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Phase Response of Fine Frequency Grid Reconstruction of Sampling Oscilloscopes Based on the NTN Calibration

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Abstract: This study describes an algorithm for determining the fine phase response of equivalent sampling oscilloscopes of a linear time-invariant response function from its magnitude. The Nose-to-Nose (NTN) calibration method can give the phase response of the equivalent sampling oscilloscopes, but the phase resolution can only achieve 250 MHz because the limit of the technology. In this study the fine phase response has been reconstructed. Although the truncation of the Kramers-Kronig transform using three basic functions may approximate gives rise to large errors in estimated phase, these errors may be approximated by using three basic functions. This result rests on data obtained by an NTN technique in combination with a swept-sine calibration procedure. The NTN technique yields magnitude and phase information over a broad bandwidth, yet has low frequency resolution. The swept-sine procedure returns only the magnitude of the oscilloscope response function, yet can be made at any frequency at which fundamental microwave power standards are available. As an example, we get the fine phase response of equivalent sampling oscilloscopes Agilent 86100 C from dc to 40 GHz; its frequency resolution achieves 1 MHz. In the process of this analysis, we observe that the true oscilloscope response function as measured by the NTN calibration is indistinguishable from the reconstructed phase response over a very large bandwidth. At last we analyze the phase uncertainty of the phase response, and the uncertainty analysis process is provided. The results show that the algorithm can be used to get the fine phase response of the equivalent sampling oscilloscopes.

Key words: NTN calibration, equivalent-sampling oscilloscopes, phase response reconstruction, Kramers-Kronig relations, phase uncertainty

INTRODUCTION

During the 1990s, technological and theoretical advances greatly simplified the direct measurement of the complex large-signal response of devices, circuits, and systems containing nonlinear elements. Nonlinear Vector Network Analyzer (NVNA) is the newest and the most practical tool to complete the large signal network analyzes (Remley and Sohreurs, 2007; Lin *et al*., 2010). At this stage the NVNA faces on two large problems: One is how to push the highest measurement frequency (50 GHz) to higher frequency limit; another is how to increase the phase resolution of the phase calibration (Moer and Rolain, 2006). This study is the initial attempt in order to solve the second problem.

Nose-to-Nose (NTN) calibration method is using two equivalent sampling oscilloscopes connected directly, one producing the kick-out pulse as the excitation signal source, another as the receiver. We can get the response function by using the deconvolution method. But the frequency resolution can only achieve 250 MHz because the limit of the NTN technology.

The Agilent 86100 sampling oscilloscope has a limited data memory. When measuring an impulse response, one has to meet two boundary conditions. On one hand, one wants to keep the time window for the measurement as small as possible to allow high time resolution and reasonable S/N ratio. On the other hand, one must keep the window long enough to make sure that the impulse response is not truncated. In the case of an NTN measurement, a kick-out pulse is generated by one oscilloscope and measured by a second oscilloscope. However, due to the imperfect internal match of both oscilloscopes, some of the pulse is reflected back and forth between both sampling oscilloscopes. After averaging the kick-out pulses, a first reflection is clearly visible at approx. 1 ns delay with respect to the main pulse. By comparing the spectrum after averaging for different time window lengths, it was found experimentally

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that one can measure up to the third reflection. The time window for the measurement was therefore set to approx. 4 ns, so the frequency resolution can only achieve 250 MHz (Degroot *et al*., 2000).

As a contrast to the NTN calibration, swept-sine (frequency-domain) measurements can be used to determine the magnitude of the frequency response of an oscilloscope. The swept-sine calibration can be made at any frequency at which fundamental microwave power standards are available, typically from 1 MHz to greater than 50 GHz. However, since the swept-sine calibration does not give phase information, an oscilloscope calibrated using this technique alone is not adequately characterized for time-domain metrology (Dienstfrey *et al*., 2006; Wang *et al*., 2009).

The study describes a procedure for reconstructing the phase response of the oscilloscope response function from its magnitude. Phase response functions have the property that, in principal, the phase can be recovered from the Kramers-Kronig (K-K) transform of the logarithm of the magnitude. In practice, naive attempts to apply the standard theory can yield extremely large absolute errors in the computed phase. Although, the truncation error may be large in absolute scale, due to the localizing nature of the K-K transform, this error is inherently low rank in the sense that using three customized basis functions can approximate it.

