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Image Restoration Method Based on Least-Squares and Regularization and Fourth-Order Partial Differential Equations

[1]Xiaoyang Yu, [1]Yuan Gao, [1]Xue Yang, [2]Chu Shi and [3]Xiukun Yang
[1]The Higher Educational Key Laboratory for Measuring and Control Technology and Instrumentation, Heilongjiang Province, Harbin University of Science and Technology, People's Republic of China
[2]Shandong Academy of Sciences Institute of Oceanographic Instrumentation,
[3]MOTOROLA Countries of Southern California to identify R and D Center, IDENTIX. Inc., Corporate Research Center, Jersey City, NJ07302, USA

Abstract: An image restoration method based on least-squares and regularization and fourth-order partial differential equations is proposed in this study. Noise is removed by improved Yu-Li and M. Kaveh's fourth-order partial differential equations method and then improved least-squares and regularization method is adopted to restored image. The experimental results show that the method can better restore degraded image and the peak signal to noise ratio and subjective visual effects of the restored image are improved significantly.

Key words: Image degradation, steepest descent method, blocking artifacts, salt-pepper noise, Euler equation, peak signal to noise ratio

INTRODUCTION

Blur and noise is inserted by environmental conditions and image equipment etc. in process to take, transport and store image, which result in degrading image. Practical application needs to restore degraded image. Because image restoration is an inverse problem and ill-posed, we generally adopt regularization method to solve the problem. Least-squares and regularization in the regularization methods is more effective (Chen *et al*., 2000), however, restored effects are declined by noise. Because partial differential equations are an effective method to remove noise, an image restoration method based on least-squares and regularization and fourth-order partial differential equations is proposed in this study.

IMAGE RESTORATION METHOD BASED ON LEAST-SQUARES AND REGULARIZATION

Image degradation model can be expressed as:

$$u = fH+n \tag{1}$$

In Eq. 1, u is degraded image. f is original image. n is additive noise. H is blurred operator.

In the conditions without noise, Eq. 1 is:

$$u = fH \tag{2}$$

In regard to Eq. 2, least-squares solution $\hat{f}$ is adopted to approximate to f. If the estimate of the current image is $\tilde{f}$ and the improved amount of $\tilde{f}$ is $\Delta\tilde{f}$, namely:

$$\hat{f} = \tilde{f} + \Delta\tilde{f} \tag{3}$$

If $g = u - H\tilde{f}$, we can get:

$$\left(H^T H\right)\Delta\tilde{f} = H^T g \tag{4}$$

If H is irreversible, the process is ill-posed. Therefore, regularization method is adopted to solve the problem.

In Eq. 4, regular parameter α is added to improve approximate solution (Miao *et al*., 2005). We can get:

$$\left(H^T H + \alpha I_f^T I_f\right)\Delta\tilde{f} = H^T g \tag{5}$$

In Eq. 5, I_f and $\Delta\tilde{f}$ are the unit matrices of the same order. α is regularization parameter. When α is to be selected, we adopt a self-adaptive-correcting mathematical model of regularization parameter (Chen *et al*., 1999) is:

Corresponding Author: Xiaoyang Yu, P.O. Box 326, No. 4, Lin Yuan Road, Xiang Fang District, College of Measurement Control Technology and Communication Engineering, Harbin University of Science and Technology, Harbin City, Heilongjiang Province, 150040, People's Republic of China

$$\alpha = \lambda \frac{\|u - Hf\|_2^2}{\|u\|_2^2} \quad (6)$$

In Eq. 6, λ is step-size correction factor of α. $\lambda = 15$. Equation 5 is expressed as:

$$D\Delta x = b \quad (7)$$

In Eq. 7, $D = H^T H + \alpha I^T_F I_F$, $\Delta x = \Delta \tilde{f}$, $b = H^T s$. If the size of matrix D is great, steepest descent method is adopted to get iterative solution. Iterative formula is:

$$\Delta x_{k+1} = \Delta x_k + \beta_k (D\Delta x_k - b) \quad (8)$$

In Eq. 7, initial value is:

$$\Delta x_0 = b, \quad \beta_k = \frac{-d^T D d}{d^T D^T D d}, \quad d = D\Delta x_k - b$$

In summary, image restoration method based on least-squares and regularization is proposed in the conditions without noise. If noise exists, restored effects are decreased. Therefore, degraded image firstly need to remove noise.

