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Reasoning about the Inverse of Cardinal Direction Relation

[1]Miao Wang and [1,2]Zhongxiao Hao
[1]Department of Computer Science and Technology,
Harbin University of Science and Technology, Harbin 150080, China
[2]Department of Computer Science and Engineering, Harbin Institute of Technology, Harbin 150001, China

Abstract: Reasoning about the inverse of cardinal direction relation is a fundamental problem in qualitative spatial reasoning and plays an important role in reasoning with cardinal direction relation and consistency checking. In order to fullfill reasoning about the inverse of the basic cardinal direction relations between regions themselves defined in the model of Goyal, after deeply researching on reasoning about the inverse of cardinal direction relation in MBR model, we propose the concept of original relation of rectangular cardinal direction relation, propose an algorithm to computer it and then develop an algorithm to reason about the inverse of basic cardinal direction relation between regions defined in the model of Goyal. The results of both analyzing in theory and verifying through comparing the result of the algorithm with the actual situation for each of basic cardinal direction relations demonstrate that the algorithm is correct and complete.

Key words: Inverse of cardinal direction relation, qualitative spatial reasoning, reasoning with cardinal direction, consistency checking, MBR model

INTRODUCTION

Research in the field of Qualitative spatial reasoning is very active from more than a decade. Qualitative spatial reasoning has received a lot of attention in the areas of Geographic Information Systems (Papadias and Egenhofer, 1997), Artificial Intelligence (Gerevini and Renz, 2002; Liu *et al*., 2008), Databases (Skiadopoulos and Koubarakis, 2001; Hao and Li, 2009) and Multimedia (Huang *et al*., 2008; Punitha and Guru, 2006). Several kinds of useful spatial relations have been studied so far, e.g., topological relations (Gerevini and Renz, 2002; Schneider and Behr, 2006), cardinal direction relations (Frank, 1996; Skiadopoulos and Koubarakis, 2001, 2004) and qualitative distance relations (Clementini *et al*., 1997; Frank, 1992).

The present study, concentrates on reasoning about the inverse of cardinal direction relations which allows the inference of the cardinal direction relation of b with respect to a, given the cardinal direction relation of object a with respect to object b. It is a fundamental problem in qualitative spatial reasoning and plays an important role in reasoning and consistence checking with cardinal direction relations. In the last few years several research contributions have appeared on the topic. Frank (1996) introduced a system for reasoning about the inverse of cardinal direction relations between points from an algebraic point of view. Papadias and Egenhofer (1997) applied the inverse operation for cardinal direction relations between points in hierarchical reasoning about direction relations, but do not address the problem of reasoning about the inverse of cardinal direction relation between regions. Liu and Hao (2006) use Interval Algebra and Rectangle Algebra to reason about the inverse of basic cardinal direction relations defined in MBR model. Wang *et al.* (2008) provide a method to reason with the inverse of basic cardinal direction relations defined in MBR model on the basis of reasoning about the inverse of direction relation between points. From the previous study presented above, regions are modeled either in terms of their Minimum Bounding Rectangle (MBR) or as a point in the previous work, to a certain extent, which reduces the complexity of reasoning, but sharply lowers the precision of representation and reasoning at the same time.

To the best of our knowledge, there has not been a method to reason about the inverse of cardinal direction relation between general regions. In this study, we take effort to solve this problem. Until now, the model presented by Goyal and Egenhofer (2001) is currently one of the most expressive models for qualitative representation and reasoning with cardinal directions because it is formally defined and can be applied to a wide set of regions (e.g., disconnected regions and regions with holes). After deeply studying reasoning about the inverse of the cardinal direction relations between rectangles, we employ the aforementioned model and then propose an algorithm to reason about the inverse of the

Corresponding Author: Miao Wang, Department of Computer Science and Technology,
Harbin University of Science and Technology, Harbin 150080, China

basic cardinal direction relations between regions. The results of analyzing in theory demonstrate that this algorithm is correct and complete which is also confirmed by comparing the result of this algorithm with the actual situation for any basic cardinal direction relation defined in the aforementioned model.

