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# Comparison of User Selection Methods for Multiuser MIMO-OFDM Downlink with Limited Feedback

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**Abstract:** In this study, we have proposed a net throughput as performance measure that maximizing limited feedback scheme for multiuser MIMO-OFDM downlink based on per time-frequency cluster user selection, in which system parameters determine the number of data streams to be assigned to one user to receive these streams through eigenmode transmission, while the remaining data streams are assigned to other users with Zero-Forcing (ZF) Receiver (Rx) processors and then compared its average net throughput with a number of Limited Feedback (LF) MIMO-OFDM downlink schemes. We obtain the net throughput maximizing cluster size for scheme I using analytical and a semi-analytical derivations approach. We compared two schemes using average net throughput and average sum rate: scheme I per-cluster user selection and scheme II ZF receiver processing with per-antenna user selection, both are Spatial Multiplexing (SM) MIMO-OFDM downlink transmission schemes based on limited feedback. The results showed scheme I has a greater average net throughput while, scheme II achieves a higher average sum rate over scheme I.

**Key words:** MIMO-OFDM, limited feedback, cluster, zero-forcing, throughput

## INTRODUCTION

In the downlink frequency selective fading channels, multiuser Orthogonal Frequency Division Multiplexing (OFDM) is very promising because of its low complexity and high flexibility, as the data on individual subcarriers can be modulated independently (Wong *et al*., 1999). In general, any Multiple-Input Multiple-Output (MIMO) downlink scheme can be applied to individual subcarriers of MIMO-OFDM with additional complexity of finding the optimum subcarrier antenna power allocation strategy compared to only antenna power allocation. But for MIMO downlink schemes which do not consider the power allocation problem, their extension to MIMO-OFDM is very simple. Furthermore, MIMO-OFDM has been selected as the underlying transmission method for many emerging high data-rate wireless standards, e.g., WiMAX (IEEE 802.16e) and 3GPP Long Term Evolution (LTE). For example, For MIMO system reliable communication in a slowly varying channel, A alamouti-based HARQ transmission scheme is proposed (Wang *et al*., 2010a), this technique increases the efficiency of HARQ packet transmission by exploiting both the spatial and time diversity of the MIMO channel, the Packet Error Rate (PER) analysis of space-time coded MIMO-HARQ is presented, an n-dimension Pair Wise Error Probability (PWEP) analysis of the optimal Alamouti-based HARQ protocol is derived.

For practical lower complexity MIMO downlink transmission schemes based on limited feedback compared with that of full Channel State Information at the Transmitter (CSIT), opportunistic beamforming with feedback carrying Signal-to-Interference-plus-Noise Ratio (SINR) information (Viswanath *et al*., 2002) and spatial multiplexing with opportunistic feedback and linear receivers (Tang *et al*., 2007) are two examples. In multiuser MIMO downlink communications, it is necessary to design precoding schemes that are able to suppress co-channel interference. In multi-cell multi-antennas networks with uplink training, the channel estimate at the base station in one cell jammed by users from other cells, the performance of systems suffers tremendous losses due to the use of corrupted pilots. In addition, as the MMSE precoding matrix exists singularity problem when coordinative method is used, A multi-cell MMSE precoding depending on the pilots assigned to the users

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for downlink transmission to mitigate this corruption and thus, increase achievable rates is proposed (Wang *et al*., 2010b), which does not need coordination. This precoding is the optimal solution of an optimization problem, which consists of the mean-square error of signals and the mean-square interference. Simulation results showed that this precoding method can effectively reduce intra/inter-cell interference. Designing precoders by maximizing Signal-to-Leakage-and-Noise Ratio (SLNR) for all users simultaneously is proposed (Sadek *et al*., 2007). Compared with Zero-Forcing (ZF) solutions, the proposed method does not impose a condition on the relation between the number of transmit and receive antennas and it also avoids noise enhancement.

A fundamental problem in the design of next generation wireless systems is selection among users through allocating limited resources at Medium Access Control (MAC) layer and physical (PHY) layer. In general, this user selection problem can be formulated as some kind of optimization problem, where the objective is to maximize/minimize some system performance measure under PHY layer constraints as well as Quality of Service (QoS) constraints on the MAC layer. In the multiuser OFDM scheme (Svedman *et al*., 2007), the Base Station (BS) modulates the data of each selected user onto a subcarrier designated exclusively to that user and due to the orthogonality of the subcarriers there is no inter-user interference.

The capacity of the multiuser MIMO downlink can be achieved by Dirty Paper Coding (DPC) (Weingarten *et al*., 2004), which is a transmitter multiuser encoding strategy employing interference presubtraction. The DPC requires nonlinear search for optimal precoding matrices as well as non-causal channel coding for the users, which is practically impossible in real-time systems. Therefore, suboptimum transmission strategies such as different forms of beamforming have been considered. In multiuser MIMO beamforming, linear or non-linear transmitter precoding algorithms together with user selection are designed to maximize the system's sum rate or some other related objective function (e.g., sum rate under fairness constraint). Unfortunately, most beamforming algorithms considered assume availability of perfect channel state information at the transmitter, which presents a big challenge to their practical implementation. One example of eigenmode transmission is (Boccardi *et al*., 2007), which employs Singular Value Decomposition (SVD) of user channel matrices and data transmission on the eigenmode with the largest gain. Another example is Bayesteh and Khandani (2007), who proposed a combination of zero-forcing beamforming (ZF-BF) with eigenmode transmission.