The study solves a linear least squares problem for the difference between the NTN measured values of the phase and the values computed via a phase assumption as an expansion in the specialized three basis functions by using the phase response of the NTN calibration and the magnitude of the swept-sine method. This expansion corrects the absolute size and coarse trends in the truncated phase approximation. Reconstruct the phase response of the Agilent 86100C equivalent sampling oscilloscopes. Then analyze the phase uncertainty of the oscilloscopes.

PHASE RECONSTRUCTION

An arbitrary frequency-domain response function can be factored as:

$$h(f) = \tilde{h}(f)\exp(j2\pi f\tau)\,B(f) \tag{1}$$

where, $\tilde{h}(f)$ is the complex frequency response, τ is a real time offset and B (f) is an "all-pass filter". One distinguishing characteristic of a phase function is that its phase is determined by its magnitude via a K-K transform. There are several ways to express this, here we use the simplest: $\tilde{h}(f) = |\tilde{h}(f)|\exp(j\varphi(f))$, then we can get:

$$\varphi(f) = \frac{2f}{\pi}\int_0^{\infty}\frac{1}{f^2 - s^2}\ln\left(\left|\tilde{h}(s)\right|\right)ds \tag{2}$$

The expression above is the K-K transform. This equation is strict right when the integral ceiling is infinity. But infinity in frequency measurement is unattainable in actual operation and the signal generator cannot reach the infinite frequency either therefore we should cut off the integral ceiling. So a pervasive problem in the application of any K-K analysis is to estimate the error due to the finite bandwidth of a measurement. The integral in (2) should be truncated:

$$\varphi_{\Omega}(f) = \frac{2f}{\pi}\int_0^{\Omega}\frac{1}{f^2 - s^2}\ln\left(\left|\tilde{h}(s)\right|\right)ds \tag{3}$$

where, Ω is the maximum frequency attainable by experiment. The problem now is how to estimate the effect of Ω. Using the method of Dienstfrey *et al.* (2006) we get Eq. 4 and 5:

$$\Delta(f) \approx \alpha_1\Psi_1(f) + \alpha_2\Psi_2(f) + \alpha_3\Psi_3(f) \tag{4}$$

$$\begin{aligned}
\Psi_1(f) &= \sqrt{3}f \\
\Psi_2(f) &= \frac{1}{\sqrt{\frac{\pi^2}{3} - 3}}\cdot\left(\ln\left(\frac{1+f}{1-f}\right) - 3f\right) \\
\Psi_3(f) &= 18.0102\cdot\sum_{n=0}^{\infty}\frac{4}{(2n+1)^2}f^{2n+1} - 4.1224\cdot\ln\left(\frac{1+f}{1-f}\right) - 66.9176\cdot f
\end{aligned} \tag{5}$$

where, $\{\Psi_1(f), \Psi_2(f), \Psi_3(f)\}$ are the orthonormal basis functions. And the condition number of the matrix $A_{mj} = \Psi_j(f'_m)$ can be reduced, so the results will be stable when the test data change and the reliability will also be enhanced. Normalization of orthonormal $\{\Psi_1, \Psi_2, \Psi_3\}$ is shown for the case $\Omega = 1$ in Fig. 1.Then substitute f/Ω for f in Eq. 5 and the value of $\left(1/\sqrt{\Omega}\right)\Psi_i(f/\Omega)$ is shown in Fig. 1.

DATA COLLECTION

We do the experimentation by using the Agilent 86100C equivalent sampling oscilloscopes. First we connect the two same equivalent sampling oscilloscopes module directly in order to get the amplitude and phase responses of the 86100 C from 1-50 GHz using the NTN calibration and the frequency resolution is 250 MHz (shown in Fig. 2, 3) (Qinghua *et al*., 2008; Zhang *et al*., 2006). In Fig. 2, the points are the amplitude response of the 86100 C measured by the NTN calibration by 0.25 GHz interval. And the points in Fig. 3 are phase response.