The major contribution of this study is that an algorithm is developed to reason about the inverse of basic cardinal direction relation between general regions. Compared with the previous methods, our algorithm can be applied to a wide set of objects (not restricted to two particular classes of objects, points and rectangles) and improves the precision of reasoning. Therefore, our algorithm better meets the demand of practical application of spatial reasoning.

A FORMAL MODEL FOR CARDINAL DIRECTION RELATION

The model presented in Goyal and Egenhofer (2001) and Skiadopoulos and Koubarakis (2004) is currently one of the most expressive models for qualitative representation and reasoning with cardinal directions. Now, we show this model by means of related definitions. Throughout this study, we will consider regions that are closed, connected and have connected boundaries. The set of these regions will be denoted by REG.

Definition 1: Let $a \in REG$. The greatest lower bound of the projection of a on the x-axis (respectively y-axis) is denoted by $inf_x(a)$ (respectively $inf_y(a)$). The least upper bound or the supremum of the projection of a on the x-axis (respectively y-axis) is denoted by $sup_x(a)$ (respectively $sup_y(a)$). The minimum bounding rectangle of a region a, denoted by mbr(a), is the rectangle formed by the straight lines $x=inf_x(a)$, $x=sup_x(a)$, $y=inf_y(a)$ and $y=sup_y(a)$.

Definition 2: Let $a \in REG$ and a be reference object. The straight lines $x=inf_x(a)$, $x=sup_x(a)$, $y=inf_y(a)$ and $y=sup_y(a)$ forming the minimum bounding rectangle of the reference region a divide the space into 9 areas which we call tiles of a (Fig. 1). These tiles will be denoted by NW(a), N(a), NE(a), W(a), B(a), E(a), SW(a), S(a) and SE(a), respectively, where $NW(a) = \{(x,y)| -\infty < x \le inf_x(a)\ sup_y(a) \le y < +\infty\}$, $N(a)=\{(x,y)|\ inf_x(a) \le x \le sup_x(a)\ \le sup_y(a) \le y+\infty\}$, $NE(a)=\{(x,y)|\ sup_x(a) \le x < +\infty \wedge sup_y(a)\ \le y < +\infty\}$, $W(a)=\{(x, y)| -\infty < x \le inf_x(a) \wedge inf_y(a) \le y \le sup_y(a)\}$, $B(a)=\{(x,y)| inf_x(a) \le x \le sup_x(a) \wedge inf_y(a) \le y \le sup_y(a))\}$, $E(a)=\{(x,y)| sup_x(a) \le x < +\infty \wedge inf_y(a) \le y \le sup_y(a)\}$, $SW(a)=\{(x,y)| -\infty < x \le inf_x(a) \wedge -\infty < y \le inf_y(a)\}$, $S(a)=\{(x,y)|=inf_x(a) \le x \le sup_x(a) \wedge -\infty < y \le inf_y(a)\}$, $SE(a)= \{(x, y)|\ sup_x(a) \le x < +\infty \wedge -\infty < y \le\ inf_y(a)\}$.

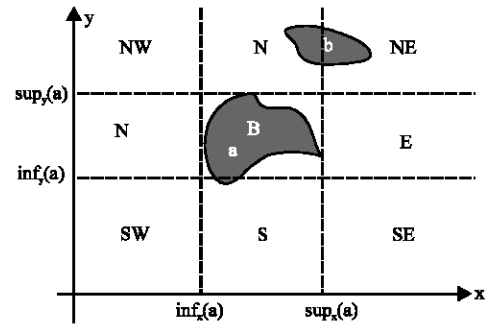

Fig. 1: Space division

Definition 3: Let $a, b \in REG$ where a is the reference region and b is the primary region. If b is included in tile NW(a) of a then we say that b is northwest of a and we write b NW a. Similarly, we can define North (N), Northeast (NE), West (W), bounding rectangle (B), Southwest (SW), East (E) and Southeast (SE) relations.