A similar scheme involving MIMO Spatial Multiplexing (SM) was proposed (Peng *et al*., 2007), the data rate on each given subcarrier is increased by eigen-beamforming, which creates parallel spatial channels. In the these schemes, the user which achieves the highest rate on a given subcarrier is selected by the BS's scheduler to be served on that subcarrier. This type of resource allocation is a case of Frequency Division Multiplexing (FDM). A selection and beamforming scheme is proposed for MIMO-OFDM downlink (Pun *et al*., 2008), in which by using a new channel decomposition technique and also grouping adjacent subcarriers into clusters, the sum rate is slightly increased and required feedback is reduced compared to per subcarrier eigen-beamforming. Three low complexity, high throughput schemes are selected for comparison from existing limited feedback MIMO downlink techniques, each being a representative member of a class of limited feedback MIMO-downlink transmission methods. They are briefly described in the following:

The MIMO spatial multiplexing wireless systems achieve high spectral efficiencies by demultiplexing the incoming bit stream into multiple substreams. Spatial multiplexing is of practical importance because the multiple substreams can be decoded using linear receivers, but it degrades the probability of error performance. To overcome this difficulty, error rate performance of spatial multiplexing systems can be improved by sending fewer substreams via low rate feedback channels than the number of transmit antennas by linear preceding. Thus, a quantized preceding scheme is proposed (Love and Heath Jr., 2003), where, a codebook based precoding approach that allows the receiver to find the optimal precoding matrix and then send this matrix to the transmitter in the form of a codebook index, i.e., the receiver sends back a fixed number of bits to the transmitter. This bit pattern corresponds to an index within a finite set of preceding matrices. A proposed criterion is used to determine the matrix in this precoder codebook to choose. A design method for these codebooks using techniques from Grassmannian subspace packing, Simulation results showed spatial multiplexing with limited feedback outperforms typical antenna selection.

A Transmit Beam Matching (TBM) multiuser MIMO downlink scheme is introduced (Kim *et al*., 2008), which extends to multiple antenna users the per-user unitary rate control (Huang *et al*., 2009) scheme, which has the relatively low complexity and it uses the channel matrix pseudo-inverse operation in order to minimize inter-stream interference at each user terminal.

Exploiting the independence of fading realizations across the user population, Opportunistic (multiuser diversity) selection based on Channel State Information at the Transmitter (CSIT) can provide significant performance gains for transmission over broadcast channels. For a multiple antenna broadcast channel, this opportunism takes the form of selecting either a single best user (opportunistic time-sharing) or a collection of users whose channels jointly maximize the sum of allocated rates during every transmission (multiuser multiplexing). A multiuser MIMO downlink scheme with spatial multiplexing at the BS is proposed (Airy *et al*., 2004), in which users use linear processors and send back the Signal to Noise Ratio (SNR) of each spatial channel to the BS and the BS selects the user with the highest SNR for each data stream.

The control overhead and signal processing complexity of multiuser MIMO-OFDM systems can be quite large. Therefore, we focus on how to reduce the overhead and complexity, It is shown that the available time-frequency resources are divided into tiles or clusters (Jorswieck *et al*., 2008). The clusters are considered two-dimensional and each cluster consists of a number of adjacent subcarriers in frequency domain and a number of consecutive OFDM symbols in time domain. For all subcarriers and all OFDM symbols within the cluster, the same spatial signal processing is applied, reducing the signal processing complexity and the feedback overhead considerably.

In this study, a limited feedback MIMO-downlink scheme based on eigenvector precoding and Zero Forcing (ZF) receiver processing is considered, in which system parameters determine the number of data streams to be assigned to one user to receive these streams through eigenmode transmission, while the remaining data streams are assigned to other users with ZF receiver (Rx) processors. We proposed per-cluster user selection (scheme I) for limited feedback multiuser MIMO-OFDM downlink with optimized cluster size and obtain a very close approximation for the sum rate of the system. We also showed net throughput advantage of per-cluster user selection with optimized cluster size over a number of limited feedback multiuser MIMO-OFDM downlink schemes and we showed that per-antenna selection on each subcarrier based on limited feedback from users with Zero Forcing (ZF) linear processors (scheme II) leads to a higher average sum rate when the number of users in the system is sufficient. However, scheme I has a much higher net throughput, as it requires far less feedback, especially when optimum cluster size is used. Further reduction of the feedback requirement of the proposed scheme is also investigated when opportunistic feedback is applied.

