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ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 9 (5): 968-973, 2010
ISSN 1812-5638

Membership-Dependent Stability Conditions for Takagi-Sugeno Fuzzy Systems

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Abstract: This study deals with the stability analysis and stabilization of fuzzy systems from the viewpoint of membership functions. Firstly, some linear matrix inequality conditions for the stability analysis and stabilization of Takagi-Sugeno (T-S) fuzzy systems are derived, which are dependent on the upper bound and the lower bound of the membership functions. Secondly, the local stability problem of the T-S fuzzy systems is considered and some sufficient conditions for the local stability analysis and stabilization are provided. Finally, some numerical examples are given to illustrate the effectiveness of the proposed results.

Key words: Fuzzy control, stability analysis, stabilization, membership function

INTRODUCTION

Much attention has been paid to the study of fuzzy systems which can model many nonlinear systems (Feng *et al*., 2005; Linfeng *et al*., 2009; Ajlouni and Al-Hamouz, 2004). The Takagi-Sugeno (T-S) fuzzy system is an important fuzzy model. Hence, many research efforts have been devoted to stability and performance analysis of Takagi-Sugeno (T-S) fuzzy systems over the past few years in the literature by Takagi and Sugeno (1985), Sugeno (1999) and Wang *et al*. (1996) and the references therein. A great number of the significant results were derived via the Linear Matrix Inequality (LMI) approach. Some basic stability conditions were presented by Wang *et al*. (1996) and some relaxed results were provided by Guerra and Vermeiren (2004), Kim and Lee (2000), Liu and Zhang (2003) and Tanaka *et al*. (1996, 2003). Currently, much attention is focused on reducing the conservativeness of the LMI stability and performance conditions. An important way to reduce the conservativeness is using the fuzzy or piecewise Lyapunov functional technique (Feng *et al*., 2005; Mozelli *et al*., 2009; Rhee and Won, 2006; Tanaka *et al*., 2007; Yuan *et al*., 2008; Zhou and Li, 2005; Zhou *et al*., 2007). However, some conservativeness still exists and the stability of many stable fuzzy systems can not be proved by the existing results. Take the system:

$$\dot{x}(t) = h_1(z(t))x(t) - (1 - h_1(z(t)))x(t)$$

for example. It can not be proved stable by the aforementioned results. But it is stable when $0 \le h_1(z(t)) < 0.5$.

The LMI conditions in the aforementioned work do not depend on the membership functions, which may be a source of conservativeness. Indeed, the stability of fuzzy systems may depend on the bound of the membership functions sometimes, such as the system mentioned in the last paragraph. On the other hand, if some knowledge on the bound of the membership functions in a region around the equilibrium is known, some relaxed conditions may be derived on the local stability of the fuzzy systems. Motivated by this, Sala and Arino (2006, 2007) considered the local stability problem and the stability problem of fuzzy systems from the viewpoint of the membership functions.

In this study, the problems studied by Sala and Arino (2006, 2007) are considered and new results are proposed. Membership-dependent conditions on the stability of the fuzzy systems are derived, which are dependent on both the upper bound and the lower bound of the membership functions. Sufficient conditions for the local stability of the fuzzy systems are also derived.

For real symmetric matrices X and Y, the notation $X<Y$ means that the matrix Y-X is positive definite. I is the identity matrix with appropriate dimensions. The superscript T represents the transpose.

PRELIMINARIES

Consider the T-S fuzzy control system described by the following rules:

Plant rule i: If $z_1(T)$ is F^i_1, … and $z_g(t)$ is F^i_g, then:

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$$\dot{x}(t) = A_i x(t) + B_i u(t)$$

where, $x(t)\in\mathbb{R}^n$ is the state vector; $u(t)\in\mathbb{R}^n$ is the control input; $z_1(t), ..., z_g(t)$ are the premise variables; $F^i_1, ... F^i_g$ are the fuzzy sets; A_i and B_i are the system matrices. The normalized membership functions are then given by:

$$h_i(z(t)) = \frac{\prod_{j=1}^{g} \mu_{ij}[z_j(t)]}{\sum_{1=1}^{s}\prod_{j=1}^{g} \mu_{ij}[z_j(t)]}, \quad i = 1,...,s$$

