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Delay Independence Stability Analysis and Switching Law Design for the Switched Time-delay Systems

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Abstract: The study presents an innovative representation modeling of the switched time-delay system and study the problem of delay-independence stability and switching law design for switched time-delay systems. The methods based on Lyapunov-Razumikhin stability theorem to study the stability and switching law design for the switched time-delay systems with state-driven switching method are presented. There are two basic problems in stability and switching law design of the switched time-delay system. These problems are stability for arbitrary switching sequences and construction of stabilizing switching sequences. This study derives stability conditions that guarantee the switched time-delay system is delay-independent asymptotically stable for two kinds of switching law. This study constructs linear matrix inequalities based design procedures for stability analysis. Finally, numerical examples are given to show the merits of the proposed approach.

Key words: Switched system, time-delay, lyapunov function, razumikhin theorem, stability, switching law, linear matrix inequalities

INTRODUCTION

Recently, switched systems have attracted great attention research in academic and industrial application. It also has been widely and successfully applied to a variety of industrial processes such as the automotive industry, chemical procedure control systems, navigation systems, automobile speed change system, aircraft control system, air traffic control etc, can be appropriately described by the switched model (Liberzon and Morse, 1999; Morse, 1996; Wu *et al.*, 2010). Switched systems are a class of hybrid dynamical systems composing of several continuous-time or discrete-time subsystems and a rule that orchestrates the switching sequences between them. In the study of switched systems, most works have been centralized on the problem of stability. Therefore, there have been many researches about the stable analysis and design of switched systems (Chiou, 2006; Chiou and Cheng, 2009a, b; Chiou *et al.*, 2009, 2010; Chiou and Wang, 2010; Chiou *et al.*, 2011) and the references cited therein.

Two important methods are used to construct the switching law for the stability analysis of the switched systems. One is the state-driven switching strategy (Chiou and Cheng, 2009a, b; Chiou *et al.*, 2010, 2011); the other is the time-driven switching strategy (Chiou, 2006). The time-driven switching method is formed based on the main concept of a dwell time. If at least one stable subsystem exits, then the switched system is stable with a proper dwell time switching law. However, in reality, it is not unusual to encounter cases in which all subsystems are unstable. Therefore, Li *et al.* (2005) proposed that all subsystem can be unstable for time-driven switching method. We would have to turn to the state-driven switching strategy, from which many choices of switching laws ensuring stability exist, even if all the subsystems are unstable. Zhai *et al.* (2003) proposed quadratic stabilizability via state feedback for both continuous-time and discrete-time switched linear systems that are composed of polytopic uncertain subsystems. For continuous-time switched linear systems, if there exists a common positive definite matrix for stability of all convex combinations of the extreme points which belong to different subsystem matrices, then the switched system is quadratically stabilizable via state feedback. The switching rules can be obtained by using the obtained common positive definite matrix.

Furthermore, the time-delay phenomenon is also unavoidable in practical systems, for instance, chemical process, long distance transmission line, hybrid procedure, electron network etc. Time-delays may cause instability and poor performance for practical systems (Jasem *et al.*, 2010; Li and Fang, 2009; Liu *et al.*, 2011). In view of the aforementioned facts, the stability of switched

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systems with time delay is very worthy of research. Basically, current efforts to achieve stability in time-delay systems can be divided into two categories, namely delay-independent criteria and delay-dependent criteria. In this study, the delay-independent stability of switched time-delay system is considered. For the delay-independent criteria, Lyapunov-Razumikhin functional technique is the most suitable for application (Hale, 1977). In view of the state-driven switching method, Lyapunov-Razumikhin functional approach is applied to analyze the stability problem for switched time-delay systems. By Lyapunov-Razumikhin functional technique (Hsiao *et al*., 2010) to construct a state-driven switching strategy such that the switched time-delay system is delay-independent asymptotically stable. And we combine with linear matrix inequalities techniques to study the stability of switched time-delay systems.

There are two basic problems in stability and switching law design of the switched time-delay system. These problems are: stability for arbitrary switching sequences and construction of stabilizing switching sequences. We derive stability conditions that guarantee the switched time-delay system is asymptotically stable for two kinds of switching law.