In this study, we use average sum rate in multiuser MIMO downlink as a comparison benchmark. Usually, the average sum rate increase of different schemes with respect to the increase in the number of users is compared. However, the average sum rate does not account for the amount of feedback required for any given scheme. Furthermore, wireless channels are time-varying in nature and only approximately a wireless channel keeps almost unchanged over coherence time of limited duration (frame). When considering Time Division Duplex (TDD) multiuser MIMO-downlink, it is required that CSIT from all users be available at the BS in a time much shorter than the frame duration, the user pool can not be arbitrarily large. This is due to the fact that uplink channel has a limited capacity when the channel is reciprocal only pilot symbols for channel estimation are required to be sent on the uplink channel and as the number of users increases, the number of pilot symbols required to be send back reliably to the BS increases, which itself results in a longer time necessary to deliver to the transmitter full CSI for all users. Since, data transmission can not begin before having CSI for all users, transmission time is effectively reduced to only the portion of the frame duration that is left after pilot transmission is completed. This reduction of time for data transmission results in a lower average sum rate, also known as net throughput and choose it as performance measure.

Throughout the study, the following notations will be used. Matrices and vectors are denoted with bold capital and lowercase letters, respectively. $(A)_{i,j}$ denotes the element of matrix A at the i-th row and j-th column, $(\cdot)^*$ denotes complex conjugate and $\delta(\cdot)$ denotes the Dirac's delta function. $E[\cdot]$ denotes the expectation of a random variable. $a^*$ denotes the conjugate of the complex number a and $a^T$ is the transpose of vector a. Tr(A) and $\|A\|$ denotes the trace and Frobenius norm of matrix A, respectively. $A^H$ denotes the Hermitian of the complex matrix A. $\mathrm{vec}(\cdot)$ is the vectorization operation. $|C|$ denotes the cardinality of the set C. $\langle A, B\rangle = \mathrm{Tr}(AB^H)$ denotes the frobenius inner product of matrices A and B of the same dimension.

## MULTIUSER OFDM-MIMO SYSTEM MODEL AND PERFORMANCE MEASURE

We consider MIMO-OFDM downlink in a single cell, in which the base station is equipped with M antennas and there are K homogeneous users each equipped with N ($N = M$) antennas. The system block diagram is shown in Fig. 1.

The channel is assumed to be frequency selective and is modeled as a length P Finite Impulse Response

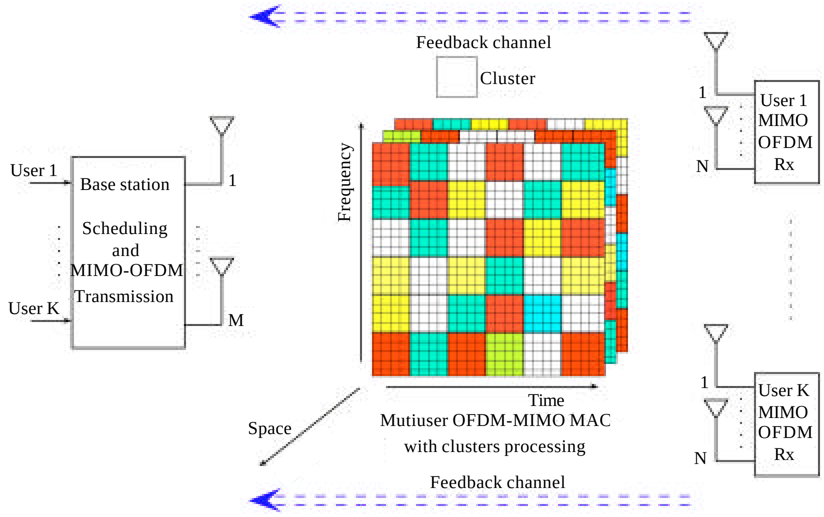

Fig. 1: Multiuser OFDM-MIMO MAC with cluster processing

(FIR) filter (Bolcskei *et al.*, 2002). The space frequency channel at the l-th tone for the k-th user is then obtained as:

$$H_l^{(k)} = \sum_{p=0}^{P-1} \sigma_p H[k,p] \exp\left(-j2\pi \frac{1}{L} p\right), \quad 0 \le l \le L-1 \tag{1}$$

where, exp (·) denotes the exponential function, L is the total number of subcarriers in the system and the size M×N H[k, p] matrices for p = 0,..., P-1, represent the MIMO channel impulse responses of the k-th user. The H[k, p] matrices are assumed to be mutually uncorrelated. It is also assumed that the channel elements exhibit spatially uncorrelated Rayleigh fading, in which case each H[k, p] contains independent CV (0, 1) elements. Also, $\sigma_p$ for p = 0,..., P-1, represents the channel power delay profile and is normalized according to $\sum_{p=0}^{P-1} \sigma_p^2 = 1$. The channel is assumed to be quasi-static, remaining constant for duration of one frame, but changing independently between frames. The correlation coefficients between the channel elements on two arbitrary subcarriers l and u are obtained according to McKay *et al.* (2008):

$$\begin{aligned}\alpha_{l,u} &= E[(H_l^{(k)})_{i,j} (H_u^{(k)})^*_{i',j'}] \\ &= \sum_{p=0}^{P-1} \sigma_p \exp(-j2\pi(l-u)p/L)\delta[i-i']\delta[j-j']\end{aligned} \tag{2}$$