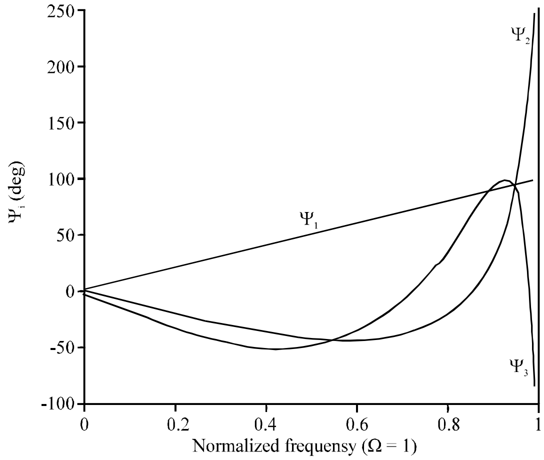

Fig. 1: Plot of orthonormal basis functions

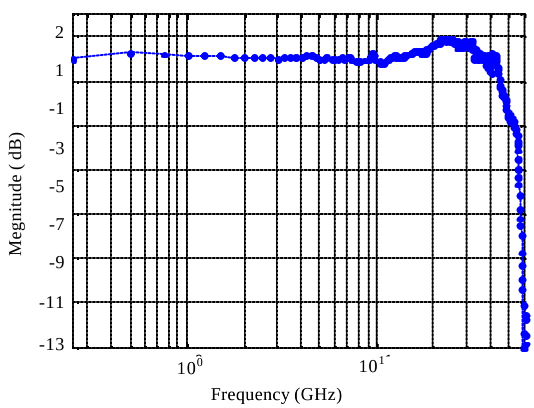

Fig. 2: Amplitude response of the NTN calibration

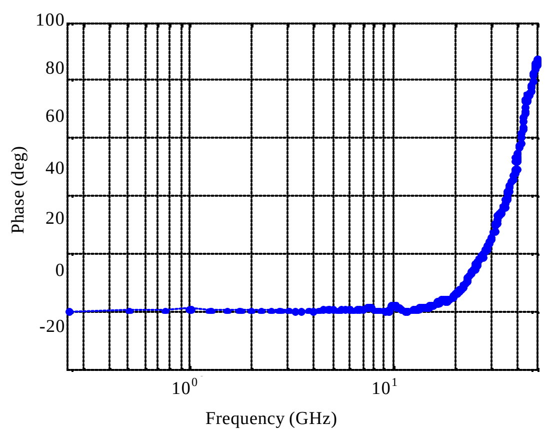

Fig. 3: Phase response of the NTN calibration

Use the IFFT method in order to get the time-domain response of the equivalent sampling oscilloscopes from the frequency response which we get using the NTN calibration. Then do Z-transform to analyze the zero-pole point distribution of the system. In order to know whether we can use the K-K transform to analyze the oscilloscopes. The result is shown in Fig. 4.

In Fig. 4, the circles show the positions of the zero points of the transfer function of the Agilent 86100C sampling oscilloscope and the crosses show the positions of the pole points. From the Fig. 4 we can see all the zero-pole points of the frequency response of the 86100C equivalent sampling oscilloscopes are in the unit circle. It verifies that the K-K transform can be used to get the phase response through the amplitude response.

Second get the magnitudes of the frequency response of the 83484A module of the oscilloscopes using swept-sine measurements. Use the Agilent E8257D PSG Analog Signal Generator (250 KHz-40 GHz) to be the trigger source and the AV1487 Wideband Synthesis Frequency Sweep Signal Generator (made by CETC No. 41 institute 0.01-40 GHz) to be the signal source. Both the trigger source and the signal source use the same reference (10 MHz). The calibration kits are HP 437B Power Meter and Agilent 8487A Power Sensor (50 MHz-50 GHz 1 μW-100mW). The mean of the 83484A background noise is 566.03 μV and variance is 10.74 μV.

The magnitude collection is in three intervals, they are 1-200 MHz 1MHz interval; 10-1000 MHz 10 MHz interval; 0.2-40 GHz 0.2 GHz interval, totally 476 sampling frequencies. Finally unite three swept-sine measurements to an arbitrary frequency grid data using the merging method. The blue, pink and red points in Fig. 5 represent the three different amplitude response of the 86100 C measured by the swept-sine measurements.

This study selects the 1-40 GHz magnitude data of NTN calibration and the 1MHz-40 GHz magnitude data of the swept-sine measurements to data merging. The global data set is formed by computing the average of the magnitude measurements at all frequencies where experiments overlapped. The average is formed by weighting each measurement by the inverse of its associated variance, i.e., the square of its standard uncertainty. In Fig. 6 the red points are the amplitude response of the 86100 C by merging the results of NTN calibration and swept-sine measurements. Analyze the data with the 1-30 GHz phase response of the NTN calibration.