Definition 4: A basic cardinal direction relation is an expression R_1: ... :R_k where $R_1, \ldots, R_k \in \{B, S, SW, W, NW, N, NE, E, SE\}$, $1 \le k \le 9$ and $R_i \ne R_j$ for every i, j such that $1 \le i, j \le k$ and $i \ne j$ and there exist regions $a_1, \ldots, a_k \in REG$ such that $a_1 \in R_1(b), \ldots, a_k \in R_k(b)$ and $a_1 \cup \cdots \cup a_k \in REG$ for any reference region $b \in REG$. A cardinal direction relation $R_1:\cdots:R_k$ is called single-tile if k = 1, otherwise it is called multi-tile. The set of basic cardinal direction relations in this model that contains 218 elements is denoted by D. For instance, b lies partly in the tile N(a) and partly in the tile NE(a) of a (Fig. 1) then, a N:NE b and we say that b is partly North and partly Northeast of a.

Using the 218 relations of D as our basis, we can define the powerset 2^D of D which contains 2^{218} relations. Elements of 2^D are called cardinal direction relations and can be used to represent not only definite but also indefinite information about cardinal directions, e.g., a {N, S} b denotes that region a is north or south of region b.

Definition 5: Let $R \in 2^D$. The inverse of relation R, denoted by inv(R), is another cardinal direction relation which satisfies the following. For arbitrary regions $a, b \in$ REG, a inv(R) b holds, iff b R a holds.

Definition 6: A basic cardinal direction relation R is called rectangular iff there exist two rectangles (with sides parallel to the x- and y-axes) a and b such that a R b is satisfied; otherwise it is called non-rectangular. The set of these relations that contains 36 elements is denoted by D_{rec}.

Definition 7: Let $R_1 = R_{11}$: ...:R_{1k} and $R_2 = R_{21}$:...:R_{2l} be two cardinal direction relations. R_1 includes R_2 iff $\{R_{21}, \ldots, R_{2l}\} \subseteq \{R_{11}, \ldots, R_{1k}\}$ holds.

Definition 8: Let R be a basic cardinal direction relation. The bounding relation of R, denoted by Br(R) is the smallest rectangular relation (with respect to the number of tiles) that includes R.

REASONING ABOUT THE INVERSE OF DIRECTION RELATION BETWEEN RECTANGLES

MBR model (Skiadopoulos and Koubarakis, 2001) is essentially that the actual target object is taken into account instead of approximating it with its minimum bounding rectangle in the model presented by Goyal and Egenhofer (2001). The set of basic cardinal direction relations in MBR model contains 36 elements. Liu and Hao (2006) researched on reasoning about the inverse of basic cardinal direction relations in MBR model by means of Interval Algebra and Rectangle Algebra. Allen's Interval Algebra (Allen, 1983) models the relative position between any two intervals as a set of thirteen basic relations, namely before, meets, overlaps, starts, during, finishes (b, m, o, s, d, f) together with theirs inverses (bi, mi, oi, si, di, fi) and the basic relation equal(eq).The set {p, m, o, s, d, f, pi, mi, oi, si, di, fi, eq} is denoted by A_{int}. Similarly, the domain considered in the rectangle algebra introduced by Balbiani et al. (1998) is the set of rectangles. A basic relation between two rectangles is a pair (R_x, R_y) of basic IA-relations: the x-axis relation and the y-axis relation. The set of basic RA-relations that contains 169 elements is denoted by A_{rec}.

Liu and Hao (2006) reveal that there exists a natural connection between basic relations in MBR model and basic RA-relations. This connection is shown in Table 1 where any basic relation in MBR model corresponds to a Cartesian product (×) of the power set of the interval relations.

Theorem 1: Let R, $S \in 2^{Aint}$. The inverse operation for RA-relations (IA-relations) is denoted by -1. Then $(R\times S)^{-1}=R^{-1}\times S^{-1}$ (Balbiani *et al*., 1998).