The closer the subcarriers are, the larger the magnitude of the correlation coefficient $|\alpha_{l,u}|$ is. The k-th user receives the following signal vector on the l-th subcarrier:

$$y_l^{(k)} = H_l^{(k)} x_l + n_l^{(k)} \tag{3}$$

where, $H_l^{(k)}$ is given by Eq. 1 and $n_l$ is the noise vector with CN (0, 1) elements. The vector $x_l$ is the transmitted signal vector on the l-th subcarrier. Hence, the average Signal to Noise Ratio (SNR) $\gamma = P_T/M$, Impose a constraint upon the power of the base station, we have:

$$\frac{1}{L} \sum_{l=0}^{L-1} \mathrm{Tr}(E[x_l x_l^H]) = P_T$$

Let, the total transmitted data rate from the base station to the users during one time slot T be $R_T$. Taking ensemble average of $R_T$ over $H_l^{(k)}$, we obtain the expected throughput of the system:

$$R = E_{H_0^{(k)},\ldots,H_{L-1}^{(k)}, k=1,\ldots,K}[R_T]$$

It is assumed that the feedback overhead reduces the transmission rate R to the net (effective) throughput $R_{net}$, given by Jorswieck *et al.* (2008):

$$R_{net} = R\left(1 - N_f \frac{\xi}{R_d T}\right) \tag{4}$$

where, $N_f$ is the average number of feedback terms per subcarrier sent back by all users during the channel's coherence time T. $\zeta$ is the number of bits used to quantize each feedback term and $R_d$ is the feedback channel's bit rate (in bits $\sec^{-1}$).

## MULTI-USER MIMO-OFDM DOWNLINK TRANSMISSION SCHEMES BASED ON LIMITED FEEDBACK

In the following subsection, we outline two low complexity limited feedback schemes for multi-user MIMO-OFDM downlink and compare their average sum rate and net throughput.

**Scheme I: Per-cluster user delection:** Assuming the number of subcarriers in each cluster is $L_c$ (for simplicity we assume all clusters have the same number of subcarriers and L is an integer multiple of $L_c$, the highest instantaneous reliable rate on the q-th cluster in bits per second per Hertz (b/s/Hz) for user k is given by:

$$r_{cls,q}^{(k)} = \frac{1}{L_c} \sum\nolimits_{l_c=(q-1)L_c}^{qL_c-1} r_l^{(k)}, \quad 1 \le q \le Q \tag{5}$$

where, Q = L/$L_c$ denotes the number of clusters in the system and $R_l^{(k)}$ is defined by David and Viswanath (2005):

$$r_l^{(k)} \triangleq \log_2 \det(I + \gamma H_l^{(k)} H_l^{(k)H}), \quad 0 \le l \le L-1$$

This rate is achieved by spatial multiplexing transmission and successive interference cancellation at each receiver (David and Viswanath, 2005). In scheme I, each user sends back the achievable rate of each cluster, Q in total, to the base station and the base station assigns each cluster to the user, which has reported highest rate for that cluster. It is expected that by grouping subcarriers into clusters, especially clusters with small number of adjacent subcarriers, which have higher correlation compared to subcarriers further apart, the sum rate would not significantly decrease, while the amount of feedback is reduced by a factor of $L_c$.

The average sum rate of scheme I is given by:

$$R_I = E\left[\frac{1}{Q}\sum_{q=1}^{Q} \max_{1\le k\le K} r_{cls,q}^{(k)}\right] \tag{6}$$

As $r_l^{(k)}$ is well approximated to be Gaussian (Smith and Shafi, 2002), $r_{cls,q}$ can also be approximated by Gaussian distribution. A close approximation for the expected value of the maximum of a set of K independent and identically distributed (i.i.d.,) Gaussian random variables each having mean μ and variance $\sigma^2$, is given by:

$$E\left[\max_{1\le k\le K} N^{(k)}(\mu,\sigma^2)\right] \approx \sigma G(K) + \mu \tag{7}$$

where,

$$G(K) = \frac{1}{0.1975}[0.5264^{\frac{0.135}{K}} - (1 - 0.5264^{\frac{1}{K}})^{0.135}]$$

Therefore, in order to be able to use Eq. 7 as an approximation for $R_I$, the mean μ and variance $\sigma^2$ of $r^{(k)}{}_{cls,q}$ are required. In McKay *et al.* (2008), it has been shown that the mean of the rate of each cluster, $E[r_{cls,q}]$ (the user index, k, has been discarded), is the same for all clusters, i.e., $E[r_{cls,q}] = E[r_{cls,u}]$, q≠u, even if the cluster sizes are not equal and is given by Kang and Alouini (2006):

$$E[r_{cls,q}] = \frac{1}{\ln(2)\prod_{m=1}^{M}[\Gamma(M-m+1)]^2} \sum_{k=1}^{M} \det(\Psi(k)) \tag{8}$$

where, Ψ(k), k = 1,...,M, are M×M matrices whose entries are defined by:

$$\{\Psi(k)\}_{i,j} = \begin{cases} u_{i,j}!\exp\left(\frac{1}{\gamma}\right)\sum_{s=1}^{u_{i,j}} \frac{\Gamma\left(s-u_{i,j},\frac{1}{\gamma}\right)}{\gamma^{(u_{i,j}-s)}}, & j=k \\ u_{i,j}!, & j\ne k \end{cases} \tag{9}$$

where, $u_{i,j}$ = 2M-i-j+1, n! denotes factorial of n and Γ(·,·) denotes the incomplete Gamma function (Simon, 2006).