where, $\mu_{ij}[z_j(t)]$ is the grade of membership function of $z_j(t)$ in F^i_j. In what follows, the argument of $h_i(z(t))$ will be dropped for simplicity. A more compact presentation of the fuzzy model is given by:

$$\dot{x}(t) = A(h)x(t) + B(h)u(t) \tag{1}$$

Where:

$$A(h) = \sum_{i=1}^{s} h_i A_i, \quad B(h) = \sum_{i=1}^{s} h_i B_i$$

The widely used controllers for T-S fuzzy systems are so-called Parallel Distributed Compensators (PDC) defined by:

$$u(t) = \sum_{i=1}^{s} h_i K_i x(t) \tag{2}$$

where, K_i is a constant matrix to be designed. Combine Eq. 1 and 2 the closed-loop system becomes:

$$\dot{x}(t) = \sum_{i=1}^{s}\sum_{j=1}^{s} h_i h_j G_{ij} x(t) \tag{3}$$

where, $G_{ij} = A_i + B_i K_j$.

Stability of fuzzy systems: Let us review some existing results on the stability of system . When $u(t) \equiv 0$, Eq. 3 becomes:

$$\dot{x}(t) = A(h)x(t) \tag{4}$$

The following sufficient conditions are proposed by Tanaka *et al.* (1996) based on the Lyapunov function $V(x(t)) = x^T(t)Px(t)$.

Lemma 1: Tanaka *et al.* (1996) system in Eq. 4 is asymptotically stable if there exists P>0 such that:

$$A_i^T P + P A_i < 0, \quad i = 1, \cdots, s \tag{5}$$

To reduce the conservativeness of the conditions in Lemma 1, a fuzzy Lyapunov function approach is provided.

Lemma 2: Tanaka *et al.* (2003) assume that:

$$|\dot{h}_i| \le \varphi_i, \quad i = 1, \cdots, s-1$$

System in Eq. 4 is asymptotically stable if there exist $P_i>0$ such that:

$$\begin{aligned} & P_i \ge P_s, \quad i = 1, \cdots, s-1, \\ & \sum_{i=1}^{s-1} \varphi_i (P_i - P_s) + \frac{1}{2}\{A_j^T P_i + P_i A_j + A_i^T P_j + P_j A_i\} \\ & < 0, 1 \le i \le j \le s \end{aligned}$$

Stability of PDC fuzzy control systems: For the stability of system , much more sufficient conditions have been reported. Let's review two main results in those works.

Lemma 3: Tanaka *et al.* (1998) system in Eq. 3 is asymptotically stable if there exist X>0 and M_i such that:

$$\begin{aligned} & A_i X + X A_i^T - M_i^T B_i^T - B_i M_i < 0, \ i = 1, \cdots, s, \\ & A_i X + X A_i^T + A_j X + X A_j^T - B_i M_j - M_j^T B_i^T \\ & \qquad - B_j M_i - M_i^T B_j^T < 0, \quad 1 \le i < j \le s \end{aligned}$$

In such case, the controller gain matrix can be given by $K_i = M_i X^{-1}$.

Lemma 4: Liu and Zhang (2003) system in Eq. 3 is asymptotically stable if there exist matrices X>0, M_i and Y_{ij} such that:

$$A_i X + X A_i^T - M_i^T B_i^T - B_i M_i - Y_{ii} < 0, \ i = 1, \cdots, s,$$

$$\begin{aligned} & A_i X + X A_i^T + A_j X + X A_j^T - B_i M_j - M_j^T B_i^T \\ & \quad - B_j M_i - M_i^T B_j^T - Y_{ij} - Y_{ij}^T < 0, \quad 1 \le i < j \le s, \\ & \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1s} \\ Y_{21} & Y_{22} & \cdots & Y_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{s1} & Y_{s2} & \cdots & Y_{ss} \end{bmatrix} < 0 \end{aligned}$$

In this case, the controller gain matrix can be given by $K_i = M_i X^{-1}$.

Membership-dependent stability conditions: When the above conditions are infeasible, we should seek some other conditions. Note that the membership functions do not appear in the aforementioned LMI conditions. This may be a source of conservativeness sometimes. In fact, some conservativeness may be reduced if the knowledge on the membership functions for a particular T-S fuzzy system is introduced in the LMI framework. Motivated by this, some researches have been done to seek the stability condition for system in Eq. 3 and 4 with the assumption that the upper bound of h_i and $h_i h_j$ is known. The main results on this problem are restated in the following.