THEORY AND RESULT OF THE PHASE FITTING

The frequencies at which the phase is measured $\{f'_m|m=1,...,M\}$ need not be the same as those of the magnitude measurements $\{f_n|n=1,...,N\}$, nor even as

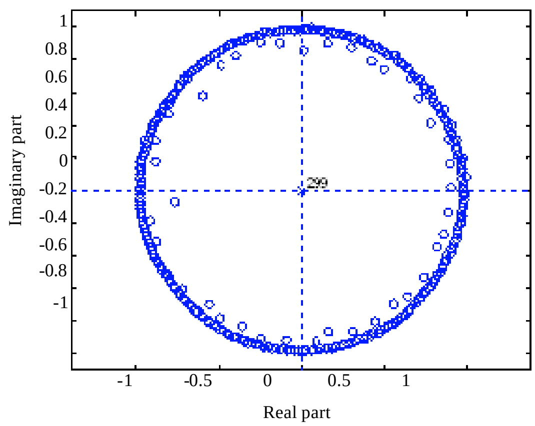

Fig. 4: The zero-pole point distribution of 86100C

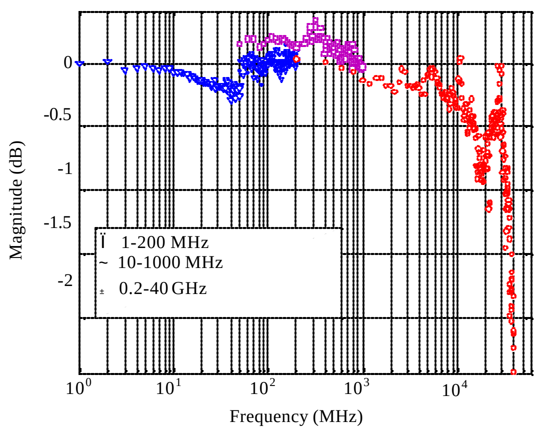

Fig. 5: Magnitude of the swept-sine measurements

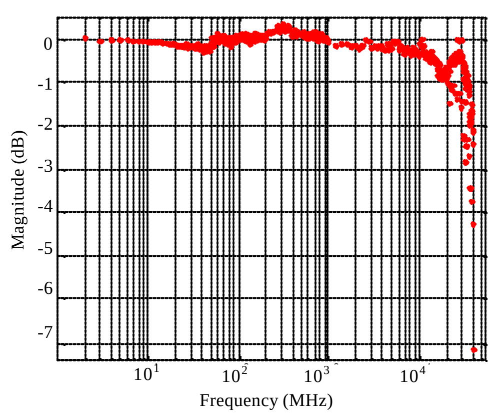

Fig. 6: Magnitude of merging the NTN calibration and the swept-sine measurements

dense. The frequencies for which we have magnitude data should "cover" the frequencies associated with the phase measurements, or it will make the data divergent.

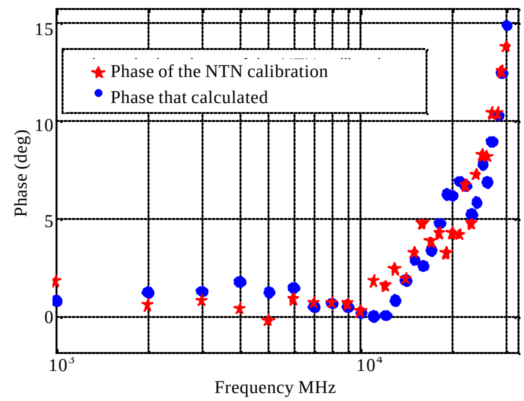

Fig. 7: Compare the phase of NTN with the result of reconstruction

$\varphi_{NTN}(f'_m)$ is the NTN calibration result. The result is a dense matrix K (f'_m, f_n) of order M×N = 30×476 (Dienstfrey *et al*., 2006). Here data of the NTN calibration is 30; data of the magnitude is 476, suppose:

$$\Delta(f'_m) = \varphi_{NTN}(f'_m) - \mathbf{K}(f'_m, f_n)\mathbf{h} = \alpha_1\Psi_1(f'_m) + \alpha_2\Psi_2(f'_m) + \alpha_3\Psi_3(f'_m) \quad (6)$$

where, h is the vector of logarithms of the magnitude response measurements. Then solve for the undetermined coefficients α_1, α_2, α_3 in a least squares sense. Finally, given sufficiently low residual in the least squares fit, an indicator of the validity of the phase assumption, we compute the phase of the oscilloscope response function as:

$$\varphi_{K-K}(f) = \mathbf{K}(f, f_n)\mathbf{h} + \alpha_1\Psi_1(f) + \alpha_2\Psi_2(f) + \alpha_3\Psi_3(f) \quad (7)$$

