http://ansinet.com/itj

ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 9 (4): 804-810, 2010
ISSN 1812-5638

Using Simple and Efficient Algorithm Involving Ordering Cost Reduction and Backorder Price Discount on Inventory System under Variable Lead Time

Shien-Ping Huang
Department of International Trade, Taipei College of Maritime Technology, Taiwan

Abstract: This study investigated lead time can be decomposed into several components; each having a crashing cost for the reduced lead time and the associated crashing expenses a fixed cost. If an item is out of stock in an inventory system in which shortage is allowed, the supplier may offer a negotiable price discount to the loyal, tolerant and obliged customers to pay off the inconvenience of backordering. On the other hand, the purposed model in which the ordering cost is regarded as a decision variable and introduces the option of investing in reducing the ordering cost parameter in the classical undiscounted EOQ system. This study investigates the integrated inventory systems with the objective to simultaneously optimizing the ordering cost, order quantity, lead time, backorder price discount and reorder point. Numerical example is included to illustrate the procedures of the simple and efficient algorithm. We take many parameters into account and help the decision maker to take choice.

Key words: Ordering cost reduction, lead time, backorder price discount, order quantity, reorder point, crashing cost

INTRODUCTION

Inventory management is mainly about specifying the size and placement of stocked goods. Inventory management is required at different locations within a facility or within multiple locations of a supply network to protect the regular and planned course of production against the random disturbance of running out of materials or goods. The scope of inventory management also concerns the fine lines between replenishment lead time, carrying costs of inventory, asset management, inventory forecasting, inventory valuation, inventory visibility, future inventory price forecasting, physical inventory, available physical space for inventory, quality management, replenishment, returns and defective goods and demand forecasting. A JIT inventory system focuses on finding ways to greatly reduce the setup costs so that the optimal order quantity will be small. Such a system, also seeks ways to reduce the lead time for the delivery of an order, since this reduces the uncertainty about the number of units that will be needed when, the delivery occurs. Firms can shorten lead times by storing inventory or having excess capacity. Liao and Shyu (1991) proposed an inventory model with predetermined order quantity and normally distributed demand with lead time being the only variable to minimize the expected total cost. Ben-Daya and Raouf (1994) presented more general models by including both lead time and order quantity as decision variables. Based on Ben-Daya and Raouf's (1994) results, Moon and Choi (1997) pointed out that the inclusion of both the service level constraint. Hariga and Ben-Daya (1999) also extended the Ouyang *et al.* (1999) model by relaxing the assumption of a given service and treated the reorder point as a decision variable. Kim and Benton (1995) established a relationship between lot size and lead time and using iterative algorithm that determines lot size and safety stock simultaneously and compare it against a more conventional sequential approach. Hariga (2000) modified Kim and Benton's model (1995) by rectifying the expression of the annual backorder cost and proposing another relation for the revised lot size to generate a smaller lot size than that of Kim and Benton (1995).

In traditional Economic Order Quantity (EOQ) models, ordering cost is treated as a constant. However, in practice, ordering cost can be reduced through Electronic Data Interchange (EDI) technology. Hariga (2000) extended the Hariga (1999) study by considering the investment in setup time reduction and the relationship between lead time, lot size and setup time. Porteus (1985) developed an extension of the EOQ model in which setup cost is viewed as a decision variable by introducing the option of investing in reducing the setup cost parameter. Billington (1987) presented an EOQ model with the setup cost parameter replaced by a function of capital investment and obtained closed form solutions for linear and exponential (with technological bounds) relationships between investment and setup costs. Trevino *et al.* (1993) presented a setup time reduction cost function to find the optimal lot size and optimal percentage of setup time reduction. Hong and Hayya (1993) considered the lot sizing problem in Material Requirements Planning (MRP) systems by assuming that setup cost can be reduced by capital investment. Ouyang *et al*. (1999) extended Moon and Choi's (1998) model by considering the ordering cost as a decision variable.

A stockout occurs whenever, insufficient stock exists to fulfill a replenishment order. During the stockout period, either all the demand is backordered, in which all customers wait until their demand is satisfied; or all the demand is lost. However, in many real inventory systems, demand can be captive partially. For customers whose needs are not crucial at that time can wait for the item to be satisfied, while others cannot wait will fill their demands from some other sources. The cost for a lost sale ranges form profit loss on the sale to some generic loss of good will. On the other hand, the backordering could result in handling cost and expediting cost to reduce the lead time. In order to compensate customers for the inconvenience of waiting, the idleness of equipment, or even lost production during the stockout period, the supplier may offer a variable price discount on the stockout item depending on the seriousness of the backorder condition. Thus, both the backorder price discount and the lead time appear to be negotiable (Pan and Hsiao, 2001) in such a way that the supplier may cut down the present and future profit losses and the customers may be able to get the item as soon as possible to resume the production. Pan *et al*. (2004) studied the integrated inventory systems with the objective to simultaneously optimizing the order quantity, backordering and reorder point under variable lead time and that the crashing cost of lead time related with quantity. Lo *et al*. (2008) considered backorder price discount and safety factor for decision variable. Lo (2007) builds a decision support system to deal with the integrated inventory model and help the decision maker to make decisions. Ijioui *et al*. (2006) used simulation to analyze the dynamic behavior of ultimate critical time orders with multiple priorities and results help them to understand the dynamic behavior of orders with multiple priorities.