Lemma 5: Suppose that $0 \le h_i \le \beta_i$, system is asymptotically stable if there exist symmetric positive matrices P_i and N_i such that:

$$A_i^T P + PA_i - N_i + \sum_{j=1}^{s} \beta_j N_j < 0, \quad i = i, \cdots, s \tag{6}$$

Lemma 6: Suppose that $0 \le h_i h_j \le \beta_{ij}$, system in 3 is asymptotically stable if there exist matrices X, $X_{ij} = X^T_{ij}$ and symmetric matrix R_{ij}, such that:

$$\begin{aligned} &X > 0, \\ &R_{ii} \ge 0, \\ &A_i X + XA_i^T - M_i^T B_i^T - B_i M_i - R_{ii} + \Lambda - X_{ii} \le 0, \ i = 1, \cdots, s, \end{aligned}$$

$$\begin{aligned} &A_i X + XA_i^T + A_j X + XA_j^T - B_i M_j - M_j^T B_i^T - B_j M_i - M_i^T B_j^T - R_{ij} \\ &+2\Lambda - X_{ij} - X_{ji} \le 0, 1 \le i < j \le s, \end{aligned}$$

$$X = \begin{bmatrix} X_{11} & \cdots & X_{1s} \\ \vdots & \ddots & \vdots \\ X_{s1} & \cdots & X_{ss} \end{bmatrix} < 0$$

Where:

$$\Lambda = \sum_{k=1}^{s} \sum_{k \le l \le s} \beta_{kl} R_{kl}$$

In this case, the controller gain matrix can be given by $K_i = M_i X^{-1}$.

Lemma 5 and 6 are corollaries of Theorem 2 and 3 in Sala and Arino (2006, 2007), respectively. It is easy to see that if there exists P satisfying , let $N_i = \varepsilon I$, and P satisfy if ε is small enough. So, is more relaxed than. By a similar process, one can see that the conditions in Lemma 6 are less conservative than those in Lemma 4. It is worthy pointing out that the results in Lemma 5 can be extended to a fuzzy Lyapunov functional approach to further reduce the conservativeness.

When $A^T_i P + PA_i$ is unfeasible for some i, system in Eq. 3 cannot be proved stable by Lemma 1 and 2. But it may be proved stable by Lemma 5. It is noted that the condition in Lemma 5 depend on the upper bound of h_i only, but has no relationship with the lower bound of h_i.

In the next section, we will seek to introduce the lower bound of the membership functions into the LMI stability conditions of the fuzzy systems.

RESULTS

Here, we presents some sufficient conditions on the local stability and stabilization of system in Eq. 3 and 4 when both the lower bound and the upper bound of h_i are known.

Stability of fuzzy systems: The following theorem gives sufficient stability conditions of system in Eq. 4.

Theorem 1: Assume that $0 \le \alpha_i \le h_i \le \beta_i \le 1$. System in Eq. 4 is asymptotically stable if there exist matrices P>0 and i = 1,...,s such that:

$$A_i^T P + PA_i - Y_i < 0, \quad i = 1, \cdots, s \tag{7}$$

$$\sum_{i=1}^{s} \beta_i Y_i + \sum_{i=1}^{s} \alpha_i \left(A_i^T P + PA_i - Y_i \right) < 0 \tag{8}$$

Proof: Choose a Lyapunov function as:

$$V(x(t)) = x^T(t) P x(t)$$

Note that:

$$\begin{aligned} \dot V(x(t)) &= 2x^T(t) P \dot x(t) \\ &= x^T(t) \left\{ A^T(h) P + PA(h) \right\} x(t) \\ &= x^T(t) \sum_{i=1}^{s} h_i Y_i + \sum_{i=1}^{s} h_i (A_i^T P + PA_i - Y_i) \} x(t) \\ &\le x^T(t) \left\{ \sum_{i=1}^{s} \beta_i Y_i + \sum_{i=1}^{s} \alpha_i \left(A_i^T P + PA_i - Y_i \right) \right\} x(t) \end{aligned}$$

where, $\dot V(x(t))$ denotes the time derivative of V(x(t)) along the trajectory of Eq. 4. It follows from Eq. 7 and 8 that $\dot V(x(t)) < 0$ for any $x(t) \ne 0$. So, system in Eq. 4 is asymptotically stable if and are satisfied. This completes the proof.