IMAGE DE-NOISING METHOD BASED ON FOURTH-ORDER PARTIAL DIFFERENTIAL EQUATIONS

Image de-noising methods include a combination of species: Wiener filtering, median filtering, average filtering, order statistics filtering, low-pass filtering and so on (Miyata and Taguchi, 2002; Komatsu and Saito, 2004; Shui, 2005). However, these methods don't only remove noise and often cause blur to edges and textures. Recently, Partial Differential Equations (PDE) methods are often used to remove noise, which can suppress noise, while maintain edges and texture information.

Traditional second-order partial differential equations remove noise, which sometimes results in blocking artifacts and some regions have the same gray after processing image. By eliminating noise repeatedly, the image will transit to blocking image. The image will look like the combination of different brightness regions after several iterations. Fourth-order partial differential equations method proposed by You and Kaveh (2000), can avoid the blocking artifacts, while remove noise and maintain edge. However, the problem of these equations can not remove salt-pepper noise. Modifying diffusion in Yu-Li and M. Kaveh's equations can obtain improved equations, which can not only maintain their original capability, but also remove the salt-pepper noise. The process is as follows:

Yu and Kaveh (2000) proposed fourth-order partial differential equations based on Laplacian diversification is:

$$\frac{\partial u}{\partial t} = -\nabla^2 \left[c\left(\left|\nabla^2 u\right|\right) \left|\nabla^2 u\right| \right] \quad (9)$$

In the space Ω of supported domain, energy function of Eq. 9 is:

$$E(u) = \int f\left(\left|\nabla^2 u\right|\right) d\Omega \quad (10)$$

The degree of image smoothing is measured by $|\nabla^2 u|$, so, the minimization of E(u) is equivalent to smooth the image.

Euler equation of Eq. 10 is:

$$\nabla^2 \left[c\left(\left|\nabla^2 u\right|\right) \nabla^2 u \right] = 0 \quad (11)$$

In Eq. 11, ∇^2 is Laplacian operator, s is complex variable, $c(s) \geq 0$, $f(\cdot)$ satisfies:

$$c(s) = \frac{f'(s)}{s}$$

and $f(\cdot) \geq 0$. In Eq. 9,

$$c(s) = \frac{1}{1 + (s/k)^2}$$

therefore,

$$f(s) = \frac{k^2}{2} \ln\left(1 + \frac{s^2}{k^2}\right)$$

k is a constant. Because f(s) is non-concave function, it is considered that f(s) doesn't change in the salt-pepper noise. Namely, E(u) doesn't change in the salt-pepper noise. Therefore, salt-pepper noise can't be removed.

To concluded, the diffusion in Eq. 9 is modified to:

$$c(s) = \frac{1}{\sqrt{1 + s^2}}$$

Chen *et al*. (2008) Eq. 9 is:

$$\frac{\partial u}{\partial t} = -\nabla^2 \left(\frac{1}{\sqrt{1 + \left(\nabla^2 u\right)^2}} \nabla^2 u \right) \quad (12)$$

f(s) in the corresponding energy function is revised to:

$$f(s)=\sqrt{1+s^2}$$

So, f(s) must be concave function, E(u) has only one global minimum solution. In a word, Eq. 12 can't only sustain the de-noising capability of Eq. 9, but also remove salt-pepper noise.

IMAGE RESTORED ALGORITHM BASED ON LEAST-SQUARES AND REGULARIZATION AND FOURTH-ORDER PARTIAL DIFFERENTIAL EQUATIONS

To combine fourth-order partial differential equations with least-squares and regularization is used to restore image, which constructs an image restoration algorithm based on least-squares and regularization and fourth-order partial differential equations. Specific algorithm steps are as follows:

Step 1: Enter the initial degraded image u^0. Assumed that the time interval is Δt, the grid size of space is Δh. To quantify the time and space coordinates are as follows:

$$t=n\Delta t \quad (n=0,1,2,\cdots)$$

$$x=i\Delta h \quad (i=0,1,2,\cdots,I)$$

$$y=j\Delta h \quad (j=0,1,2,\cdots J)$$

In equation, $I\Delta h\times J\Delta h$ is the size of image.