According to Theorem1 and the correspondence between a basic cardinal direction relation in MBR model and a basic relation in rectangle algebra, the algorithm for reasoning about the inverse of basic cardinal direction relations in MBR model is shown as follows:

```
Algorithm: Invmbr (R)
  input: a basic cardinal direction relation R in MBR model
  output: the inverse relation of R
  begin
    get the RA-relation S×T (S,T∈{{p,m},{o,fi},{di},{eq,s,d, f},{si,oi},
    {pi,mi}}) corresponding to R from Table 1;
    R*=ø;
    for each s∈S
          for each t∈T
          {get a basic cardinal direction relations R* in MBR model
             corresponding to (s⁻¹, t⁻¹) from Table 1;
             R0=:R0∪R*; }
    return R0
  end
```

REASONING ABOUT THE INVERSE OF DIRECTION RELATION BETWEEN REGIONS

This study proposes an algorithm to reason about the inverse relation of basic cardinal direction relation defined in the model presented by Goyal and Egenhofer (2001). The main idea of this algorithm is reasoning about the inverse of cardinal direction relation between the primary object and the reference object from the inverse of cardinal direction between the minimum bounding rectangle of the primary object and the minimum bounding rectangle of the reference object. We first give the related definitions and theorems, before the algorithm is presented.

Definition 9: Let $R\in D_{rec}$, $r\in D$. If Br(r)=R holds, then we say that r is the original relation of R. The set of the original relations of R is denoted by ORG(R).

Next, we will focus on computing ORG(R) of a basic rectangular cardinal direction relation R. We first introduce many symbols as fallows. A rectangle is denoted by rec(p, q) where p and q are the endpoints of a diagonal of the rectangle. The number of single tiles included in a basic cardinal direction relation R is denoted by R_{num}.

Theorem 2: Let $R\in D_{rec}$. If $R_{num}=1$ or $R_{num}=2$ or $R_{num}=3$, then ORG(R) ={R}.

Proof: If $R_{num}=1$, then R is a single-tile. Then there exists only one r (r=R) such that Br(r) =R holds. Therefore ORG(R) ={R} holds when $R_{num}=1$.

If $R_{num}=2$, assume that R is $r_1:r_2$ where r_1, r_2 are single-tile cardinal direction relations, then $Br(r_1)=r_1\neq R$,

Table 1: The mapping between basic relations in MBR model and basic RA-relations

Y×X	{p,m}	{o, fi}	{di}	{eq,s,d, f}	{si,oi}	{pi,mi}
{p,m}	SW	SW:S	SW:S:SE	S	S:SE	SE
{o, fi}	W:SW	W:B:SW:S	W:B:E:SW:S:SE	B:S	B:E:S:SE	E:SE
{di}	NW:W:SW	NW:N:W:B:SW:S	all	N:B:W	N:NE:B:E:S:SE	NE:E:SE
{eq,s,d, f}	W	W:B	W:B:E	B	B:E E	
{si,oi}	NW:W	NW:N:W:B	NW:N:NE:W:B:E	N:B	N:NE:B:E	NE:E
{pi,mi}	NW	NW:N	NW:N:NE	N	N:NE	NE

$Br(r_2)=r_2 \neq R$ and $Br(r_1:r_2)=R$ hold. According to definition 9, $r_1, r_2 \notin ORG(R)$ and $R \in ORG(R)$ hold. So $Br(r)=R$ holds iff $r=R$ holds. Therefore $ORG(R)=\{R\}$ holds when $R_{num}=2$.

If $R_{num}=3$, assume that R is $r_1{:}r_2{:}\ r_3$, similarly, we have $r_1, r_2, r_3, r_1{:}\ r_2, r_2{:}\ r_3 \notin ORG(R)$ and $R \in ORG(R)$. It is easy to see that $r_1{:}\ r_3$ is not a basic cardinal direction relation. So, there exists only one $r=R$ such that $Br(r)=R$ holds. Therefore $ORG(R)=\{R\}$ when $R_{num}=3$ and thus the theorem holds.