This study considers an inventory system that agrees to shortage and the total amount of stock out is a combination of backorder and lost sale. It is assumed that the vendor may offer a backorder price discount to the patient buyer with exceptional orders during the shortage period and the backorder ratio is proportion to the price discount (Pan and Hsiao, 2001). Furthermore, it is assumed that the inventory lead time is controllable and the crashing cost can be represented as a function of reduced lead time. On the one hand, ordering cost parameter replaced by a function of capital investment. Since, the shortage cost is explicitly included, the reorder point is also treated as a decision variable in this study.

NOTATIONS AND ASSUMPTIONS

The notations used in the study are listed as following:

L = The length of lead time (decision variable)
Q = Order quantity (decision variable)
π_x = Backorder price discount offered by the supplier per unit (decision variable)
k = Safety factor (decision variable)
r = The reorder point
π_0 = The gross marginal profit per unit
D = Average demand year^{-1}
A = Ordering cost per order (decision variable)
A_0 = Original ordering cost
$I(A)$ = Capital investment required to achieve ordering cost A, $0 \le A \le A_0$
θ = Fractional opportunity cost of capital
δ = Percentage decrease in ordering cost A per dollar increase in investment I(A)
h = Inventory holding cost per unit year^{-1}
μ = The average demand rate in units day^{-1}
β = The backorder ratio
β_0 = The upper bound of the backorder ratio

The following assumptions are made on the models in the study:

- Lead time is deterministic and lead time demand X has finite mean μL and variance$\sigma_L^2 = \sigma^2 L$
- The reorder point $r = \mu L + k\sigma_L$, where k is the safety factor
- Inventory is continuously reviewed and replenishments are made whenever the inventory level falls to the reorder point r
- The lead time consists of n mutually independent components. The ith component has a normal duration U_i and a minimum duration u_i, i = 1, 2, ..., n, with a crashing cost per unit time a_i. These a_i's are arranged such that $a_1 \le a_2 \le \dots \le a_n$. The lead times are crashed one component at a time starting with the one of least a_i and so on
- Let, L_i be the length of lead time with component 1, 2, ..., i crashed to their minimum values and Li can be expressed as $L_i = \sum_{j=1}^{n} U_j - \sum_{j=1}^{i}(U_j - u_j)$. Thus, the lead time crashing cost R(L) per replenishment cycle is given by:

$$R(L) = a_i (L_{i-1}-L) + \sum_{j=1}^{i-1} a_j(U_j - u_j), \text{ for } L \in (L_i, L_{i-1})$$

- The backorder ratio β is variable and is in proportion to the price discount offered by the supplier per unit π_x; thus, $\beta = \beta_0\pi_x/\pi_0$, for $0 \le \beta_0 \le 1$, $0 \le \pi_x \le \pi_0$ (Pan and Hsiao, 2001)
- The capital investment, I(A) for reducing the ordering cost is a logarithmic function of the ordering cost A (Porteus, 1985), that is:

$$I(A) = \alpha \ln\left(\frac{A_0}{A}\right) \quad \text{for } 0 < A \le A0, \text{ where, } \alpha = \frac{1}{\delta}$$

A MODEL WITH NORMALLY DISTRIBUTED DEMAND

The lead time demand X is assumed to be normally distributed with mean μL and standard deviation $\sigma\sqrt{L}$. While shortage is allowed, the expected inventory shortage at the end of a cycle is given by $B(r) = \sigma\sqrt{L}\,\Psi(k)$, where, $\Psi(k) = \phi(k)-k[1-\Phi(k)]$ and ϕ, Φ are the standard normal distribution and cumulative distribution function, respectively (Ravindran *et al*., 1987). For backorder ratio β, the expected number of backorders per cycle is $\beta B(r)$, the expected demand lost per cycle is $(1-\beta)B(r)$ and the annual stockout cost is $D/Q[\beta\pi_x + \pi_0(1-\beta)]B(r)$ (Pan and Hsiao, 2001). Therefore, the expected net inventory level at the end of a cycle is $(r-\mu L + (1-\beta)B(r))$ and at the beginning of the cycle is $(Q + r-\mu L + (1-\beta)B(r))$. Consequently, the expected annual cost comprising of ordering cost, holding cost, stockout cost and lead time crashing cost can be represented by:

$$\begin{aligned} &EAC(Q,\pi_x,r,L) \\ &= \frac{AD}{Q} + h\left[\frac{Q}{2} + r - \mu L + (1-\beta)B(r)\right] + \frac{D}{Q}[\pi_x\beta + \pi_0 \\ &(1-\beta)]B(r) + \frac{D}{Q}\left[a_i(L_{i-1}-L) + \sum_{j=1}^{i-1} a_j(U_j - u_j)\right] \end{aligned} \tag{1}$$