Theorem 1 provides a new condition to check the stability of system. When $\alpha_i > 0$, Eq. 7 and 8 may guarantee larger stability region than sometimes. This will be illustrated in the following example.

Example 1: Consider the fuzzy system with the following rules:

- R_1: If $x_1(t)$ is M_1 then $\dot x(t) = A_1 x(t)$
- R_1 : If $x_1(t)$ is M_2 then $\dot x(t) = A_2 x(t)$

Where:

$$A_1 = \begin{bmatrix} -5 & -4 \\ -1 & -2 \end{bmatrix},\quad A_2 = \begin{bmatrix} -2 & -4 \\ 20 & a \end{bmatrix}$$

Assume that $0 \le h_2 \le \alpha$, then $1-\alpha \le h_1 \le 1$. Let $\alpha = 4$. Using the LMI tool box, one has that the LMI conditions in both Lemma 1 and 2 are infeasible. When $\alpha = 0.4$, solving, one has:

$$P = \begin{bmatrix} 1.9382 & 0.4497 \\ 0.4497 & 0.8072 \end{bmatrix}$$
$$N_1 = \begin{bmatrix} 1.5267 & -0.8059 \\ -0.8059 & 0.7936 \end{bmatrix}$$
$$N_2 = \begin{bmatrix} 33.2259 & 19.1956 \\ 19.1956 & 11.1163 \end{bmatrix}$$

Solving Eq. 7 and 8, one has:

$$P = \begin{bmatrix} 0.2747 & 0.0795 \\ 0.0795 & 0.1705 \end{bmatrix}$$
$$N_1 = \begin{bmatrix} 1.5508 & -0.3428 \\ -0.3428 & 1.8394 \end{bmatrix}$$
$$N_2 = \begin{bmatrix} 3.3249 & 2.2121 \\ 2.2121 & 1.5189 \end{bmatrix}$$

When $\alpha = 0.3$, by solving Eq. 7 and 8, the following results can be obtained.

$$P = \begin{bmatrix} 0.2729 & 0.0761 \\ 0.0761 & 0.1527 \end{bmatrix}$$
$$N_1 = \begin{bmatrix} 1.4312 & -0.3177 \\ -0.3177 & 1.7192 \end{bmatrix}$$
$$N_2 = \begin{bmatrix} 2.9768 & 1.9183 \\ 1.9183 & 1.2579 \end{bmatrix}$$

Next we consider the stability of system. The following theorem provides sufficient conditions for the stability of system when the bound of h_i is known.

Theorem 2: Suppose that $0 \le \alpha_i \alpha \le h_i \le \beta_i \le 1$. System (3) is asymptotically stable if there exist matrices $X>0$, $Y_{ij}>0$ and M_i, $i,j = 1,\ldots,s$ such that:

$$A_i X + XA_i^T - M_j^T B_i^T - B_i M_j - Y_{ij} < 0 \quad i = 1,\cdots,s,\ j = 1,\cdots,s \tag{9}$$

$$\sum_{i=1}^{s} \alpha_i \left(A_i X + XA_i^T - M_j^T B_i^T - B_i M_j - Y_{ij}\right) + \sum_{i=1}^{s} \beta_i Y_{ij} < 0,\ j = 1,\cdots,s \tag{10}$$

In such a case, the controller gain matrix can be given by $Ki = M_i X^{-1}$.

Proof: Define $P = X^{-1}$ and $\bar{Y}_{ij} = PY_{ij}P$.