REPRESENTATION MODELING OF SWITCHED TIME-DELAY SYSTEM

Consider the following switched time-delay system:

$$\dot{x}(t) = A_{\sigma(x(t))}x(t) + B_{\sigma(x(t))}x(t-\tau) \tag{1}$$

$$x(t_0) = x_0 \text{ and } x(t) = \psi(t),\ t\in[-\tau, 0]$$

where, $x(t)\in R^n$ is state, $A_{\sigma(x(t))}\, x(t)\in R^{n\times n}$, $B_{\sigma(x(t))}\, x(t)\, R^{n\times n}$, $t_0\geq 0$ is the initial time, x_0 is the initial state, $\sigma(x(t))$: $R^n\rightarrow\{1, 2, ..., N\}$ is a piecewise constant function of time, called a switch signal, i.e., the matrix $A_{\sigma(x(t))}$ switches between matrices $A_1, A_2, ..., A_N$ belonging to the set $A = \{A_1, A_2, ..., A_N\}$ and A_i, $i\in\{1, 2, ..., N\}$, the matrix $B_{\sigma(x(t))}$ switches between matrices $B_1, B_2, ..., B_N$ belonging to the set $B = \{B_1, B_2, ..., B_N\}$ and B_i, $i\in\{1, 2, ..., N\}$. $\tau>0$ is the time-delay duration. $\psi(t)$ is a vector-valued initial continuous function defined on the interval $[-\tau, 0]$ and finally $\psi(t)$, defined on $-\tau\leq t\leq 0$, is the initial condition of the state.

Therefore, the switched discrete time-delay system (1) can be described as follows:

$$\dot{x}(t) = A_l x(t) + B_l x(t-\tau) \tag{2}$$

where, $l\in\{1, 2, ..., N\}$.

There are two basic problems in stability and switching law design of the switched system. These problems are: stability for arbitrary switching sequences and construction of stabilizing switching sequences.

Definition 1: Stability for Arbitrary Switching Sequences (SASS):There exist conditions that guarantee that the switched time-delay system is asymptotically Stable for Arbitrary Switching Sequences (SASS).

Definition 2: Construction of Stabilizing Switching Sequences (CSSS): There exist conditions that guarantee that the switched time-delay system is asymptotically stable for Construction of Stabilizing Switching Sequences (CSSS).

STABILITY ANALYSIS

Sufficient conditions for ensuring delay-independent stability of switched time-delay system (1) will be derived using Lyapunov stability approach with two kinds of switching law.

Stability for arbitrary switching sequences

Theorem 1: There exists a switching law (SASS), the switched time-delay system (1) is delay-independent asymptotically stable if there exist matrices p>0 and Q>0 such that:

$$A_l^T P + PA_l + PB_l Q B_l^T P + P > 0 \tag{3a}$$

$$P \geq Q^{-1} \tag{3b}$$

for l = 1, 2, ..., N.

Proof: We select the Lyapunov function as:

$$V(x(t)) = x^T(t)Px(t) \tag{4}$$

The derivative of the Lyapunov function V (x (t)) along the trajectories of switched time-delay system is:

$$\begin{aligned}\dot{V}(x(t)) &= x^T(t)(A_l^T P + PA_l)x(t) + 2x^T(t)PB_l x(t-\tau)\\ &\leq x^T(t)(A_l^T P + PA_l + PB_l Q B_l^T P)x(t) + x^T(t-\tau)Q^{-1}x(t-\tau)\\ &\leq x^T(t)(A_l^T P + PA_l + PB_l Q B_l^T P)x(t) + V(x(t-\tau))\end{aligned} \tag{5}$$

By using Razumikhin theorem (Hale, 1977), we assume that there exists a real $\upsilon>1$ such that:

$$V(x(t-\theta))>\upsilon V(x(t)), \text{ for } \theta\in[0, \tau] \tag{6}$$

Then,

$$\dot{V}(x(t)) \le x^T(t)(A_l^T P + PA_l + PB_l Q B_l^T P)x(t) + \upsilon V(x(t)) \quad (7)$$

If:

$$\Theta(\upsilon) := A_l^T P + PA_l + PB_l Q B_l^T P + \upsilon p < 0$$

hold for all i and l, then $\dot{V}(x(t)) < 0$.