Substituting $r = \mu L + k\sigma_L$ into Eq. 1, we have:

$$\begin{aligned} EAC(Q,\pi_x,k,L) &= \frac{AD}{Q} + h\left[\frac{Q}{2} + k\sigma\sqrt{L}\right] + \left[h\left(1-\frac{\beta_0}{\pi_0}\pi_x\right) + \frac{D}{Q}\left(\frac{\beta_0}{\pi_0}\pi_x^2 + \pi_0 - \beta_0\pi_x\right)\right] \\ &\sigma\sqrt{L}\Psi(k) + \frac{D}{Q}\left[a_i(L_{i-1}-L) + \sum_{j=1}^{i-1} a_j(U_j - u_j)\right] \\ &L \in (L_i, L_{i-1}) \end{aligned} \tag{2}$$

As previously stated, the ordering cost A is a decision variable and the problem under study seeks to minimize the sum of the capital investment cost on the reduction of A and the inventory costs by optimizing over Q, π_x, k, L and A constrained by $0 < A \le A_0$. The objective was to minimize the following expected annual cost:

$$\begin{aligned} &EAC(Q,\pi_x,k,A,L) \\ &= \theta\alpha\ln\left(\frac{A_0}{A}\right) + \frac{AD}{Q} + h\left[\frac{Q}{2} + k\sigma\sqrt{L}\right] + \left[h\left(1-\frac{\beta_0}{\pi_0}\pi_x\right) + \frac{D}{Q}\left(\frac{\beta_0}{\pi_0}\pi_x^2 + \pi_0 - \beta_0\pi_x\right)\right] \\ &\sigma\sqrt{L}\Psi(k) \quad + \frac{D}{Q}\left[a_i(L_{i-1}-L) + \sum_{j=1}^{i-1} a_j(U_j - u_j)\right] \\ &L \in (L_i, L_{i-1}) \end{aligned} \tag{3}$$

Subject to $\quad 0 < A \le A_0$

Taking partial derivatives of EAC(Q, π_x, k, A, L) with respect to Q, π_x, k, A and L, respectively, we get:

$$\begin{aligned} \frac{\partial EAC(Q,\pi_x,k,A,L)}{\partial Q} &= -\frac{AD}{Q^2} + \frac{h}{2} - \frac{D}{Q^2}\left[\frac{\beta_0}{\pi_0}\pi_x^2 + \pi_0 - \beta_0\pi_x\right] \\ &\sigma\sqrt{L}\Psi(k) - \frac{D}{Q^2}\left[a_i(L_{i-1}-L) + \sum_{j=1}^{i-1} a_i(U_j - u_j)\right] \end{aligned} \tag{4}$$

$$\begin{aligned} \frac{\partial EAC(Q,\pi_x,k,A,L)}{\partial \pi_x} &= -\frac{\beta_0}{\pi_0}h\sigma\sqrt{L}\,\Psi(k) + \\ &\frac{D}{Q}\left[\frac{2\beta_0}{\pi_0}\pi_x - \beta_0\right]\sigma\sqrt{L}\Psi(k) \end{aligned} \tag{5}$$

$$\begin{aligned} \frac{\partial EAC(Q,\pi_x,k,A,L)}{\partial k} &= h\sigma\sqrt{L} - \\ &\left[h\left(1-\frac{\beta_0}{\pi_0}\pi_x\right) + \frac{D}{Q}\left(\frac{\beta_0}{\pi_0}\pi_x^2 + \pi_0 - \beta_0\pi_x\right)\right] \times \sigma\sqrt{L}(1-\Phi(k)) \end{aligned} \tag{6}$$

$$\frac{\partial EAC(Q,\pi_x,k,A,L)}{\partial A} = -\frac{\theta\alpha}{A} + \frac{D}{Q} \tag{7}$$

and

$$\begin{aligned} \frac{\partial EAC(Q,\pi_x,k,A,L)}{\partial L} &= \frac{1}{2}\left[h\left(1-\frac{\beta_0}{\pi_0}\pi_x\right) + \frac{D}{Q}\left(\frac{\beta_0}{\pi_0}\pi_x^2 + \pi_0 - \beta_0\pi_x\right)\right] \\ &\sigma L^{-1/2}\Psi(k) + \frac{1}{2}hk\sigma L^{-1/2} - \frac{D}{Q}a_i \end{aligned} \tag{8}$$