Pre-multiply and post-multiply P to Eq. 9 and 10, respectively, one has:

$$G_{ij}^T P + PG_{ij} - \bar{Y}_{ij} < 0,\ i,j = 1,\cdots,s \tag{11}$$

$$\sum_{i=1}^{s} \alpha_i \left(G_{ij}^T P + PG_{ij} - \bar{Y}_{ij}\right) + \sum_{i=1}^{s} \beta_i \bar{Y}_{ij} < 0 \quad j = 1,\cdots,s. \tag{12}$$

Choose the Lyapunov function as $V(x(t)) = x^T(t)Px(t)$. From, one has that:

$$\begin{aligned}
\dot{V}(x(t)) &= 2x^T(t)P\dot{x}(t) \\
&= x^T(t)\left\{\sum_{i=1}^{s}\sum_{j=1}^{s} h_i h_j (G_{ij}^T P + PG_{ij})\right\}x(t) \\
&= x^T(t)\left\{\sum_{i=1}^{s}\sum_{j=1}^{s} h_i h_j (G_{ij}^T P + PG_{ij} - Y_{ij}) + \sum_{i=1}^{s}\sum_{j=1}^{s} h_i h_j Y_{ij}\right\}x(t) \\
&\le x^T(t)\left\{\sum_{i=1}^{s}\sum_{j=1}^{s} \alpha_i h_j (G_{ij}^T P + PG_{ij} - Y_{ij}) + \sum_{i=1}^{s}\sum_{j=1}^{s} \beta_i h_j Y_{ij}\right\}x(t) \\
&= \sum_{j=1}^{s} h_j x^T(t)\left\{\sum_{i=1}^{s} \alpha_i (G_{ij}^T P + PG_{ij} - Y_{ij}) + \sum_{i=1}^{s} \beta_i Y_{ij}\right\}x(t).
\end{aligned} \tag{13}$$

It follows from Eq. 12 and 13 that for any $\dot{V}(x(t)) < 0$. Therefore, system (3) is asymptotically stable if Eq. 9-10 are satisfied. This completes the proof.

Example 2: To show the effectiveness of Theorem 2, we consider the fuzzy control system with the following rules:

- R_i: If $x_i(t)$ is M_1 then,

$$\dot{x}(t) = A_i x(t) + B_i u(t) \quad i = 1,2,3$$

where:

$$A_1 = \begin{bmatrix} 1.59 & -7.29 \\ 0.01 & 0 \end{bmatrix},\quad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0.02 & -4.64 \\ 0.35 & 0.21 \end{bmatrix},\quad B_2 = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$$
$$A_3 = \begin{bmatrix} -a & -4.33 \\ 0 & 0.05 \end{bmatrix},\quad B_3 = \begin{bmatrix} 6-b \\ -1 \end{bmatrix}$$

Assume that $\alpha_1 \le h_1 \le \beta_1$, $\alpha_2 \le h_2 \le \beta_2$, $\alpha_3 \le h_3 \le \beta_3$ then:

$$\alpha_1\alpha_2 \le h_1h_2 \le \frac{1}{4}(1-\alpha_3)^2,\ \ \alpha_1\alpha_3 \le h_1h_3 \le \frac{1}{4}(1-\alpha_2)^2$$

and

$$\alpha_2\alpha_3 \le h_2h_3,\ \le \frac{1}{4}(1-\alpha_1)^2$$

When $\alpha_1 = \alpha_2 = \alpha_3 = 0.1$ and $\beta_1 = \beta_2 = \beta_3 = 0.8$, it can be shown that Theorem 2 can guarantee the stability of the closed-loop system and the controller can be designed.

Local stability conditions: Unfortunately, the lower bound of h_i is usually equivalent to 0. In such a case, the LMIs in Theorem 1 and 2 are not feasible. However, α_i may be bigger than 0 in a zone around the equilibrium of system in Eq 3 and 4. In such a case, we have the following corollaries on the local stability of system and system.

Corollary 1: Assume that h_i satisfies $0 < \alpha_i \le h_i \le \beta_i \le 1$ in a region Ω that contains the equilibrium x = 0. System in Eq. 4 is locally asymptotically stable at the equilibrium x = 0 if there exist matrices P>0 and $Y_i \ge 0, i = 1,\cdots,s$ satisfying Eq. 7 and 8. In such a case, the set of the initial state $\Omega^* = \{x \in \mathbb{R}^n \mid x^TPx \le V_M\}$ is a domain of attraction, where,

$$V_M = \min\{x^TPx \mid x \in \partial\Omega\}$$

with $\partial\Omega$ denoting the boundary of Ω.