Step 2: Calculate Laplacian value of all the pixels in the image u^0

$$\nabla^2 u_{i,j}^n=\frac{u_{i+1,j}^n+u_{i-1,j}^n+u_{i,j+1}^n+u_{i,j-1}^n-4u_{i,j}^n}{h^2}$$

And the corresponding symmetry boundary conditions are:

$$u_{-1,j}^n=u_{o,j}^n,\quad u_{I+1,j}^n=u_{I,j}^n,\quad (j=0,1,2,\cdots J)$$

$$u_{i,-1}^n=u_{i,0}^n,\quad u_{i,J+1}^n=u_{i,J}^n,\quad (i=0,1,2,\cdots,I)$$

In equation, $u_{i,j}^n$ is the image to be iterated n times, (i,j) is the location of the current pixel in the image.

Step 3: According to the fourth-order partial differential equations, we supposed:

$$\nabla^2 u_{i-1,j},\ \nabla^2 u_{i+1,j},\ \nabla^2 u_{i,j-1},\ \nabla^2 u_{i,j}$$

are Lapliacian values of four adjacent pixels. Image u^{n+1} is calculated as the following formula.

$$u_{i,j}^{n+1}=u_{i,j}^n-\Delta t*p*\left[c_N*\nabla_N^2\Delta u+c_S*\nabla_S^2\Delta u+c_E*\nabla_E^2\Delta u+c_W*\nabla_W^2\Delta u\right]$$

In equation,

$$p\in\left[0,\frac{1}{4}\right],\quad \nabla_N^2\Delta u=\nabla^2 u_{i-1,j}-\nabla^2 u_{i,j},\quad \nabla_S^2\Delta u=\nabla^2 u_{i+1,j}-\nabla^2 u_{i,j}$$
$$\nabla_E^2\Delta u=\nabla^2 u_{i,j+1}-\nabla^2 u_{i,j},\quad \nabla_W^2\Delta u=\nabla^2 u_{i,j-1}-\nabla^2 u_{i,j},\quad c_N=c\left(\nabla_N^2\Delta u\right)=c\left(\left\|\nabla^2 u_{i-1,j}-\nabla^2 u_{i,j}\right\|\right)$$
$$c_S=c\left(\nabla_S^2\Delta u\right)=c\left(\left\|\nabla^2 u_{i+1,j}-\nabla^2 u_{i,j}\right\|\right),\quad c_E=c\left(\nabla_E^2\Delta u\right)=c\left(\left\|\nabla^2 u_{i,j+1}-\nabla^2 u_{i,j}\right\|\right)$$
$$c_W=c\left(\nabla_W^2\Delta u\right)=c\left(\left\|\nabla^2 u_{i,j-1}-\nabla^2 u_{i,j}\right\|\right).$$

Step 4: If iterative times n is less than set iterative times N, the procedure enter step3 and n = n+1. Go on iterative loop. If not, the procedure enter step 5

Step 5: Random matrix is produced with the same size of blurred operator and it is normalized to be H_0. Treated blurred image produces estimation $\tilde{f}_0$, therefore, initial value is calculated by blurred image

$$u.\ \Delta x_0=H_0^T\left(u-H_0\tilde{f}_0\right)$$

Step 6: In the internal iterative loop, regularization parameter α_k is automatically corrected by:

$$\alpha_k=15\cdot\frac{\left\|u-Hx_k\right\|_2^2}{\left\|u\right\|_2^2}$$

and then, Δx_k is obtained by Eq. 8.

Step 7: Enter the external loop and then update $x_{k+1}=x_k+\Delta x_k$

Step 8: If :

$$\frac{\left\|x_{k+1}-x_k\right\|_2^2}{\left\|x_k\right\|_2^2}\le 10^{-7}$$

the program enters step 9; otherwise, enter step 6.

Step 9: Show restored image and calculate the Peak Signal to Noise Ratio (PSNR) of restored image.

$$PSNR=10\lg\frac{IJa_{max}^2}{\sum_{i=0}^{I-1}\sum_{j=0}^{J-1}\left(\hat{f}_{i,j}-f_{i,j}\right)^2}$$

In equation, $\hat{f}_{i,j}$, $f_{i,j}$ are, respectively represented the gray value of restored image and original image.