For a MIMO-OFDM link, the variance of the instantaneous reliable rate has been derived in McKay *et al.* (2008). We use the derivations of McKay *et al.* (2008) to obtain the variance of reliable rate for individual clusters of a MIMO-OFDM link. For the variance of the rate of each cluster we provide the following theorem.

**Theorem 1:** For a MIMO-OFDM link with channel gains exhibiting spatially uncorrelated Rayleigh fading, the variance of the instantaneous reliable rate on each cluster, variance of $r_{cls,q}$, for all clusters with equal number of subcarriers, $L_c$, is the same and is given by:

$$\begin{aligned} \mathrm{Var}(r_{cls,q}) = {} & \frac{2(\log_2(e))^2 \exp(2/\gamma)}{(L_c)^2[\prod_{m=1}^{M}\Gamma(M-m+1)]^2} \times \sum_{d=1}^{L_c-1}\left[(L_c-d)\sum_{r=1}^{M}\sum_{s=1}^{M}\det(C_{r,s}(\alpha_{0,d}))\right] \\ & + \frac{1}{L_c \ln^2(2)[\prod_{m=1}^{M}\Gamma(M-m+1)]^2}\sum_{k=1}^{M}\sum_{l=1}^{M}\det(\Psi_{k,l}) - E^2[r_{cls,q}] \end{aligned} \tag{10}$$

where, the matrices $C_{r,s}$ ($\alpha_{0,d}$) and $\Psi_{kl}$ are given by Eq. 11 (McKay *et al.*, 2008) and Eq. 14 (Kang and Alouini, 2006), respectively. $\alpha_{0,d}$ is defined by Eq. 2.

$$[C_{r,s}(\alpha_{0,d})]_{i,j}=\begin{cases}\eta_{i,j}(1,\alpha_{0,d}), & i\neq r, j\neq s\\ \eta_{i,j}(g_1(i+j-1),\alpha_{0,d}), & i=r, j\neq s\\ |\alpha_{0,d}|^{2(i-j)}\,\eta_{j,i}(g_1(i+j-1),\alpha_{0,d}), & i\neq r, j=s\\ \dfrac{(1-|\alpha_{0,d}|^2)^{i+j-1}}{|\alpha_{0,d}|^{2(j-1)}}\exp\left(\dfrac{2|\alpha_{0,d}|^2}{\gamma(1-|\alpha_{0,d}|^2)}\right) & \\ \times\displaystyle\sum_{t=0}^{\infty}\dfrac{|\alpha_{0,d}|^{2t}\,\Gamma(i+t)\Gamma(j+t)g_2(i+t)g_2(j+t)}{(t!)^2}, & i=r, j=s\end{cases}\tag{11}$$

Where, $1\le i, j\le M$,

$$g_1\triangleq\sum\nolimits_{n=1}^{z}E_n(\gamma)$$

and

$$g_2\triangleq\sum\nolimits_{n=1}^{z}E_n((\gamma(1-|\alpha_{0,d}|^2))^{-1})$$

wherein, the exponential integral of x defined as:

$$E_n(x)\triangleq\begin{cases}\displaystyle\int_x^{\infty}\frac{\exp(-t)}{t}dt, & n=1\\ \displaystyle\int_{t=1}^{\infty}\frac{\exp(-xt)}{t^n}dt=\frac{1}{n-1}[\exp(-x)-xE_{n-1}(x)], & n>1\end{cases}\tag{12}$$

For an arbitrary f function, we have:

$$\eta_{i,j}(f(z),\alpha_{0,d})=\Gamma(j)\sum_{t=0}^{j-1}\binom{j-1}{t}\left(\frac{1-|\alpha_{0,d}|^2}{|\alpha_{0,d}|^2}\right)^t\frac{\Gamma(i+j-t-1)}{\Gamma(j-t)}f(z-t)\tag{13}$$

and

$$\{\Psi_{k,l}\}_{i,j}=\begin{cases}u_{i,j}!\exp\left(\dfrac{1}{\gamma}\right)\displaystyle\sum_{s=1}^{u_{i,j}}\dfrac{\Gamma\left(s-u_{i,j},\dfrac{1}{\gamma}\right)}{\gamma^{(u_{i,j}-s)}}, & j=k\\ u_{i,j}!, & j\neq k\end{cases}\tag{14}$$