Proof: It is easy to see that if $x(0) \in \Omega^*$, then $x(0) \in \Omega$. From the proof of Theorem 1 one has that:

$$\dot{V}(x) \le x^T\left\{\sum_{i=1}^{s}\beta_iY_i + \sum_{i=1}^{s}\alpha_i\left(A_i^TP + PA_i - Y_i\right)\right\}x \qquad (14)$$

for any $sx \in \Omega^*$. Suppose that Eq. 7 and 8 hold, one has that $\dot{V}(x) \le 0$ for any $sx \in \Omega^*$ and $\dot{V}(x) = 0$ only when x = 0. Therefore, Ω^* is an invariant set. LaSalle's theorem ensures that every solution of system in Eq. 4 starting in Ω^* will approach x = 0. And the proof is completed.

Corollary 2: Assume that h_i satisfies $0 < \alpha_i \le h_i \le \beta_i \le 1$ in a region Ω that contains x = 0. System in Eq. 3 locally asymptotically stable at the equilibrium x = 0 if there exist matrices X>0, Y_{ij}>0 and M, i, j = 1,..,s satisfying Eq. 9 and 10 . In such a case, the controller gain matrix can be given by $K_i = M_iX^{-1}$ and $\Omega^* = \{x \mid x^TX^{-1}x \le V_M\}$ is a domain of attraction, where, $V_M = \min\{x^TPx \mid x \in \partial\Omega\}$ with $\partial\Omega$ denoting the boundary of Ω.

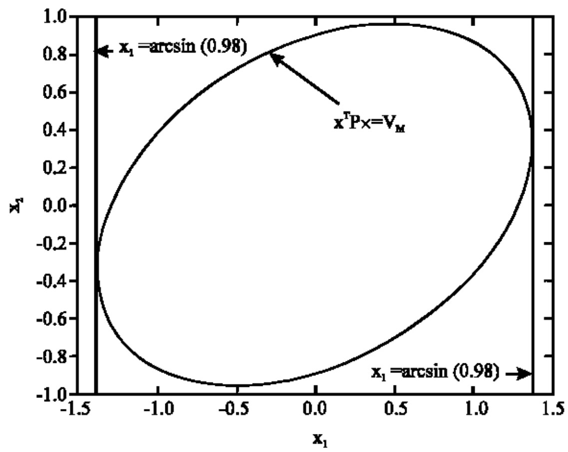

Fig. 1: Basin of attraction in example 3

Proof: Corollary 2 can be derived easily based on Theorem 2, which is similar to the derivation of Corollary 1.

To illustrate the effectiveness of Corollary 1, we give the following example. The application of Corollary 2 is similar to that of Corollary 1. Hence, it is omitted here.

Example 3: Consider the system investigated in example 1. Suppose that:

$$h_1 = \frac{1+\sin x_1(t)}{2},\quad h_2 = \frac{1-\sin x_1(t)}{2}$$

then h_1 hand h_1 satisfy $0.01 \le h_1 \le 0.99$ and $0.01 \le h_2 \le 0.99$ when -arcsin $0.98 \le x_1 \le$ arcsin 0.98. Solving Eq. 7and 8 with $\alpha_1 = \alpha_2 = 0.01$ and $\beta_1 = \beta_2 = 0.99$, one has that:

$$P = \begin{bmatrix} 11.8091 & -5.9513 \\ -5.9513 & 24.0503 \end{bmatrix}$$

$$N_1 = \begin{bmatrix} 54.2115 & -1.8099 \\ -1.8099 & 52.9116 \end{bmatrix}$$

$$N_2 = \begin{bmatrix} 59.7889 & 59.5099 \\ 59.5099 & 70.5902 \end{bmatrix}$$

So, the system is locally stable at the equilibrium x =0. Let,

$$V_M = \min\{x^TPx \mid x_1 = \pm\arcsin 0.98\}$$

According to Corollary 1, one has that the basin of attraction of x = 0 is $\Omega^* = \{x \mid x^TPx \le V_M\}$, which is shown in Fig. 1.

CONCLUSION

Some membership-dependent LMI conditions have been derived on the stability analysis and stabilization for continuous T-S fuzzy systems. It has been shown that if the lower bound and the upper bound of the membership functions are known, some new stability conditions can be obtained. Furthermore, when the bound of the membership functions in a region of x = 0 is known, relaxed conditions can be obtained on the local stability of the fuzzy systems. In such a case, a basin of attraction of x = 0 is also provided. The given numerical examples have shown the effectiveness of the proposed approaches.

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