Theorem 3: Let $R \in D_{rec}$ and $R_{num}=4$. Assume that R is $r_1{:}\ r_2{:}\ r_3{:}\ r_4$. Then, ORG(R) can be computed using the formula:

$$ORG(R)=\{r_i{:}\ r_j{:}\ r_k | r_i, r_j, r_k \in \{r_1, r_2, r_3, r_4\}\} \cup \{R\} \quad (1)$$

Proof: If $R \in D_{rec}$ and $R_{num}=4$, then R will be in one of the following relations: W:B:SW:S, B:E:S:SE, NW:N:W:B, N:NE:B:E.

Let us first consider that R=W:B:SW:S. It is easy to see that W:S and B:SW are not basic cardinal relations and $Br(r)=R$ holds for any $r \in$ {W:B:SW, W:B:S, W:SW:S, B:SW:S, W:B:SW:S}. Then, according to Definition 9, ORG(R)={W:B:SW, W:B:S, W:SW:S, B:SW:S, W:B:SW:S} holds, that is to say, Eq. 1 holds.

The cases where, R=B:E:S:SE, R=NW:N:W:B and R= N:NE:B:E can be proved similarly and thus the theorem holds.

Theorem 4: Let $R \in D_{rec}$, $R_{num}=6$, $S \in D_{rec}$, $T \in D_{rec}$, $S_{num}=3$ and $T_{num}=3$. Assume that S is $s_1{:}\ s_2{:}\ s_3$ and T is $t_1{:}\ t_2{:}\ t_3$. Then there exists only one pair (S, T) such that $R= s_1{:}\ s_2{:}\ s_3{:}t_1{:}\ t_2{:}\ t_3$ holds. Then, the correspondence between this R and this pair(S, T) is shown in Table 2.

Proof: The theorem can be proved easily from 36 basic rectangular cardinal direction relations.

Theorem 5: Let $R \in D_{rec}$ and R_{num} = 6. Assume that there exist $S=s_1{:}\ s_2{:}\ s_3$ and $T= t_1{:}\ t_2{:}\ t_3$ where S and T are in D_{rec} such that $R= s_1{:}\ s_2{:}\ s_3{:}t_1{:}\ t_2{:}\ t_3$ holds. Then, ORG(R) can be computed using the formula:

$$ORG(R)=\{s_1{:}\ s_2{:}\ s_3{:}\ t | t \in \{t_1, t_2, t_3\}\} \cup \{s_1{:}\ s_2{:}\ s_3{:}\ t{:}\ r | t, r \in \{t_1, t_2, t_3\}\} \cup\ t_1{:}\ t_2{:}\ t_3{:}\ s | s \in \{s_1, s_2, s_3\}\} \cup \{t_1{:}\ t_2{:}\ t_3{:}\ s{:}\ r | s, r\ \{s_1, s_2, s_3\}\} \cup \{R, s_1{:}\ s_2{:}\ t_2{:}\ t_3\ , s_2{:}\ s_3{:}\ t_1{:}\ t_2\} \quad (2)$$

Proof: If $R \in D_{rec}$ and $R_{num}=6$, then R will be in one of the following relations: NW:N:NE:W:B:E, W:B:E:SW:S:SE, N:NE:B:E:S:SE, NW:N:W:B:SW:S. We can prove that Eq. 2 holds for each of these cases above similarly with Theorem 3 and thus the theorem holds.

Theorem 6: Let R be NW:N:NE:W:B:E:SW:S:SE. Then, ORG(R) can be computed using the formula:

$$ORG(R)=D - \bigcup_{r \in D_{rec}, r \neq R} ORG(r) \quad (3)$$

Proof: According to the definition of original relation (Definition 9) and the set of basic cardinal direction relations, it is easy to see that:

$$\bigcup_{r \in D_{rec}} ORG(r)=D$$

holds and if $r \neq s$ where r and s are in $D_{rec,}$ then $ORG(r) \cap ORG(s)=\varnothing$ holds. Therefore, Eq. 3 holds and thus the theorem holds.