Setting Eq. 5 to zero and solving for π_x, it follows that:

$$\pi_x = \frac{hQ}{2D} + \frac{\pi_0}{2} \tag{9}$$

Setting Eq. 7 to zero and solving for A, it follows that:

$$A = \frac{\theta\alpha Q}{D} \tag{10}$$

Solving for Q by setting Eq. 4 to zero and substituting Eq. 9 and 10 into Eq. 4, we obtain:

$$Q=\frac{\theta\alpha+\sqrt{\begin{array}{l}(\theta\alpha)^2+2Dh\left[1-\frac{h\beta_0}{2D\pi_0}\sigma L^{1/2}\Psi(k)\right]\times\\ \left[\pi_0\left(1-\frac{\beta_0}{4}\right)\sigma L^{1/2}\Psi(k)+a_i(L_{i-1}-L)+\sum_{j=1}^{i-1}a_j(U_j-u_j)\right]\end{array}}}{h\left[1-\frac{h\beta_0}{2D\pi_0}\sigma L^{1/2}\Psi(k)\right]} \quad (11)$$

Setting Eq. 6 to zero and substituting Eq. 9 into 6 to solve for k, then:

$$\Phi(k)=1-\frac{h}{\left[h\left(1-\frac{\beta_0 hQ}{2\pi_0 D}-\frac{\beta_0}{2}\right)+\frac{D}{Q}\left(\frac{\beta_0}{4\pi_0}\left(\frac{hQ}{D}\right)^2+\pi_0\left(1-\frac{\beta_0}{4}\right)\right)\right]} \quad (12)$$

It can be verified from Eq. 11 that $hQ/D\leq\pi_0$ so that, the relationship $\Phi(k)\geq 0.5$ holds for nonnegative safety factor k. Therefore, the value of π_x derived in Eq. 9 will automatically satisfy the requirement that it is between 0 and π_0 in assumption Eq. 6.

For fixed values of Q, π_x, k and A, EAC(Q, π_x, k, A, L) is concave in $L\in(L_i, L_{i-1})$, since:

$$\frac{\partial EAC(Q,\pi_x,k,A,L)}{\partial L^2}=-\frac{1}{4}hk\sigma L^{-\frac{3}{2}}-\frac{1}{4}\left[h\left(1-\frac{\beta_0}{\pi_0}\pi_x\right)+\frac{D}{Q}\left(\frac{\beta_0}{\pi_0}\pi_x^2+\pi_0-\beta_0\pi_x\right)\right]\sigma L^{-\frac{3}{2}}\Psi(k)<0 \quad (13)$$

For fixed $L\in(L_i, L_{i-1})$, the values of π_x, A, Q and k can be obtained from Eq. 9, 10, 11 and 12 theoretically. Denote these values by Q^*, π_x^*, k^* and A^*. In addition, for fixed $L\in(L_i, L_{i-1})$, the determinant of Hessian matrix for EAC(Q, π_x, k, A, L) is positive definite at (Q^*, π_x^*, k^*, A^*) as shown in Appendix.

The following algorithm can be used to find the optimal values of the order quantity, backorder discount, reorder point, ordering cost and lead time.

Step 1: For i = 0, 1, 2, ..., n.

- Set k_{io} = 0 (implies $Ø(k_{io})$ = 0.39894)
- Substitute $Ø(k_{io})$ into Eq. 11 to evaluate Q_{io}
- Use Q_{io} to determine $\Phi(k_{in})$ from Eq. 12, then find kin from $\Phi(k_{in})$ by checking the normal table. Let $k_{io} = k_{in}$
- Repeat (ii) and (iii) until no change occurs in the values of Q_i and k_i. Denote these resulting solutions by Q_i and k_i

Step 2: Use Q_i and Eq. 10 to compute the A_i and Compare A_i and A_0.

- If $A_i<A_0$, then the solution found in step 1 is optimal for given L_i. Denote the solution by . Q_i, k_i. and A_i
- If $A_i\geq A_0$, then for this given L_i take $A_i = A_0$ and the corresponding Q_i can be obtained by substituting A_i into Eq. 10 and then solving Eq. 11 and 12 iteratively until convergence (the solution procedure is similar to that given in step 1)

Step 3: Use Q_i and Eq. 9 to compute the backorder price discount π_{xi}.

Step 4: Use Eq. 3 to compute the expected total annual cost EAC(Q_i, π_{xi}, k_i, A_i, L_i).

Step 5: Set EAC(Q^*, π_x^*, k^*, A^*, L^*)= Min{ EAC(Q_i, π_{xi}, k_i, A_i, L_i), i = 0, 1, 2, ..., n }.

Step 6: (Q^*, π_x^*, k^*, A^*, L^*) is a set of optimal solutions.

Numerical example: Suppose, an item has the following characteristics: A_0 = \$200 per order, D = 600 units year^{-1}, h = \$20 per unit per year, π_0 = \$150 per unit, σ = 7 units/week. For ordering cost reduction, take θ = 0.1 per dollar per year and α = 5,800. Assume that the lead time demand follows a normal distribution. Apply the proposed algorithm to solve the problem for the upper bound of the backorder ratio β_0 = 0.95, 0.80, 0.65, 0.50, 0.35 and 0.20 and the lead time has three components as shown in Table 1 (Pan and Hsiao, 2001).

Apply the proposed algorithm to solve the problem for the upper bound of the backorder ratio β_0 = 0.95, 0.80, 0.65, 0.50, 0.35 and 0.20, respectively. The resulting optimal solutions are summarized in Table 2.