**Proof:** Following the same lines as the McKay *et al.* (2008) in deriving the variance of mutual information for a MIMO-OFDM system, we drive the variance of mutual information for each cluster of MIMO-OFDM by showing that the variances of mutual information for two arbitrary clusters of equal size are equal, then we conclude that the variances of all clusters are equal. Denote the achievable rate of each cluster, $r_{cls,q}$, by $I_q$, then we have:

$$\begin{aligned}\mathrm{Var}(I_q)&=E[I_q^2]-E^2[I_q]\\ &=E\left[\frac{1}{L_c^2}\sum\nolimits_{k=(q-1)L_c}^{qL_c-1}\sum\nolimits_{l=(q-1)L_c}^{qL_c-1}I_kI_l\right]-E^2[\mathcal{I}_q]\\ &=\frac{1}{L_c^2}\left(\sum\nolimits_{k=(q-1)L_c}^{qL_c-1}\sum\nolimits_{l=(q-1)L_c,l\neq k}^{qL_c-1}E[I_kI_l]+\sum\nolimits_{k=(q-1)L_c}^{qL_c-1}E[I_k^2]\right)-E^2[I_q]\\ &=\frac{1}{L_c^2}\left(\sum_{k=0}^{L_c-1}\sum_{l=0,l\neq k}^{L_c-1}E[I_kI_l]\right)+\frac{1}{L_c}E[I_q^2]-E^2[I_q]\\ &=\mathrm{Var}(I_0),\end{aligned}\tag{15}$$

where, the second last line is obtained considering the fact that $E[I_kI_l]$ only depends on $[|k-l|]$ and not k and l ($\alpha_{k,l}=\alpha_{0,|k,l|}$) and is given by McKay *et al.* (2008):

$$E[I_kI_l]=\frac{(\log_2(e))^2\exp(2/\beta)}{[\prod_{m=1}^{M}\Gamma(M-m+1)]^2}\sum_{r=1}^{M}\sum_{s=1}^{M}\det(C_{r,s}(\alpha_{k,l}))\tag{16}$$

where, the matrices $C_{r,s}(\alpha_{k,l})$ are taken from McKay *et al.* (2008).

Using Eq. 7, 8 and 10, the average sum rate of scheme I is approximated by:

$$R_I\approx\sqrt{\mathrm{Var}(r_{cls,q})}G(K)+E[r_{cls,q}]\tag{17}$$

The unique feature of Eq. 17 is that the effect of the cluster size and the effect of the number of available users in the system on the sum rate are clearly separate through $\sqrt{\mathrm{Var}(r_{cls,q})}$ and G (k). The maximum net throughput for this scheme is then approximated by:

$$R_{net,I}^{max}\approx R_I\left(1-\frac{N_f^{(max,I)}\xi}{R_dT}\right)\tag{18}$$

where, $Nf^{(max,I)}=KL_c^{max}/L$ is the average number of feedback terms per subcarrier. $L_c^{max}$ is the cluster size which maximizes $R_{net,I}$, the net throughput given by Eq. 18 for an arbitrary cluster size. In our work, $R_{net,I}$ is obtained numerically.

**Scheme II: Per-antenna user selection upon each subcarrier:** For this scheme, we consider spatial multiplexing transmission and post-processing SNR feedback from users with linear ZF processors on each subcarrier. Each user adopts a linear ZF processor given by $G_l^{(k)}=(H_l^{(k)})^{-1}$ on the l-th subcarrier and evaluates M SNR values each obtained as Chen and Wang (2007) $\beta_{l,m}^{(k)}=\gamma/[(H_l^{(k)H}H_l^{(k)})^{-1}]_{mm}$, $1\le m\le M$, where, $[\cdot]_{mm}$ denotes the m-th diagonal term of the matrix argument and $\beta_{l,m}^{(k)}$ is the post-processing SNR of the m-th data stream on the l-th subcarrier of user k's channel. After receiving M×L post-processing SNRs from each user, for each spatial data stream and subcarrier, the base station selects the user which has reported the largest post-processing SNR for that data stream and subcarrier.

As mentioned in earlier, we assume M = N. The average sum rate of scheme II in b/s/Hz is Chen and Wang (2007):

$$\begin{aligned}R_{II}&=E\left[\frac{1}{L}\sum_{l=1}^{L}\sum_{m=1}^{M}\log_2(1+\gamma\beta^{max})\right]\\ &=\frac{KM}{\ln(2)}\sum_{q=0}^{K-1}\frac{1}{q+1}\binom{K-1}{q}(-1)^q\exp\left(\frac{q+1}{\gamma}\right)E_1\left(\frac{q+1}{\gamma}\right)\end{aligned}\tag{19}$$