According to the theorems above, an algorithm is proposed to compute the ORG(R) of any basic rectangular cardinal direction relation R which is shown as follows:

Algorithm: ORG (R)

```
input: A basic rectangular cardinal direction relation R
output: The set of orginal relations of R
begin
   if (R_num=1 or R_num=2 or R_num=3) then
      {R*=R; return R* ;}
   if (R_num=4) then
      {R*={r_i: r_j: r_k | r_i, r_j, r_k∈{r_1, r_2, r_3, r_4}}∪{R};/* R= r_1: r_2: r_3: r_4*/
      return R*; }
   if (R_num=6) then
      {Get S=s_1: s_2: s_3 and T= t_1: t_2: t_3 where S and T are in D_rec such
      that R= s_1: s_2: s_3:t_1: t_2: t_3 holds from Table 2;
      Assign the result obtained according to Theorem 5 to R*;
      return R*;}
   if (R_num=9) then
      {R*=D;
       for all r∈(D_rec -{R})
          R*= R* - ORG(r);
       return R*;}
 end
```

Theorem 7: Algorithm ORG( ) for computing the set of original relations of basic rectangular cardinal direction relations is correct and complete.

Proof: $\forall R \in D_{rec}$, R_{num} may be every value in {1,2,3,4,6,9}. Theorem 2-6 offer methods to compute ORG (R) and prove themselves when $R_{num}=1$ or $R_{num}=2$ or $R_{num}=3$, when $R_{num}=4$, when $R_{num}=6$ and when $R_{num}=9$, respectively. The algorithm

Table 2: The correspondence between the aforementioned relation R and pair (S, T)

R	NW:N:NE:W:B:E	W:B:E:SW:S:SE	N:NE:B:E:S:SE	NW:N:W:B:SW:S
S,T	NW:N:NE,W:B:E	W:B:E,SW:S:SE	N:B:S,NE:E:SE	NW:W:SW,N:B:S

fulfills the methods offered by these theorems. Therefore the algorithm can correctly compute ORG (R) for any R in D_{rec}. Thus the algorithm is correct and complete.

Definition 10: Let R_1 and R_2 be basic cardinal direction relation. The bounding relation of R_1 and R_2, denoted by Br (R_1, R_2) is the smallest rectangular relation (with respect to the number of tiles) that includes R_1 and R_2 simultaneously.

Theorem 8: Let a and b be regions in REG and b be a rectangle. Assume that p and q are bottom-left point and top-right point of b, respectively. If p R_1 a and q R_2 a hold, then b Br(R_1, R_2) a holds.

Proof: Since, p is bottom-left point of b and q is top-right point of b, we have the follows: if R_1=SW, then $R_2 \in$ {N,NE,E,SE,S,SW,W,NW,B}; if R_1=S, then $R_2 \in$ {N, B, S, NE,E,SE}; if R_1=SE, then $R_2 \in$ {NE,E,SE}; if R_1=W, then $R_2 \in$ {W,B,E,NW,N,NE}; if R_1= B, then $R_2 \in$ {B,E,N,NE}; if R_1=E, then $R_2 \in$ {E,NE}; if R_1=NW, then $R_2 \in$ {NW, N,NE}; if R_1=N, then $R_2 \in$ {N,NE}; if R_1=NE, then R_2 =NE.

Let us first consider the case that R_1=SW and R_2= B. Assume that rectangle b is formed by the straight lines x =l, x =r, y =d and y =t. Since, R_1=SW $\wedge R_2$= B$\wedge$ p R_1 a$\wedge$q R_2 a holds, $l<inf_x(a) \wedge inf_x(a)<r<sup_x(a) \wedge d<inf_y(a) \wedge inf_y(a) <t<sup_y(a)$ holds. Thus, rec((l,d), (inf_x(a),inf_y(a))) SWa$\wedge$ rec((inf_x(a), d), (r,inf_y(a)) S a$\wedge$rec((l, inf_y(a)), (inf_x(a), t)) Wa$\wedge$rec((inf_x(a), inf_y(a), (r,t)) Ba$\wedge$rec((l,d), (inf_x(a), inf_y(a))$\cup$ rec((inf_x(a),d), (r,inf_y(a))$\cup$rec((l,inf_y(a)), (inf_x(a),t))$\cup$rec ((inf_x (a), inf_y(a), (r,t))=b holds. Then, according to Definition 4, b SW:S:W:B a holds. Since, Br (SW, B) is equal to SW:S: W:B, b Br(SW, B) a holds, that is to say, b Br(R_1, R_2) a holds. Thus the theorem holds in the case.