For example β_0 = 0.95, the optimal order quantity (Q^* = 84), the optimal backorder price discount ($ð_x^*$=76.41), the optimal safety factor (k^* =1.97), the optimal ordering cost (A^* = 70.4), the optimal lead time (L^* = 4) and the corresponding total cost (EAC(Q^*, π_x^*, k^*, L^*) = 2760.94). It is interesting to observe that the upper bound of backorder ratio β_0 increases, the optimal lead time, the optimal backorder price discount and the optimal order quantity remain unchanged. As the upper bound of backorder ratio β_0 increases, the cost of lost sales becomes smaller and the motivation for reorder point diminishes. As the upper bound of backorder ratio β_0 increases, the ordering cost is allowed higher.

Table 3 shows the optimal solutions with the standard deviation equals = 2, 4, 8, 20 for β_0 = 0.5 respectively. If the demand standard deviation increases while all the other parameters remain unchanged, the expected cash cost tends to increase as illustrated in Table 3. Therefore, the lead time should be reduced. The

Table 1: Lead time data of the examples

Lead time component i	1	2	3
Normal duration T_i (days)	20.0	20.0	16.0
Minimum duration t_i (days)	6.0	6.0	9.0
Unit fixed crashing cost a_i (\$ day^{-1})	0.4	1.2	5.0

Table 2: The solution result of β_0 = 0.95, 0.80, 0.65, 0.50, 0.35 and 0.20 (L_i, L in weeks)

β_0	i	0	1	2	3
	L_i	8	6	4	3
0.95	Q	72	75	84	101
	π_x	76.21	76.41	76.41	76.69
	k	2.04	2.02	1.97	1.89
	A	70.04	72.55	81.53	97.86
	EAC(Q, π_x, k, A, L)	2865.43	2783.74	*2760.94	2900.08
0.80	Q	72	75	84	101
	π_x	76.21	76.40	76.40	76.69
	k	2.06	2.04	1.99	1.92
	A	69.94	72.46	81.47	97.80
	EAC(Q, π_x, k, A, L)	2872.57	2789.95	*2766.11	2904.69
0.65	Q	72	75	84	101
	π_x	76.20	76.40	76.40	76.69
	k	2.08	2.06	2.01	1.94
	A	69.85	72.39	81.41	97.75
	EAC(Q, π_x, k, A, L)	2879.33	2795.84	*2771.00	2909.04
0.50	Q	72	75	84	101
	π_x	76.20	76.40	76.40	76.68
	k	2.10	2.08	2.03	1.96
	A	69.76	72.31	81.35	97.70
	EAC(Q, π_x, k, A, L)	2885.76	2801.43	*2775.64	2913.18
0.35	Q	72	75	84	101
	π_x	76.20	76.40	76.40	76.68
	k	2.11	2.10	2.05	1.98
	A	69.68	72.24	81.30	97.66
	EAC(Q, π_x, k, A, L)	2891.87	2806.76	*2780.06	2917.11
0.20	Q	72	75	84	101
	π_x	76.20	76.40	76.40	76.68
	k	2.13	2.12	2.07	1.99
	A	69.60	72.18	81.25	97.62
	EAC(Q, π_x, k, A, L)	2897.71	2811.84	*2784.28	2920.86

*An optimal expected annual cost.

Table 3: The optimal solution result of σ =2, 4, 8 and 20 for β_0 = 0.5 in example (L_i, L in weeks)

σ	2	4	8	20
L_i	8.00	6.00	4.00	3.00
Q	75.00	70.00	85.00	114.00
π_x	76.03	76.34	76.42	76.90
k	2.16	2.11	2.03	1.90
A	59.92	67.47	82.55	110.41
EAC(Q, π_x, k, A, L)	2183.11	2440.30	2871.63	3954.37

Table 4: Summary of the results for example (L_i in weeks)

	The proposed model						Lo *et al.* (2008) (A = 200)					
β_0	Q^*	π_x^*	k^*	A^*	L_i^*	EAC(•)(1)	Q^*	π_x^*	k^*	L_i^*	EAC(•)(2)	Saving cost EAC(•)(2)-EAC(•)(1)
0.95	84	76.41	1.97	81.53	4	2760.94	121	77.018	1.82	4	2932.15	171.21
0.80	84	76.40	1.99	81.47	4	2766.11	121	77.0171	1.84	4	2937.62	171.51
0.65	84	76.40	2.01	81.41	4	2771.00	121	77.0164	1.86	4	2942.81	171.81
0.50	84	76.40	2.03	81.35	4	2775.64	121	77.0157	1.88	4	2947.72	172.08
0.35	84	76.40	2.05	81.30	4	2780.06	121	77.0150	1.90	4	2952.40	172.34
0.20	84	76.40	2.07	81.25	4	2784.28	121	77.0144	1.92	4	2956.85	172.57

data also show that lead time tends to be shortened as the standard deviation increases for a given β_0.