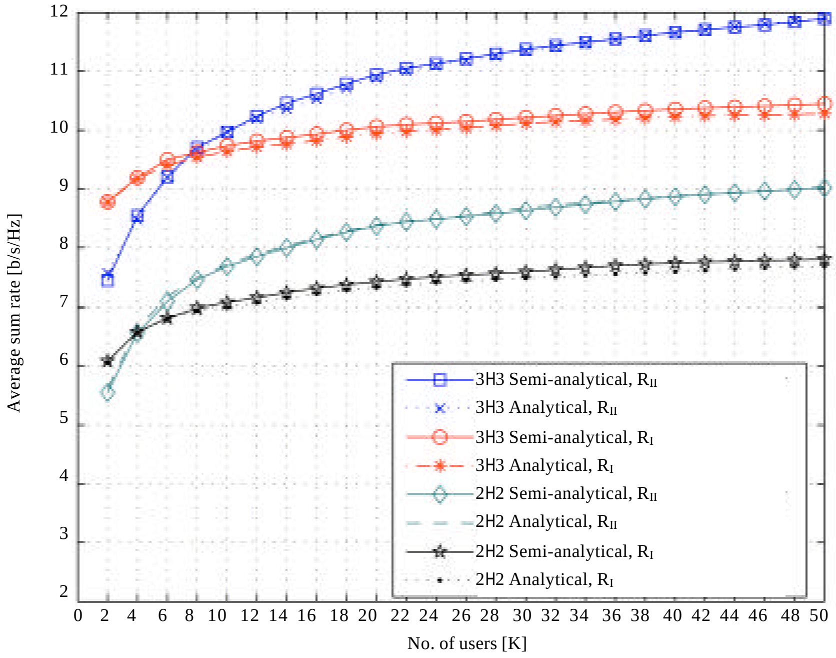

Fig. 2: Average sum rate of schemes I and II for MIMO-OFDM downlink with L = 64 subcarriers, cluster sizes of $L_c = 16$, $P_T = 10$ dB and M = N = 2 and 3

where, $E_1(x) = \int_x^\infty \exp(-t)/t\,dt$, and $\beta^{max} = \max\limits_{1 \le k \le K} [\beta_{l,m}^{(k)} / \gamma]$ (we have dropped the subscripts l and m, as the maximum post-processing SNR has the same distribution for any $0 \le l \le L\text{-}1$ and $1 \le m \le M$). The net throughput for this scheme is:

$$R_{net,II} = R_{II}\left(1 - \frac{N_f^{(II)}\xi}{R_d T}\right)$$

where, $Nf^{(II)}$ is the number of feedback terms from all users, per subcarrier. By comparing $Nf^{(max,I)}$ and $Nf^{(II)}$, it becomes clear that scheme I has a far lower feedback requirement than scheme II. Hence, we try the following feedback reduction concept.

Scheme II with opportunistic feedback. Similarly to Tang *et al.* (2007), we consider the case where, not all users send back their per-antenna rate (or SNR) values. Only users with per-antenna post processing SNRs above a pre-defined threshold are allowed to use the feedback channel. We simply use the expected value of SNR, $E[\beta^{max}]$, as the threshold as finding the optimum threshold is beyond the scope of this study:

$$\begin{aligned} E[\beta^{max}] &= \int_0^\infty x f_{\beta^{max}}(x)\,dx \\ &= K\int_0^\infty x(1-\exp(-x))^{K-1}\exp(-x)\,dx = \sum_{k=1}^{K} 1/k \end{aligned} \qquad (20)$$

Using the Cumulative Distribution Function (CDF) of $\beta^{max}$ given by:

$$F_{\beta^{max}}(x) = \Pr(\beta^{max} \le x) = (1-\exp(-x))^K \qquad (21)$$

The average number of feedback terms for scheme II is reduced to $Nf^{(rdc,II)} \approx KM\ (1\text{-}F\ \beta^{max}\ (T[K]))$ where, T[K] $\Sigma^k_{k=1} 1/k$. The net throughput is then,

$$R_{net,II}^{opp} \approx R_{II}^{opp}\left(1 - \frac{N_f^{(rdc,II)}\xi}{R_d T}\right)$$

where, $R_{II}^{opp}$ is average sum rate of scheme II with the feedback scheme described above.

## NUMERICAL RESULTS

For the numerical results presented in this section, we model the channel using the exponential power delay profile (McKay *et al.*, 2008):

$$\sigma_p^2 = \frac{1-\exp(-1/2)}{1-\exp(-7/2)}\exp(-p/2), \quad 0 \le p \le 7 \qquad (22)$$

Figure 2 shows the average sum rates of schemes I and II obtained using a semi-analytical approach for M = N = 2, or 3 antennas at both the BS and each user

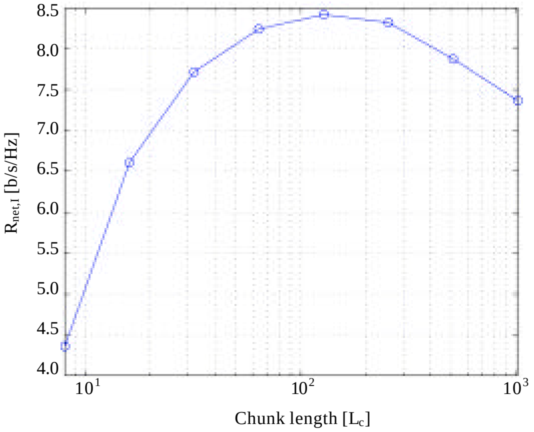

Fig. 3: Average net throughput for a system with $L = 2048$ subcarriers, $P_T = 10$ dB, $K = 100$ users and $M = N = 2$ antennas, versus cluster size