THE EXPERIMENTAL RESULTS AND ANALYSIS

The experiments are adopted 0 to 255 gray-scale image, 256×256 size of which is the original image to be experimented on.

Experiment 1: Gaussian noise and salt-pepper noise are added into original image and then degraded images are separately removed noise by improved equations and Yu-Li and M. Kaveh's fourth-order partial differential equations, the experimental results are shown in Fig. 1a-d and 2a-d.

Experimental results in Fig. 1a-d show that the removal effects of Gaussian noise through improved equation is similar to the removal effects of Yu-Li and M. Kaveh's fourth-order partial differential equations. Experimental results in Fig. 2a-d show that improved their equations can remove salt-pepper noise which is not removed by Yu-Li and M. Kaveh fourth-order partial differential equations.

Experiment 2: The original image is firstly degraded, which is separately restored by using least-squares regularization method and this method in the study, the experimental results are shown in Fig. 3a-e to 6a-e.

Experimental results of Fig. 3a-e to 6a-e shows that the visual effect of the restored image by using this method in the study is significantly better than the one by using least-squares and regularization method.

The Peak Signal to Noise Ratio (PSNR) of the restored image in Fig. 3a-e to 6a-e is shown in Table 1.

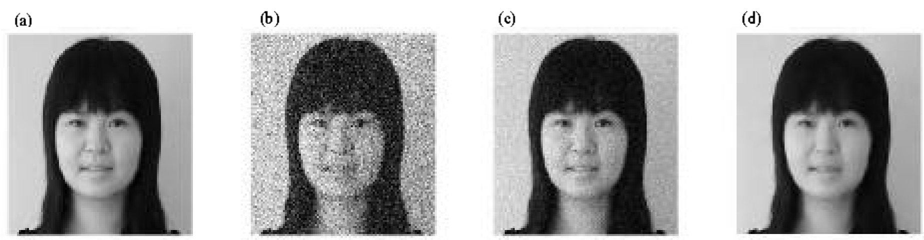

Fig. 1: Removal Gaussian noise, (a) original image, (b) image with Gaussian noise, (c) image de-noised by fourth-order PDE of Yu-Li and M. Kave and (d) image de-noised by the improved PDE

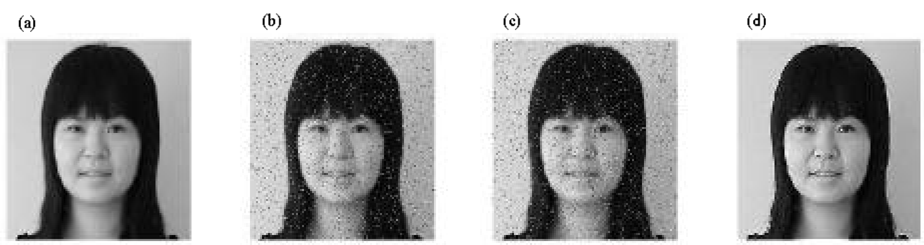

Fig. 2: Removal salt-pepper noise, (a)original image, (b) image with salt-pepper noise, (c) image de-noised by fourth-order PDE of Yu-Li and M. Kave and (d) image de-noised by the improved PDE

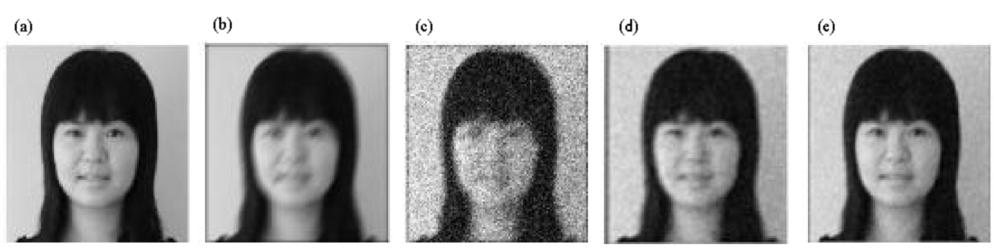

Fig. 3: Restored image degraded by motion blur and Gaussian noise, (a)original image, (b) image with motion blur, (c) image with motion blur and Gaussian noise, (d) image restored by least-squares and regularization method and (e) image restored by the method in this study