Compare the proposed model with Lo *et al*. (2008) model, the resulting optimal solutions are summarized in Table 4. Also, included in Table 4 are the results obtained from the associated model by setting A fixed at 200, along with the corresponding saving on the total expected annual cost of the proposed model over that of Lo *et al.* (2008) model. It is interesting to observe that the saving increases as β_0 decreases.

CONCLUSIONS

Lead time reduction has been one of the favorite topics for both investigators and managers. Under probabilistic demand, inventory shortage is inevitable. In order to make up for the incommoding and even the losses of royal and uncomplaining customers, the supplier may offer a backorder price discount to secure orders during the shortage period. This study proposed that tripartite the lead time, the backorder price discount and ordering cost to be negotiable and the lead time crashing cost to be represented as a function of among the order quantity, safety factor, ordering cost and lead time. In the study, the inventory model studied normally distributed demand with the objective to simultaneously optimizing the order quantity, lead time, backorder price discount, ordering cost and reorder point. Numerical results show that as the upper bound of the backorder ratio β_0 increases, the order quantity, lead time, backorder price discount stays almost fixed and the expected average inventory cost tends to decreases when all the other parameters remain unaltered. If the standard deviation increases, the lead time tends to be shortened. When shortage occurs, the backorder discount offered to the customers is the sum of half of the unit marginal profit and the unit holding cost of the item during a replenishment cycle.

APPENDIX

The Hessian matrix H of EAC (Q, π_x, k, A, L) for a given value of L can be shown as:

$$H=\begin{bmatrix} \frac{\partial^2 EAC(\bullet)}{\partial Q^2} & \frac{\partial^2 EAC(\bullet)}{\partial Q\partial \pi_x} & \frac{\partial^2 EAC(\bullet)}{\partial Q\partial k} & \frac{\partial^2 EAC(\bullet)}{\partial Q\partial A} \\ \frac{\partial^2 EAC(\bullet)}{\partial \pi_x\partial Q} & \frac{\partial^2 EAC(\bullet)}{\partial \pi_x^2} & \frac{\partial^2 EAC(\bullet)}{\partial \pi_x\partial k} & \frac{\partial^2 EAC(\bullet)}{\partial \pi_x\partial A} \\ \frac{\partial^2 EAC(\bullet)}{\partial k\partial Q} & \frac{\partial^2 EAC(\bullet)}{\partial k\partial \pi_x} & \frac{\partial^2 EAC(\bullet)}{\partial k^2} & \frac{\partial^2 EAC(\bullet)}{\partial k\partial A} \\ \frac{\partial^2 EAC(\bullet)}{\partial A\partial Q} & \frac{\partial^2 EAC(\bullet)}{\partial A\partial \pi_x} & \frac{\partial^2 EAC(\bullet)}{\partial A\partial k} & \frac{\partial^2 EAC(\bullet)}{\partial A^2} \end{bmatrix} \quad (14)$$

Where,

$$EAC(\bullet) \equiv EAC(Q,\pi_x,k,A,L)$$

$$\frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial Q^2} = \frac{2AD}{Q^3} + \frac{2D}{Q^3}\left[\frac{\beta_0}{\pi_0}\pi_x^2 + \pi_0 - \beta_0\pi_x\right] \sigma\sqrt{L}\Psi(k) + \frac{2D}{Q^3}\left[a_i\left(L_{i-1}-L\right) + \sum_{j=1}^{i-1} a_j\left(U_j - u_j\right)\right] \quad (15)$$

$$\frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial \pi_x^2} = \frac{2D\beta_0}{Q\pi_0}\sigma\sqrt{L}\Psi(k) \quad (16)$$

$$\frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial k^2} = \left[h\left(1-\frac{\beta_0}{\pi_0}\pi_x\right) + \frac{D}{Q}\left(\frac{\beta_0}{\pi_0}\pi_x^2 + \pi_0 - \beta_0\pi_x\right)\right]\sigma\sqrt{L}\phi(k) \quad (17)$$

$$\frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial A^2} = \frac{\theta\alpha}{A^2} \quad (18)$$

$$\frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial Q\ \partial \pi_x} = \frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial \pi_x\partial Q} = -\frac{D}{Q^2}\left[\frac{2\beta_0}{\pi_0}\pi_x - \beta_0\right]\sigma\sqrt{L}\Psi(k) \quad (19)$$

$$\frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial k\ \partial \pi_x} = \frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial \pi_x\partial k} = \left[h\frac{\beta_0}{\pi_0} - \frac{D}{Q}\left(\frac{2\beta_0}{\pi_0}\pi_x - \beta_0\right)\right]\sigma\sqrt{L}(1-\Phi(k)) \quad (20)$$

$$\frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial Q\partial k} = \frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial k\partial Q} = \frac{D}{Q^2}\left[\frac{\beta_0}{\pi_0}\pi_x^2 + \pi_0 - \beta_0\pi_x\right]\sigma\sqrt{L}(1-\Phi(k)) \quad (21)$$

$$\frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial Q\partial A} = \frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial A\partial Q} = \frac{D}{Q^2} \quad (22)$$

$$\frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial k\partial A} = \frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial A\partial k} = 0 \quad (23)$$

and

$$\frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial A\partial \pi_x} = \frac{\partial^2 EAC(Q,\pi_x,k,A,L)}{\partial \pi_x\partial A} = 0 \quad (24)$$