From using Razumikhin theorem, notice that if $A_l^T P+PA_l+PB_l Qb_l^T P+p<0$ holds, i.e., $\Theta(1)<0$, then by continuity, there exists a $\upsilon = 1+\delta$ with $\delta>0$ sufficiently small such that $\Theta(\upsilon)<0$ for all i and l.

Therefore, if $A_l^T P+PA_l+PB_l Qb_l^T P+p<0$ hold, then $\dot{V}(x(t)) < 0$, i. e., the switched time-delay system (1) is delay-independent asymptotically stable under the switching law 1 (SASS).

Switching law 1(SASS): Switched time-delay system with arbitrary N individual systems is switched to or stay at mode l at arbitrary switching sequences.

Using Schur complement we find that the matrix inequalities (3a) and (3b) are equivalent to the following LMIs, respectively:

$$HA_l^T+A_lH+H+B_lQB_l^T<0 \quad (8a)$$

$$Q \ge H \quad (8b)$$

where, $H= P^{-1}$

Corollary 1: There exists a switching law 1 (SASS), the switched time-delay system (1) is delay-independent asymptotically stable if there exist matrices p>0 and Q>0 satisfying LMIs (8a) and (8b) for i = 1, 2, ..., r and l = 1, 2, ... , N.

Construction of stabilizing switching sequences: For this objective of stable analysis, one helpful lemma is given below.

Switching law 2 (CSSS): Switched time-delay system with arbitrary N individual systems is switched to or stay at mode l at time t if (9) is satisfied at time t.

$$x^T(t)(A_l^T P + PA_l)x(t) + 2x^T(t)PB_l x(t-\tau) < 0,\ l \in \{1, 2, \ldots, N\} \quad (9)$$

Lemma 1: There exists a switching law 2 (CSSS) for the switched time-delay system (1) such that the system is asymptotically stable if there exist positive constants α_i $(1 \le i \le N)$ satisfying:

$$\sum_{i=1}^{N} \alpha_i = 1$$

such that the convex combination of the whole switched time-delay system:

$$\dot{x}(t) = \sum_{l=1}^{N} \alpha_l [A_l x(t) + B_l x(t-\tau)] \quad (10)$$

is an asymptotically stable system.

Proof: Since there exist positive numbers α_i $(1 \le i \le N)$ such that:

$$\dot{x}(t) = \sum_{l=1}^{N} \alpha_l [A_l x(t) + B_l x(t-\tau)]$$

is asymptotically stable, there exists a Lyapunov function V (x) such that:

$$\frac{\partial V}{\partial x}\{\sum_{l=1}^{N} \alpha_l [A_l x(t) + B_l x(t-\tau)]\} < 0 \quad (11)$$

It follows that for any t, at least there exists an $i \in \{1, 2, ..., N\}$ such that:

$$\frac{\partial V}{\partial x}[A_l x(t) + B_l x(t-\tau)] < 0 \quad (12)$$

From (11), it implies that a convex combination of the corresponding Lyapunov function is negative along the trajectory. Thus, the switched time-delay system (1) is asymptotically stable.

Theorem 2: There exists a switching law 2 (CSSS), the switched time-delay system (1) is delay-independent asymptotically stable if there exist matrices p>0 and Q>0 such that:

$$(\sum_{l=1}^{N} \alpha_l A_l^T)P + P(\sum_{l=1}^{N} \alpha_l A_l) + P(\sum_{l=1}^{N} \alpha_l B_l)Q(\sum_{l=1}^{N} \alpha_l B_l^T)P + p < 0 \quad p \ge Q^{-1} \quad (13)$$

for l =1, 2, ..., N.

Proof: By Lemma 1, the switched time-delay system (1) implies that a convex combination system:

$$\dot{x}(t) = \sum_{l=1}^{N} \alpha_l [A_l x(t) + B_l x(t-\tau)]$$

We select the Lyapunov function as:

$$V(x(t)) = x^T(t) P x(t)$$

The derivative of the Lyapunov function V (x (t)) along the trajectories of (1) is:

$$\dot{V}(x(t)) = \sum_{l=1}^{N} \alpha_l [x^T(t)(A_l^T P + PA_l) x(t) + 2x^T(t) PB_l x(t-\tau)]$$
$$\leq \sum_{l=1}^{N} \alpha_l [x^T(t)(A_l^T P + PA_l + PB_l QB_l^T P) x(t) + x^T(t-\tau) Q^{-1} x(t-\tau)]$$
$$\leq \sum_{l=1}^{N} \alpha_l [x^T(t)(A_l^T P + PA_l + PB_l QB_l^T P) x(t) + V(x(t-\tau))] \quad (14)$$