In this equation, the domain of the K-K operator consists of the same frequencies as the magnitude measurements $\{f_n\}$, the target frequencies f are some arbitrary desired frequency grid. In our experiment $\alpha_1 = -0.4175$ $\alpha_2 = -1.1156$ $\alpha_3 = 0.4798$, standard deviation of the 30 frequency points (1-30 GHz) is 1. 26. We compare the phase calculated with the phase of the NTN calibration which we observe as the true oscilloscope phase response function in Fig. 7. The red stars are the phase response measured by NTN calibration; the blue rounds are the reconstructed phase.

In Fig. 8, the red line is the phase response of fine frequency grid reconstruction and its frequency resolution achieves 1 MHz, the purple points are the phase response measured by NTN calibration by 1 GHz interval.

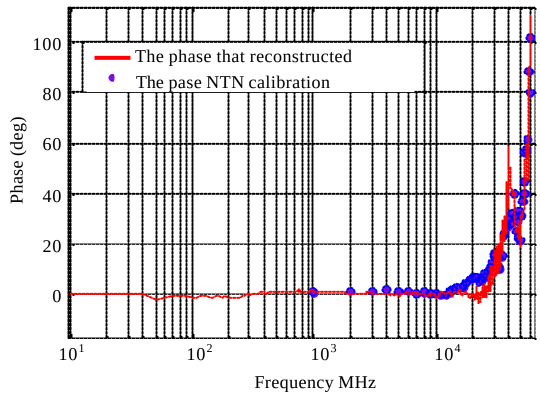

Fig. 8: Phase response result of oscilloscopes (1 MHz-40 GHz)

In the process of this analysis, we observe that the true equivalent sampling oscilloscope phase response function as measured by the NTN calibration is indistinguishable from the reconstructed phase response from 1 to 40 GHz.

UNCERTAINTY ANALYSIS

The uncertainty of the phase get from the K-K transform consists of three parts: the uncertainty of the NTN calibration: u^2 [φ_{NTN}]; the uncertainty of the frequency sweep method: u^2 [h] and the uncertainty from the transform matrix and the fitting.

As the phase is getting from K-K transform of the amplitude information:

$$\varphi(f) = \frac{2f}{\pi}\int_0^\infty \frac{1}{f^2 - s^2}\ln\left(\left|\tilde{h}(s)\right|\right)ds \tag{8}$$

We can get the expressions (9) by using some mathematical transform.

$$\varphi(f'_m) = \sum_{n=0}^{N} y_n KB_n(f'_m) \tag{9}$$

where, K_{mn} = KB_n (f'_m) is the mth row and the nth column element of the matrix. K is the operator and B_n is the piecewise linear function, as shown in Eq. 10-12.

$$K = \frac{1}{\pi}\int_\alpha^\beta \frac{2f}{f^2 - s^2}(ms + b)ds = \frac{m}{\pi} f \ln\left|\frac{\alpha^2 - f^2}{\beta^2 - f^2}\right| - \frac{b}{\pi}\ln\left|\frac{\alpha + f}{\alpha - f}\cdot\frac{\beta - f}{\beta + f}\right| \tag{10}$$

$$B_n(s;f) = \begin{cases} \dfrac{s - f_{n-1}}{f_n - f_{n-1}} & f_{n-1} \le s < f_n \\ \dfrac{f_{n+1} - s}{f_{n+1} - f_n} & f_n \le s < f_{n+1} \\ 0 & \text{elsewhere} \end{cases} \tag{11}$$

$$KB_n(f) = \frac{1}{\pi(f_n - f_{n-1})}\cdot\left(f\ln\left|\frac{f_{n-1}^2 - f^2}{f_n^2 - f^2}\right| + f_{n-1}\ln\left|\frac{f_{n-1} + f}{f_{n-1} - f}\cdot\frac{f_n - f}{f_n + f}\right|\right) - \frac{1}{\pi(f_{n+1} - f_n)}\cdot\left(f\ln\left|\frac{f_n^2 - f^2}{f_{n+1}^2 - f^2}\right| + f_{n+1}\ln\left|\frac{f_n + f}{f_n - f}\cdot\frac{f_{n+1} - f}{f_{n+1} + f}\right|\right) \tag{12}$$