Next, we'll evaluate the principal minor of H at point (Q^*, π^*, k^*, A^*). The first principal minor of H is:

$$|H_{11}| = \frac{2A^*D}{Q^{*3}} + \frac{2D}{Q^{*3}}\left[\frac{\beta_0}{\pi_0}\pi_x^{*2} + \pi_0 - \beta_0\pi_x^*\right] \sigma\sqrt{L}\Psi(k^*) + \frac{2D}{Q^{*3}}\left[a_i\left(L_{i-1}-L\right) + \sum_{j=1}^{i-1} a_j\left(U_j - u_j\right)\right] > 0 \quad (25)$$

Since, from Eq. 9 that $\pi_x^* = hQ^*/2D + \pi_0/2$, the second principal minor of H is:

$$|H_{22}| = \frac{4\beta_0 D^2\sigma L^{1/2}\Psi(k^*)}{Q^{*4}\pi_0} \times \left\{A^* + \left[a_i(L_{i-1}-L) + \sum_{j=1}^{i-1} a_j(U_j - u_j)\right]\right\} + \frac{\beta_0 D^2\sigma^2 L\Psi^2(k^*)}{Q^{*4}}(4-\beta_0) > 0 \quad (26)$$

Consequently, after substituting π_x^* from Eq. 9 and $\Phi(k^*)$ from Eq. 12, the third principal minor of H is:

$$|H_{33}| = \left\{\frac{4\beta_0 D^2\sigma L^{1/2}\Psi(k^*)}{Q^{*4}\pi_0}\left\{A^* + \left[a_i(L_{i-1}-L) + \sum_{j=1}^{i-1} a_j(U_j - u_j)\right]\right\} + \frac{\beta_0 D^2\sigma^2 L\Psi^2(k^*)}{Q^{*4}}(4-\beta_0)\right\}$$
$$\times\left[h\left(1-\frac{\beta_0}{\pi_0}\pi_x^*\right) + \frac{D}{Q^*}\left(\frac{\beta_0}{\pi_0}\pi_x^{*2} + \pi_0 - \beta_0\pi_x^*\right)\right]\sigma\sqrt{L}\phi(k^*)$$
$$-\left[\frac{D}{Q^{*2}}\left[\frac{\beta_0}{\pi_0}\pi_x^{*2} + \pi_0 - \beta_0\pi_x^*\right]\sigma\sqrt{L}P_Z(k)\right]^2 \times \frac{2D\beta_0}{Q^*\pi_0}\sigma\sqrt{L}\Psi(k^*)$$
$$= \frac{4\beta_0 D^2\sigma L^{1/2}\Psi(k^*)}{Q^{*4}\pi_0}\left\{A + \left[a_i(L_{i-1}-L) + \sum_{j=1}^{i-1} a_j(U_j - u_j)\right]\right\}$$

$$\times\left[h\left(1-\frac{\beta_0}{\pi_0}\pi_x^*\right)+\frac{D}{Q^*}\left(\frac{\beta_0}{\pi_0}\pi_x^{*2}+\pi_0-\beta_0\pi_x^*\right)\right]\sigma\sqrt{L}\phi(k^*)$$
$$+\frac{\beta_0 D^2\sigma^2 L\Psi^2(k^*)}{Q^{*4}}(4-\beta_0)\times h\left(1-\frac{\beta_0}{\pi_0}\pi_x^*\right)\times\sigma\sqrt{L}\phi(k^*)$$

$$+\frac{D^3\sigma^2 L}{Q^{*5}}\sigma\sqrt{L}\Psi(k^*)\times\left[\frac{\beta_0\pi_0}{4}\left(\frac{hQ^*}{\pi_0 D}\right)^2+\pi_0\left(1-\frac{\beta_0}{4}\right)\right]\times F(k^*) \quad (27)$$

Where, $F(k^*) = (4-\beta_0)\phi(k^*)\Psi(k^*)-2\beta_0(1-\Phi(k^*))^2$

$$\times\left[\frac{1}{4\beta_0}\times\left[2-\frac{2}{1-\Phi(k^*)}-\beta_0+\sqrt{\left(2-\frac{2}{1-\Phi(k^*)}-\beta_0\right)^2+\beta_0(4-\beta_0)}\right]^2+\left(1-\frac{\beta_0}{4}\right)\right]$$

For $\forall\ k^* \in [0, \infty)$ and $0 < \beta_0$ £1, $F(k^*)$ is positive. Hence, we have $|H_{33}| > 0$.