By using Razumikhin theorem (Hale, 1977), we assume that there exists a real $\upsilon>1$ such that:

$$V(x(t-\theta)) < \upsilon V(x(t)), \text{ for } \theta \in [0, \tau] \quad (15)$$

Then,

$$\dot{V}(x(t)) \leq \sum_{l=1}^{N} \alpha_l [x^T(t)(A_l^T P + PA_l + PB_l QB_l^T P) x(t) + \upsilon V(x(t))] \quad (16)$$

If $\Theta(\upsilon) := A_l^T P + PA_l + PB_l Qb_l^T P + p < 0$ hold for all i and l, then $\dot{V}(x(t)) < 0$.

From using Razumikhin theorem, notice that if $A_l^T P + PA_l + PB_l Qb_l^T P + p < 0$ holds, i.e., $\Theta(1) < 0$, then by continuity, there exists a $\upsilon = 1+\delta$ with $\delta>0$ sufficiently small such that $\Theta(\upsilon)<0$ for all i and l.

Therefore, if :

$$\sum_{l=1}^{N} \alpha_l [x^T(t)(A_l^T P + PA_l + PB_l QB_l^T P + P) x(t)] < 0$$

i. e.,

$$(\sum_{l=1}^{N} \alpha_l A_l^T) P + P(\sum_{l=1}^{N} \alpha_l A_l) + P(\sum_{l=1}^{N} \alpha_l B_l) Q (\sum_{l=1}^{N} \alpha_l B_l^T) P + p < 0$$

then, $\dot{V}(x(t)) < 0$ thus, T-S fuzzy switched time-delay system (1) is delay-independent asymptotically stable under the switching law 2 (CSSS).

We denote:

$$\bar{A}_l = \sum_{l=1}^{N} \alpha_l A_l \quad (17a)$$

$$\bar{B}_l = \sum_{l=1}^{N} \alpha_l B_l \quad (17b)$$

Using Schur complement we find that the matrix inequalities are equivalent to the following LMIs, respectively:

$$H\bar{A}_l^T + \bar{A}_l H + H + \bar{B}_l Q \bar{B}_l^T < 0 \quad (18a)$$

$$Q \geq H \quad (18b)$$

where, $H = P^{-1}$

Corollary 2: There exists a switching law (CSSS), the switched time-delay system (1) is delay-independent asymptotically stable if there exist matrices p>0 and Q>0 satisfying LMIs (18a) and (18b) for i = 1, 2, ..., r and l = 1, 2, ..., N.

Lemma 2: For any matrices $A_1, A_2, ..., A_N$ with the same dimensions, the following inequality folds for any positive constant ε:

$$(\sum_{l=1}^{N} A_l)^T (\sum_{l=1}^{N} A_l) \leq (1+\varepsilon) A_1^T A_1 + (1+\varepsilon^{-1})(1+\varepsilon) A_2^T A_2 + (1+\varepsilon^{-1})^2 (1+\varepsilon) A_3^T A_3 + \cdots + (1+\varepsilon^{-1})^{N-2} (1+\varepsilon) A_{N-1}^T A_{N-1} + (1+\varepsilon^{-1})^{N-1} A_N^T A_N \quad (19)$$

Proof: For a positive constant ε, A and B with the same dimension, it is a obvious fact that:

$$(A+B)^T (A+B) \leq (1+\varepsilon) A^T A + (1+\varepsilon^{-1}) B^T B \quad (20)$$