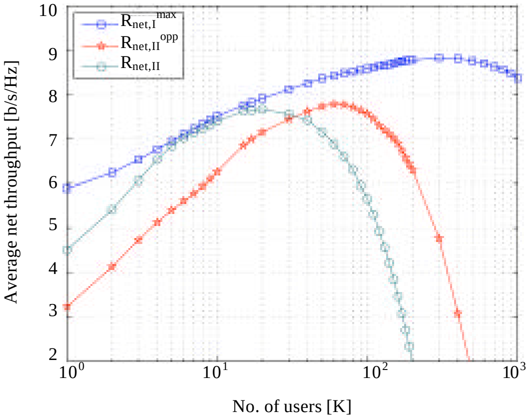

Fig. 4: Average net throughput of schemes I ($L_c^{max}$) and II for a system with $L = 2048$ subcarriers, $P_T = 10$ dB and $M = N = 2$ antennas, versus the number of users

terminal plotted versus the number of users at $P_T = 10$ dB. The system considered has $L = 64$ subcarriers with cluster size of $L_c = 16$. In the semi-analytical approach, for one system realization, a set of random channel matrices is generated and used to evaluate the sum rate of each scheme according to the scheme's selection and signal processing approach. The average sum rate is then obtained by averaging the sum rates over a large number of realizations. Analytical results have also been plotted and are in good agreement with the semi-analytical results. As expected and due to multiuser diversity, the average sum rate increases with the number of users in both schemes. Scheme II (per-antenna user selection), thanks to having more degrees of freedom, takes better advantage of multiuser diversity and achieves a higher sum rate.

In Fig. 3, the net throughput of scheme I is plotted versus the cluster size, $L_c$. The system considered has $L = 2048$ subcarriers with $M = N = 2$ as the number of antennas at the base station and each user terminal. Similarly to Jorswieck *et al.* (2008), we set $\zeta/R_dT$ for all the results presented in this section. As seen in Fig. 3, $L_c^{max}$ is the optimum cluster size that maximizes the net throughput.

In Fig. 4, the average net throughput of schemes I and II is plotted versus the number of users for a system with $L = 2048$ subcarriers. Cluster size of $L_c^{opt} = 128$ is used for scheme I and $R_{net,I}$ is obtained using the analytical approximations of Eq. 17 and 18. For scheme II with opportunistic feedback, $R^{opp}_{net,II}$ is obtained through semi-analytical approach to find $R_{II}^{opp}$ and using $Nf^{(rdc,II)}$ to evaluate the number of feedback terms. Figure 4 shows that as the number of users increases, the net throughput of scheme II approaches zero, even with opportunistic feedback. It is known that the sum rate of scheme II has an asymptotically optimum increase with the number of users, when the number of users approaches infinity (Airy *et al.*, 2004), while this is not true for scheme I. Therefore, as we can see with a more realistic net throughput definition, the throughput superiority of scheme II does not exist anymore.

## CONCLUSIONS

We investigate the problem of finding efficient limited feedback multiuser MIMO downlink transmission schemes. Limited feedback MIMO-OFDM downlink with per cluster user selection is considered. By grouping adjacent subcarriers into clusters, the amount of required feedback is reduced. Using net throughput concept, which accounts for the reduction of sum rate due to feedback overhead, we have proposed a net throughput maximizing limited feedback scheme for multiuser MIMO-OFDM downlink based on per-cluster user selection and compared its average net throughput with a number of limited feedback MIMO-OFDM downlink schemes. It is shown that there exists an optimum cluster size which maximizes the net throughput. To reduce the feedback requirement even further, an opportunistic feedback scheme is proposed and a close approximation for its net throughput is derived. By comparing two low complexity spatial multiplexing MIMO-OFDM downlink transmission schemes which require partial CSIT, we show how the amount of feedback affects the MIMO-OFDM downlink transmission schemes by reducing their net throughput. The results show that while per-antenna user selection with ZF receiver processing achieves a higher average sum rate compared to per-cluster user selection, the latter scheme has a much greater average net throughput. In fact, as the number of users in the system increases, net throughput for per-antenna user selection approaches zero much faster than per-cluster user selection, whereas, the proposed scheme's net throughput slightly increases before approaching zero at a significantly lower rate with the number of users and when that number is relatively large. The use of net throughput, that as opposed to the sum rate accounts for the amount of feedback, as the benchmark for comparing multiuser MIMO downlink schemes, enables a fairer comparison between different multiuser MIMO downlink schemes. Net throughput maximization of multiuser MIMO and multiuser MIMO-OFDM downlink seems to be an interesting area of research, currently in its early stages. It is known that DPC achieves the sum capacity for multiuser MIMO downlink. However, in multiuser MIMO downlink (single carrier or multicarrier) which scheme achieves the maximum average net throughput, this fundamental question still remains to be answered. Our preliminary work in this area suggests that the answer might be a hybrid of the existing schemes, in which depending on the number of users available in the system, one mode in the hybrid is activated.

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