Consequently, after substituting π_x^* from Eq. 9 and A^* from Eq. 10, the fourth principal minor of H is:

$$|H_{44}| = \frac{2\beta_0 D^3\sigma L^{1/2}\Psi(k^*)}{AQ^{*5}\pi_0}\left\{A+2\left[a_i(L_{i-1}-L)+\sum_{j=1}^{i-1}a_j(U_j-u_j)\right]\right\}\times$$
$$\left[h\left(1-\frac{\beta_0}{\pi_0}\pi_x^*\right)+\frac{D}{Q^*}\left(\frac{\beta_0}{\pi_0}\pi_x^{*2}+\pi_0-\beta_0\pi_x^*\right)\right]\sigma\sqrt{L}\phi(k^*)$$
$$+\frac{\beta_0 D^3\sigma^2 L\Psi^2(k^*)}{A^*Q^{*5}}(4-\beta_0)\times h\left(1-\frac{\beta_0}{\pi_0}\pi_x^*\right)\times\sigma\sqrt{L}\phi(k^*)$$
$$+\frac{D^4\sigma^2 L}{A^*Q^{*6}}\sigma\sqrt{L}\Psi(k^*)\times\left[\frac{\beta_0\pi_0}{4}\left(\frac{hQ^*}{\pi_0 D}\right)^2+\pi_0\left(1-\frac{\beta_0}{4}\right)\right]\times F(k^*)$$

$$+\frac{D^2}{Q^4}\left[h\frac{\beta_0}{\pi_0}-\frac{D}{Q}\left(\frac{2\beta_0}{\pi_0}\pi_x-\beta_0\right)\right]^2\sigma^2 L(1-\Phi(k))^2 > 0 \quad (28)$$

Therefore, from Eq. 25-28, it follows that the Hessian matrix H is positive define at (Q^*, π_x^*, k^*, A^*).

REFERENCES

Ben-Daya, M. and A. Raouf, 1994. Inventory models involving lead time as a decision variable. J. Operat. Res. Soc., 45: 579-582.

Billington, P.J., 1987. The classic economic production quantity model with setup cost as a function of capital expenditure. Dec. Sci., 18: 25-42.

Hariga, M.A., 1999. Stochastic inventory model with lead-time lot size interaction. Product. Plann Cont., 10: 434-438.

Hariga, M.A. and M. Ben-Daya, 1999. Some stochastic inventory models with deterministic variable lead time. Eur. J. Operat. Res., 113: 42-51.

Hariga, M.A., 2000. Setup cost reduction in (Q, r) policy with lot size, setup time and lead-time interactions. J. Opera. Res. Soc., 51: 1340-1345.

Hong, J.D. and J.C. Hayya, 1993. ynamic lot sizing with setup reduction. Comput. Ind. Eng., 24: 209-218.

Ijioui, R., H. Emmerich, F. Gerecke and R. Prieler, 2006. Examining the dynamic behaviour of an aeronautical specialized supply chain with multiple order priorities. Inform. Technol. J., 5: 70-73.

Kim, J. and W. Benton, 1995. Lot size dependent lead times in a (Q, r) inventory system. Int. J. Product. Res., 33: 41-48.

Liao, C.J. and C.H. Shyu, 1991. Analytical determination of lead time with normal demand. Int. J. Operat. Res. Prod. Manage., 11: 72-78.

Lo, M.C., 2007. Decision support system for the integrated inventory model with general distribution demand. Inform. Technol. J., 6: 1069-1074.

Lo, M.C., C.H.P. Jason, L. Kai-Cing Lin and H. Jia-Wei, 2008. Impact of lead time and safety factor in mixed inventory models with backorder discounts. J. Applied Sci., 8: 528-533.

Moon, I. and S. Choi, 1997. Distribution free procedures for make-to-order (MTO), make-in-advance (MIA) and composite policies. Int. J. Prod. Econ., 48: 21-28.

Moon, I. and S. Choi, 1998. A note on lead time and distribution assumptions in continuous review inventory models. Comput. Operat. Res., 25: 1007-1012.

Ouyang, L.Y., C.K. Chen and H.C. Chang, 1999. Lead time and ordering cost reductions in continuous review inventory systems with partial backorders. J. Operat. Res. Soc., 50: 1272-1279.

Pan, J.C.H. and Y.C. Hsiao, 2001. Inventory models with back-order discounts and variable lead time. Int. J. Syst. Sci., 32: 925-929.

Pan, J.C.H., M.C. Lo and Y.C. Hsiao, 2004. Optimal reorder point inventory models with variable lead time and backorder discount considerations. Eur. J. Operat. Res., 158: 488-505.

Porteus, E.L., 1985. Investing in reduced setups in the EOQ model. Manage. Sci., 31: 998-1010.

Ravindran, A., D.T. Phillips and J.J. Solberg, 1987. Operations Research. Principle and Practices. 1st Edn., John Wiley, New York.

Trevino, J., B.J. Hurley and W. Friedrich, 1993. A mathematical model for the economic justification of setup time reduction. Int. J. Prod. Res., 31: 191-202.