In view of inequality (19), we have:

$$(\sum_{l=1}^{N} A_l)^T (\sum_{l=1}^{N} A_l) \leq (1+\varepsilon) A_1^T A_1 + (1+\varepsilon^{-1}) (\sum_{l=2}^{N} A_l)^T (\sum_{l=2}^{N} A_l)$$
$$\leq (1+\varepsilon) A_1^T A_1 + (1+\varepsilon)(1+\varepsilon^{-1}) A_2^T A_2 + (1+\varepsilon^{-1})^2 (\sum_{l=3}^{N} A_l)^T (\sum_{l=3}^{N} A_l)$$
$$\vdots$$
$$\leq (1+\varepsilon) A_1^T A_1 + (1+\varepsilon)(1+\varepsilon^{-1}) A_2^T A_2 + \cdots$$
$$+ \cdots (1+\varepsilon)(1+\varepsilon^{-1})^{N-2} A_{N-1}^T A_{N-1} + (1+\varepsilon^{-1})^{N-1} A_N^T A_N$$

Lemma 3: Let $\alpha_l = 1/(1+\varepsilon^{-1})^{l-1}\,(1+\varepsilon)$, $l = 1, 2, ..., N-1$ and $\alpha_N = 1/(1+\varepsilon^{-1})^{N-1}$, there exists any positive constant ε such that $\alpha_l \in [0, 1]$ $(1 \le l \le N)$ and:

$$\sum_{l=1}^{N} \alpha_l = 1$$

Proof: Obviously, if there exists a positive constant ε then $0<\alpha_l<1$, $l = 1, 2, ..., N$ and

$$\sum_{l=1}^{N} \alpha_l = \frac{1}{1+\varepsilon} + \frac{1}{(1+\varepsilon^{-1})(1+\varepsilon)} + \cdots + \frac{1}{(1+\varepsilon^{-1})^{N-2}(1+\varepsilon)} + \frac{1}{(1+\varepsilon^{-1})^{N-1}}$$

$$= \frac{\dfrac{1}{1+\varepsilon} - \dfrac{1}{(1+\varepsilon)(1+\varepsilon^{-1})^{N-1}}}{1 - \dfrac{1}{1+\varepsilon^{-1}}} + \frac{1}{(1+\varepsilon^{-1})^{N-1}} = 1$$

$(\alpha_1, \alpha_2, ..., \alpha_{N-1})$ is a geometric progression).

In the light of Lemma 2 and 3, we have:

$$\sum_{l=1}^{N} \alpha_l = \Big(\sum_{l=1}^{N-1} \frac{1}{(1+\varepsilon^{-1})^{l-1}(1+\varepsilon)}\Big) + \frac{1}{(1+\varepsilon^{-1})^{N-1}} \tag{21}$$

Therefore, we denote:

$$\bar{\bar{A}}_l = \Big(\sum_{l=1}^{N-1} \frac{(1+\varepsilon^{-1})^{l-1}}{(1+\varepsilon)} A_l\Big) + (1+\varepsilon^{-1})^{1-N} A_N$$
$$\bar{\bar{B}}_l = \Big(\sum_{l=1}^{N-1} \frac{(1+\varepsilon^{-1})^{l-1}}{(1+\varepsilon)} B_l\Big) + (1+\varepsilon^{-1})^{1-N} B_N \tag{22}$$

Corollary 3: There exists a switching law (CSSS), the switched time-delay system (1) is delay-independent asymptotically stable if there exist matrices P>0 and Q>0 satisfying LMIs (23a) and (23b) for $l = 1, 2, ..., N$.

$$H\bar{\bar{A}}_l^T + \bar{\bar{A}}_l H + H + \bar{\bar{B}}_l Q \bar{\bar{B}}_l^T < 0 \tag{23a}$$

$$Q \ge H \tag{23b}$$

where, $H = P^{-1}$.

Therefore, the result of Corollary 3 can simplify parameter and reduce $\alpha_1, \alpha_2, ..., \alpha_N$ to ε. Thus, it is easy to analyze the switched time-delay systems.

EXAMPLE

Example 1: Consider the switched time-delay system composed of two subsystems given as:

Subsystem 1:

$$\dot{x}(t) = A_1 x(t) + B_1 x(t-\tau) = \begin{bmatrix} -5 & 2 \\ 0 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.1 \end{bmatrix} x(t-\tau) \tag{24a}$$

Subsystem 2:

$$\dot{x}(t) = A_2 x(t) + B_2 x(t-\tau) = \begin{bmatrix} -4 & 1 \\ 1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0.2 & 0 \\ 0.1 & 0.2 \end{bmatrix} x(t-\tau) \tag{24b}$$