All the other cases can be proved similarly and thus the theorem holds.

Theorem 9: Let a, b be regions in REG. If b R a holds where R is a basic cardinal direction relation, then mbr(b) Br(R) a holds.

Proof: Let us first consider that R=W:SW:S. Assume that mbr(b) is formed by the straight lines x =l, x =r, y =d and y=t. Since, R=W:SW:S holds, $l<inf_x(a) \wedge inf_x(a)< r<sup_x(a) \wedge d<inf_y(a) \wedge inf_y(a) <t<sup_y(a)$ holds. Therefore, for (l,d) the bottom-left point of mbr(b), (l,d) SW a holds; for (r,t) the top-right point of mbr(b), (r,t) B a holds. Then, according to Theorem 8, mbr(b) Br(SW,B) a holds, that is to say, mbr(b) SW:S:W: B a holds. Since Br(R) is SW:S:W: B, b Br(R)a holds and thus the theorem holds in the case.

Similarly, we can prove that the theorem holds for any other basic cardinal direction relations and thus the theorem holds.

Theorem 10: Let R$\in$D. Then the inverse of relation R can be computed using the formula:

$$inv(R) = \bigcup_{t\in invmbr(Br(R))} ORG(t) \quad (4)$$

Proof: $\forall r \in$inv(R), there exist a, b$\in$REG such that a r b and b R a hold. Then a r mbr(b) and b R mbr(a) hold. According to Theorem 9, mbr(a)Br(r)mbr(b) and mbr(b) Br(R) mbr(a) hold. Then, according to Definition 4, Br(r)$\in$invmbr(Br(R)) holds. Since, a r mbr(b) $\wedge$mbr(a) Br(r) mbr(b) holds, according to Definition 9, r$\in$ORG(Br(r)) holds. Therefore, $\forall r \in$inv(R),

$$r\in \bigcup_{t\in invmbr(Br(R))} ORG(t) \text{ holds}$$

Conversely, $\forall t \in$invmbr(Br(R)), there exist rectangle a and rectangle b such that a t b and b Br(R) a hold. Then a r mbr(b) and b R mbr(a) hold. $\forall r \in$ ORG(t), assume that r is r_1: ... :r_k, assume also that R is R_1: ... : R_j, let us form region b_0=b$\cap$ (R_1(a)$\cup$…$\cup R_j$(a)) and region a_0=a$\cap$(r_1(b)$\cup$…$\cup r_k$(b)), it is easy to see that b_0 R a_0 and a_0 r b_0 hold and thus, r$\in$inv(R) holds. Therefore,

$$\forall r\in \bigcup_{t\in invmbr(Br(R))} ORG(t)$$

r$\in$inv(R) holds and thus the theorem holds.

Using Algorithm invmbr( ), Algorithm ORG( ) and Theorem 10 as our basis, we propose an algorithm to reason the inverse of basic cardinal direction relations which is shown as follow.

```
Algorithm: inv (r)
  input: a basic cardinal direction relations r
  output: the inverse of r
      begin
      R=Br(r);
      S=invmbr(R); /* call algorithm invmbr( )*/
      R0=ø;
      for each s∈S
          {R*=ORG(s); /* call algorithm ORG( )*/
          R0= R0∪R* ;}
      return R0;
   end
```

Theorem 11: Algorithm inv( ) for reasoning about the inverse of basic cardinal direction relation is correct and complete.

