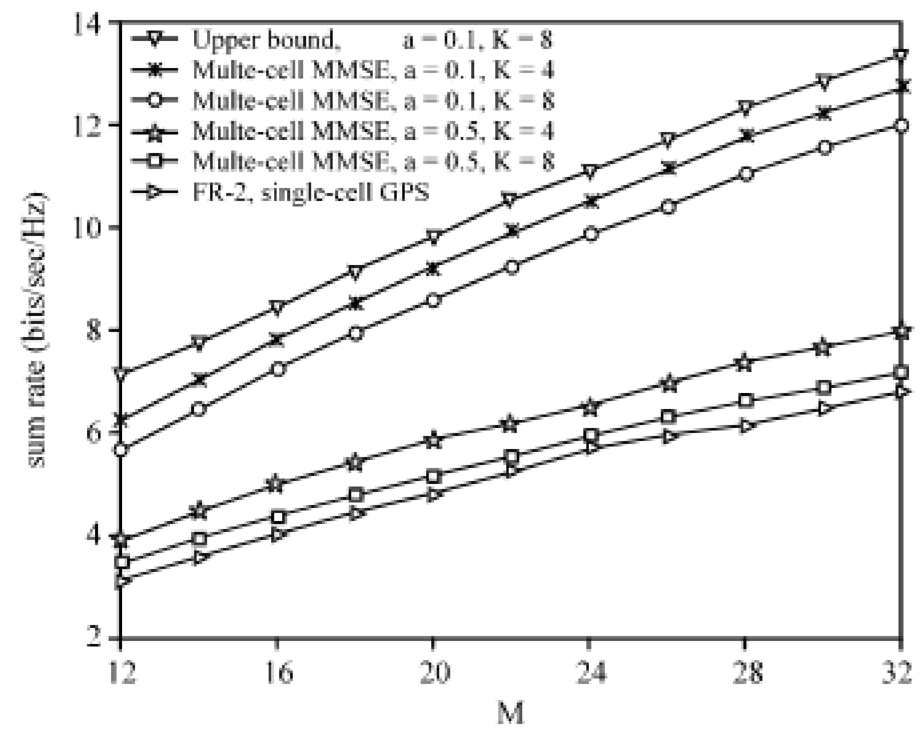

Fig. 1: Sum rate for different reused pilots

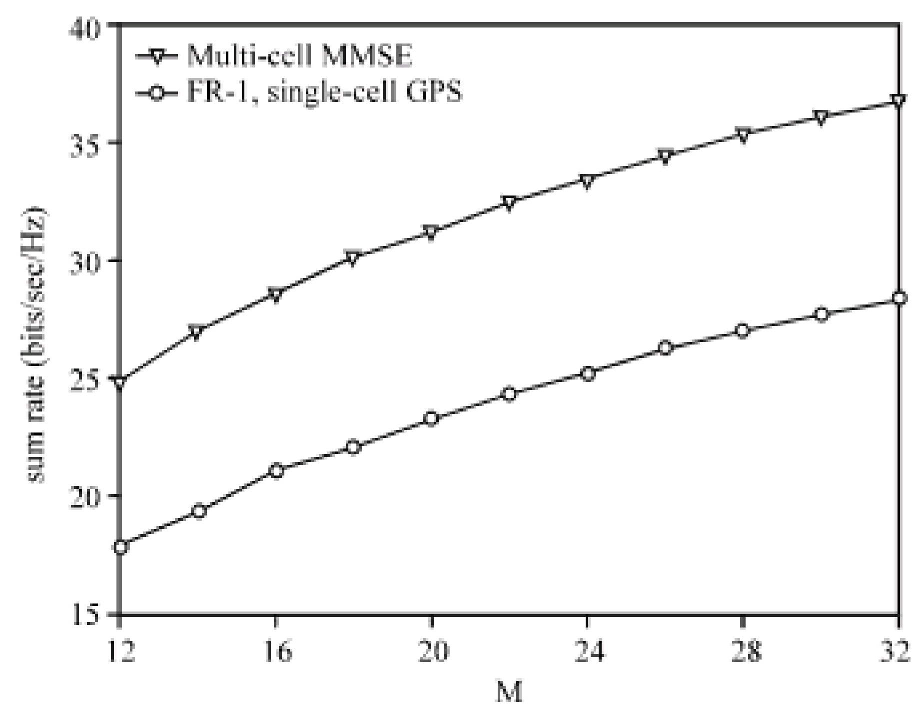

Fig. 2: Sum rates for different precoding methods

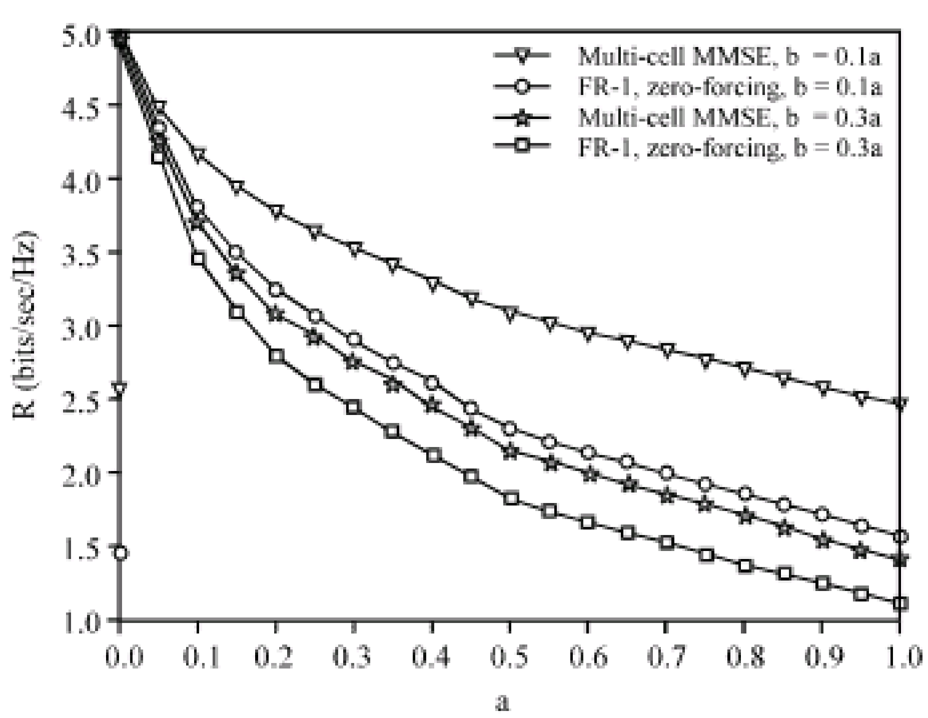

Fig. 3: Performance of ZF and multi-cell MMSE precoding

In Fig. 4, here, L = 4, a = 0.8, b = 0.1a. Minimum rates of No. 2 setting using multi-cell MMSE and GPS. precodings as functions of the numbers of antenna M at the base station, so do in Fig. 2.

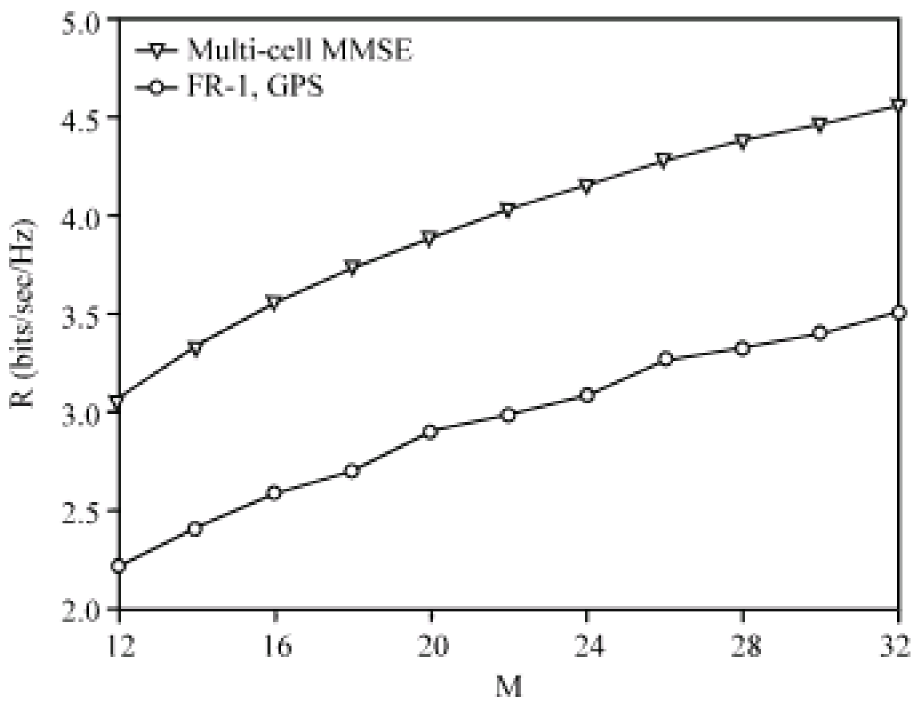

Fig. 4: Minimum rates for different precoding methods

CONCLUSION

The influence of corrupted channel estimates resulting from pilot corruption on TDD systems was discussed in this study. The precoding matrix designed at a multi-antenna base station is correlated with the users using non-orthogonal pilots in other cells. An expression of the rates achieved by all the users for downlink transmission using multi-cell MMSE precoding is presented. The results of analytical analysis and simulation show that improved rates achieved by users via increasing the number of base station antennas is not effective when pilot corruption occurs. It's necessary for utilizing some reuse techniques, e.g., frequency/time, to overcome this obstructing effect. A multi-cell MMSE precoding depending on the pilots assigned to the users is considered and obtain this optimal precoding matrix as the solution of an optimization problem, objective function consists of the mean-square error of received signals by the users in the same cell and the mean-square interference received by the users in other cells. This precoding matrix is shown to reduce intra/inter-cell interference and show that this precoding method used in all cell in the TDD system be superior to ZF precoding.

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