Given a scalar random variable X with mean E (X) = X_0 and variance var (X) = u^2 [X] and variable Y = F (X), where F is sufficiently differentiable, we assume that:

$$E[Y] = F_0[X_0]; u^2[Y] = \left(\left.\frac{\partial F}{\partial s}\right|_{s=X_0}\right)^2 u^2[X] \tag{13}$$

Using the rules for linear propagation of errors, we find that:

$$u^2[\varphi_\Omega] = \mathbf{K}\Sigma_{u^2[\mathbf{h}]}\mathbf{K}^* \tag{14}$$

where, $\Sigma_{u^2[\mathbf{h}]}$ is the diagonal matrix containing the uncertainties (systematic and random) of the vector h. (In this experiment it is diagonal matrix of order 476×476) We can obtain that the standard uncertainty of our swept-sine data is typically between 0.004-0.17 dB by using the method of Lin *et al*. (2006), Williams *et al*. (2005) and Williams *et al*. (2007).

The main diagonal elements of the matrix u^2 [φ_Ω] are the uncertainty of φ (f'_m) which we need. Since the NTN calibration and swept-sine measurements are independent, the uncertainties u^2 [φ_{NTN}] and u^2 [φ_Ω] can be added in quadrature.

It is at this point that we benefit from having pre-orthogonalized the natural set of expansion functions to instead form the orthonormal basis {Ψ_1 (f), Ψ_2 (f), Ψ_3 (f)}. It reduces the condition number of the matrix A_{mj} = Ψ_j (f'_m), therefore the calculation errors of the vector α = {α_1, α_2, α_3}, decrease observably.

Here we assume that the relevant random variables are h and the set {α_1, α_2, α_3}, by this argument, are independent. The error propagation through the integral operator and the three point-wise multiplications obey their respective linear propagation formalisms and the resulting uncertainties of each are added in quadrature (Jargon *et al*., 2010).

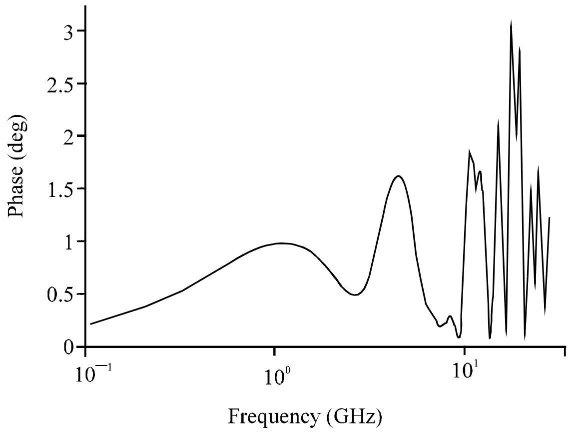

Fig. 9: Phase response uncertainty

The final result of the phase uncertainty form 100 MHz-30 GHz is shown in Fig. 9. The phase uncertainty of the oscilloscopes is defined by 2σ and σ is the standard deviation. In Fig. 9 the line is the phase response uncertainty of the reconstructed phase.

In the process of this analysis, we analyze the uncertainty of the NTN calibration, the frequency sweep method and the transform matrix and the fitting. Since the NTN calibration, swept-sine measurements and the algorithm are independent with each other, the uncertainties can be added in quadrature. Then we get the uncertainty of the reconstructed phase response.

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CONCLUSION

In this study we verify the equivalent sampling oscilloscopes reconstruct the phase response of 86100 C by using the K-K transform in combination with the harmonic phase response measurement of the NTN calibration and the magnitude of the swept-sine measurements. In the process of this analysis, we observe that the true oscilloscope response function as measured by the NTN calibration is indistinguishable from the reconstructed phase response over a very large bandwidth. Analyze the uncertainty of the reconstructed phase response.

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