Proof: $\forall r \in$D, there exists only one R$\in D_{rec}$ such that Br(r)=R holds. $\forall t \in$ invmbr(R), t$\in D_{rec}$ holds. Theorem 7

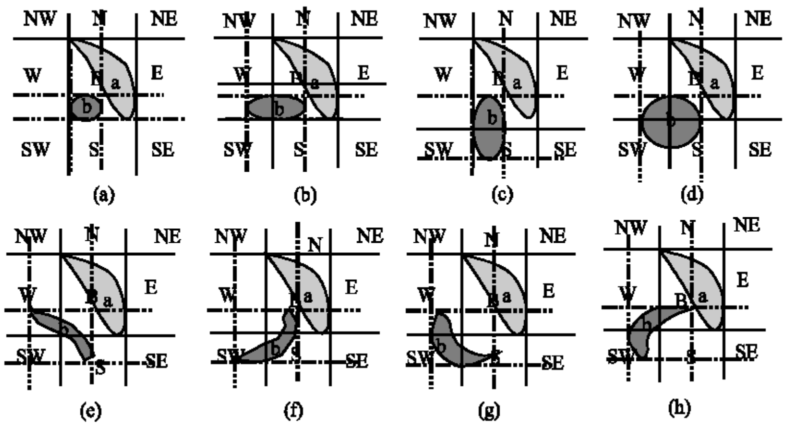

Fig. 2: All the possible spatial configurations between regions a and b. (a) bBa, (b) bW:Ba, (c) bB:Sa, (d) bB:bW:S:SW:Sa, (e) bW:B:Sa, (f) bS:S:SWa, (g) bW:SW:Sa and (h) bW:B:SWa

proves that Algorithm ORG( ) can correctly compute ORG(t) for any t in D_{rec}. Theorem 10 offers a method proved to be correct which uses Algorithm invmbr ( ) and Algorithm ORG( ) to reason about the inverse of r. Therefore the algorithm can correctly compute the inverse of r for any r in D. Thus the algorithm is correct and complete.

CONFIRMATION

The purpose of this section is to confirm that our algorithm is correct by comparing the result of our algorithm with the actual situation.

Let us first verify whether our algorithm holds or not for cardinal direction relation N:NE:E.

Let us consider two regions a and b and assume that a N:NE:E b. If a N:NE:E b, then it is possible that b B a or b B:S a or b W:B a or b W:B:SW:S a or b W:B:S a or b B:S:SW a or b W:SW:S a or b W:B:SW a (Fig. 2a-h). Therefore, inv(N:NE:E)={B, B:S, W:B, W:B:SW:S, W:B:S, B:S:SW, W:SW:S, W:B:SW}.

Next, let us adopt our algorithm to reason about the inverse of relation N:NE:E. It is easy to see that Br(N:NE:E)=N:NE:B:E. Then we have that N:NE:B:E corresponds to rectangle algebra {si,oi}×{si,oi} from Table 1. Next, we have that $(\{si,oi\}\times\{si, oi\})^{-1}= \{s,o\}\times\{s, o\} = \{(s, s), (s, o), (o, s), (o, o)\}$, according to Theorem 1. Therefore, invmbr(N:NE:B:E)={B, B:S, W:B, W:B:SW:S}. Then, we have inv(N:NE:E) = ORG(B)∪ORG(B:S)∪ORG (W:B)∪ORG (W:B:SW:S) = {B,B:S, W:B, W:B:SW:S, W:B:S, B:S:SW, W:SW:S,W:B:SW}, according to Theorem 10.

We can see that the result of our algorithm is consistent with the actual situation in the case. Therefore our algorithm holds true for cardinal direction relation N:NE:E.

Furthermore, we compare the result of our algorithm with the actual situation for each of the other basic cardinal direction relations, respectively. We find that the result of our algorithm is consistent with the actual situation for any basic cardinal direction relation. Therefore our algorithm can correctly computer the inverse of any basic cardinal direction relation.

CONCLUSIONS

In this study we developed an algorithm to solve the problem of reasoning about the inverse of the basic cardinal direction relations defined in the model of Goyal and Egenhofer (2001). We move a step forward and solve the problem of reasoning about the inverse of cardinal direction relation between regions in a more direct way that does not approximate regions too coarsely either in terms of their minimum bounding rectangle or as a point. The contribution is original and useful in itself; additionally, it will be helpful to enrich and improve the powers of spatial reasoning and spatial analysis.

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