The switched time-delay system with two subsystems (N = 2), in view of the stability conditions of Corollary 1, inequalities (8) can be written as follows:

$$HA_1^T + A_1 H + H + B_1 Q B_1^T < 0$$
$$HA_2^T + A_2 H + H + B_2 Q B_2^T < 0$$
$$Q \ge H \quad H = P^{-1}$$

By using LMI tool of MATLAB software, we find:

$$H = \begin{bmatrix} 0.2182 & 0.1046 \\ 0.1046 & 0.3564 \end{bmatrix} \quad Q = \begin{bmatrix} 1.4300 & 0.0466 \\ 0.0466 & 1.4980 \end{bmatrix} \quad P = \begin{bmatrix} 5.3352 & -1.5665 \\ -1.5665 & 3.2661 \end{bmatrix}$$

Therefore, for arbitrary constant delay τ, the switched time-delay system (24) can be stabilized by arbitrary switching sequences (SASS).

With time-delay τ = 2 sec, swithing during [0, 4] sec and initial value $x(0) = [-20\ 10]^T$, the trajectories of the switched time-delay system (24) is shown in Fig. 1a. The switching sequence is shown in Fig. 1b.

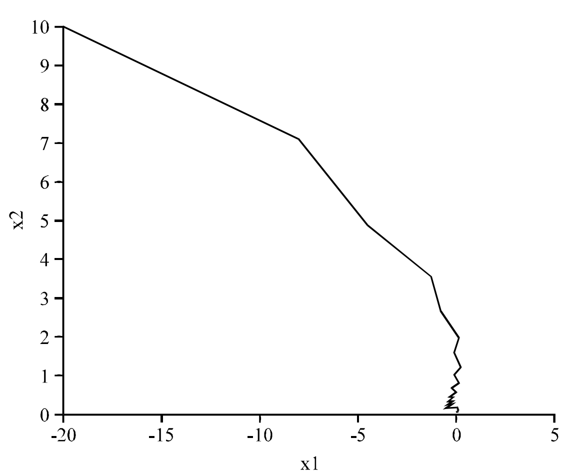

Fig. 1a: State responses of system (24)

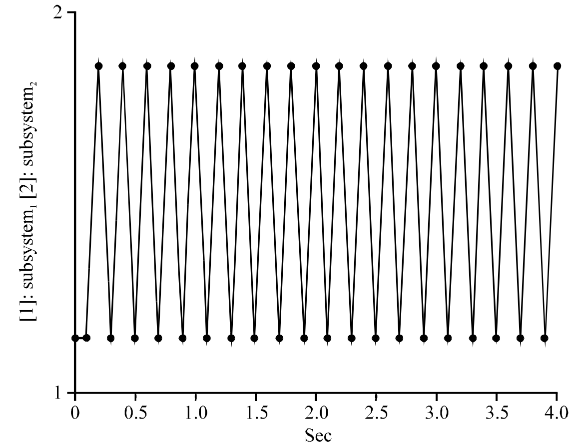

Fig. 1b: Switching sequences of system (24)

Example 2: Consider the switched time-delay system composed of two subsystems given as:

Subsystem 1:

$$\dot{x}(t) = A_1 x(t) + B_1 x(t-\tau) = \begin{bmatrix} -5 & 2 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.1 \end{bmatrix} x(t-\tau) \tag{25a}$$

Subsystem 2:

$$\dot{x}(t) = A_2 x(t) + B_2 x(t-\tau) = \begin{bmatrix} 1 & -1 \\ 1 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0.2 & 0 \\ 0.1 & 0.2 \end{bmatrix} x(t-\tau) \tag{25b}$$

The switched time-delay system with two subsystems (N = 2), in view of the stability conditions of Corollary 3, inequalities (23) can be written as follows:

$$H\bar{\bar{A}}_1^T + \bar{\bar{A}}_1 H + H + \bar{\bar{B}}_1 Q \bar{\bar{B}}_1^T < 0$$

$$H\bar{\bar{A}}_2^T + \bar{\bar{A}}_2 H + H + \bar{\bar{B}}_2 Q \bar{\bar{B}}_2^T < 0$$

$$Q \geq H \quad H = P^{-1}$$

We choose $\varepsilon = 2$ and use LMI tool of MATLAB software, then we get:

$$H = \begin{bmatrix} 34.7922 & 9.9729 \\ 9.9729 & 28.1921 \end{bmatrix}, Q = \begin{bmatrix} 65.0685 & 4.7833 \\ 4.7833 & 61.9757 \end{bmatrix} \text{ and}$$

$$P = \begin{bmatrix} 0.0320 & -0.0113 \\ -0.0113 & 0.0395 \end{bmatrix}$$

Therefore, for arbitrary constant delay τ, the switched time-delay system can be stabilized by Construction of Stabilizing Switching Sequences (CSSS).