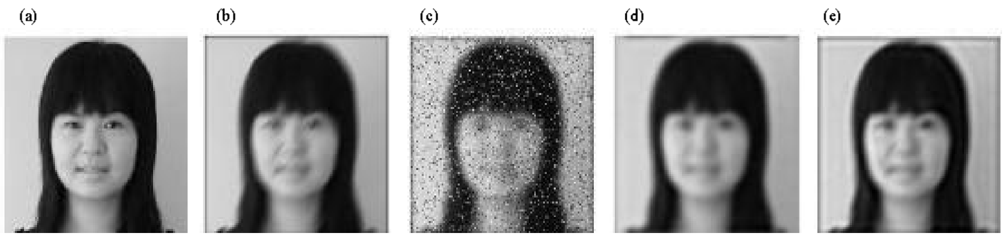

Fig. 4: Restored image degraded by motion blur and salt-pepper noise, (a)original image, (b) image with motion blur, (c) image with motion blur and salt-pepper noise, (d) image restored by least-squares and regularization method and (e) image restored by the method in this study

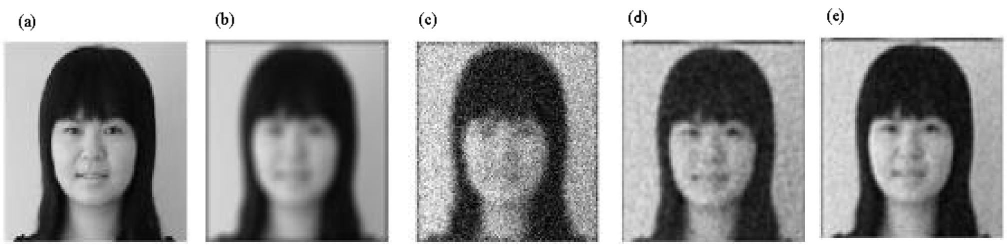

Fig. 5: Restored image degraded by defocus blur and Gaussian noise, (a)original image, (b) image with defocus blur, (c) image with defocus blur and Gaussian noise, (d) image restored by least-squares and regularization method and (e) image restored by the method in this study

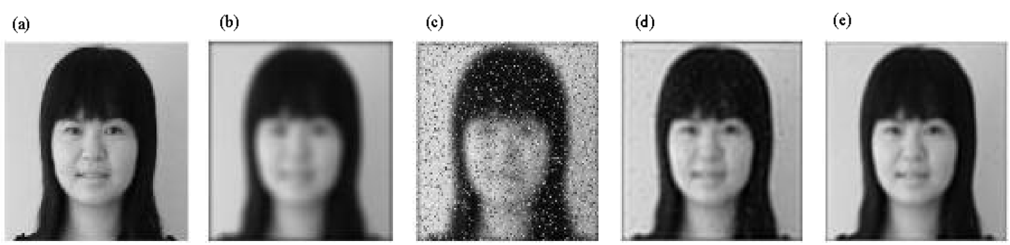

Fig. 6: Restored image degraded by defocus blur and salt-pepper noise, (a)original image, (b) image with defocus blur, (c) image with defocus blur and salt-pepper noise, (d) image restored by least-squares and regularization method and (e) image restored by the method in this study

Table 1: The comparison of PSNR

	PSNR of restored image			
Restoration methods	Motion blur and Gaussian noise	Motion blur and salt-pepper noise	Defocus blur and Gaussian noise	Defocus blur and salt-pepper noise
Least-squares and regularization method	44.5793	47.5729	43.5463	46.2716
The method in this study	47.2101	55.4625	49.1325	51.2837

Comparison of experimental data show that restored effects by using this method in the study is significantly better than least-squares and regularization method.

CONCLUSION

Image restoration method based on least-squares and regularization can restore image better. However, the restored effects are declined by noise. This method in the study combines least-squares and regularization with fourth-order partial differential equations, which don't only overcome the ill-posed of image restoration, but also eliminate noise besides salt-pepper noise. Experimental results show that this method on the peak signal to noise ratio and subjective visual effect of the restored images are better than the one by image restoration method based on least-squares regularization.

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