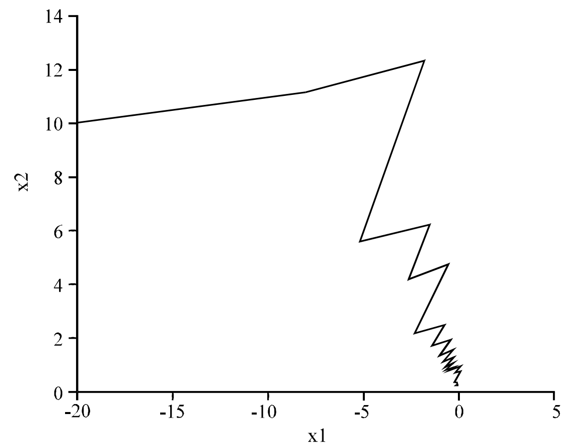

Fig. 2a: State response of system (25)

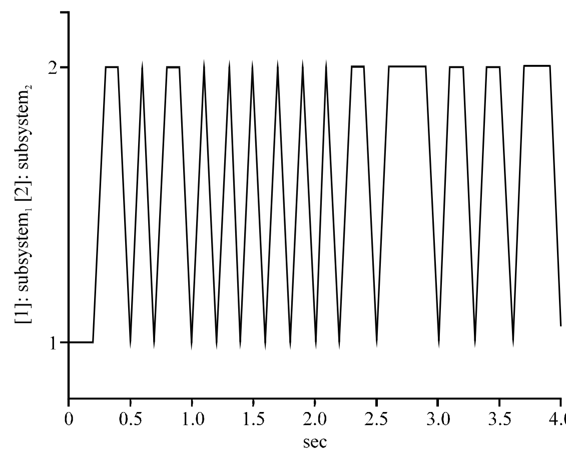

Fig. 2b: Switching sequences of system (25)

Switching law (CSSS): Switched time-delay system with arbitrary N individual systems is switched to or stay at mode l at time t if (26) is satisfied at time t.

$$x^T(t)(A_1^T P + PA_1)x(t) + 2x^T(t)PB_1 x(t-\tau) < 0 l \in \{1, 2\} \tag{26}$$

With time-delay $\tau = 2$ sec, swithing during [0, 4] sec and initial value x (0) = $[-20\ 10]^T$, the trajectories of the switched time-delay system (24) is shown in Fig. 2a. The switching sequence is shown in Fig. 2b. Therefore, if we select positive constant ε and satisfy inequalities (23) then the system is delay-independent asymptotically stable for construction of stabilizing switching sequences (CSSS).

Example 2 is exploited to illustrate the proposed schemes, stability conditions that guarantee the switched discrete time-delay system is delay-independent asymptotically stable for construction of stabilizing switching law. In the light of Corollary 2 and Corollary 3, we simplify parameter and reduce α_1, α_2, ..., α_N to ε. Thus, it is easy to analyze the switched time-delay system. And we constructively design a switching rule which can guarantee the stability of the switched systems. Otherwise, the particular method can be applied to cases whose individual subsystems are unstable.

CONCLUSION

The study adopted Lyapunov-Razumikhin stability theorem to study the delay-independence stability analysis of a class of switched time-delay system. We derive stability conditions that guarantee the switched time-delay system is delay-independent asymptotically stable for arbitrary switching sequences and construction of stabilizing switching law. The main advantages of our approach are that simplify parameter and reduce α_1, α_2, ..., α_N to ε then it is easy to analyze the switched time-delay system, can be applied to individual subsystems whose includes unstable subsystems, can extend to the case of arbitrary subsystems of switched delay system and develop the simple switching rule to stabilize the switched time-delay system and construct LMI-based design procedures for stability analysis. The subject is interesting and important and it will be attracting increasing attention in future.

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