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Single-Sensor Parametric Location Algorithm and Accuracy Analysis of TDMA Moving Target Based on TOA and DOA Measurements

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Abstract: The problem of TDMA (Time division multiple address) target tracking using the measurements of time of arrival (TOA) and direction of arrival (DOA) is addressed. According to the time synchronization property of TDMA system, the location model of quasi-periodic slot signal of TDMA target is established. The method of parametric Target Motion Analysis (TMA) is used in analyzing the observability condition of target position and deduces the theoretic analysis algorithm of location accuracy. The single-sensor location of TDMA target can be realized using measurements of more than two target positions without the linearization of nonlinear observation equation. The simulation results illustrate the validity of the proposed algorithms.

Key words: Wireless sensor networks (WSN), passive location, target motion analysis (TMA), bearing-only, range estimation, tracking

INTRODUCTION

The bearing-only target motion analysis (TMA) is a classical single-sensor passive location algorithm (Nardone and Graham, 1997; Moon and Nordone, 2000) but its application is limited because the observer must do maneuver movement for moving emitter location (Jauffret *et al.*, 2010; Hua *et al.*, 2010).

Several researchers have investigated the methods of estimating the motion trajectory by using measurements of time of arrival (TOA) and direction of arrival (DOA) which is also a kind of TMA. The characteristics of the constant-speed motion and the periodicity of signal are commonly used in TMA. In literature of Yang and Zheng (1996), the nonlinear observation equation is linearized according to Taylor expansion method and Weighted Least Square (WLS) and Weighted Extended Kalman Filter (WEKF) are used in positioning the moving target without the sensor movement. Zhong-Kang *et al.* (2008) realized the target location by estimating the navigation angle, range and velocity. The observability condition of target position based on the measurements of TOA and DOA can be analyzed through the linearization technique of nonlinear observation equation (Li *et al.*, 2004) and the observability analysis principle of nonlinear system (Xie *et al.*, 2007).

The TDMA system is a kind of time synchronization one, that is to say, the time of TDMA terminal is

synchronous with the system (Liu *et al.*, 2007). The sending time difference between different slots of TDMA terminals is an integer multiple of the slot signal cycle. A TDMA target transmits slot signals with slot interval as a benchmark but whether it can transmit signals is restricted by the slot allocation rules (Lee and Chang, 2004; Qin *et al.*, 2009). A target cannot transmit signals in each slot. So, the slot signal has quasi-periodic feature.

The closed-form TDMA target location algorithm with the variables of position coordinates, velocity and range is proposed based on the measurements of TOA and DOA in this study by parametric target motion analysis. The observability condition of target position is also analyzed according to the rank concept of matrix. The proposed algorithm simplifies the target localization process through the parametric target motion analysis.

SINGLE-SENSOR LOCATION MODEL OF TDMA TARGET

Quasi-periodic slot signal of TDMA system: Each terminal shares wireless channel with frame-slot mode in the TDMA system, as shown in Fig. 1.

The time interval of frame is the minimum distribution cycle (T_f) of TDMA system, suppose that T_f is a constant and the sending time of No. n frame is set to be t_{fn} , there by:

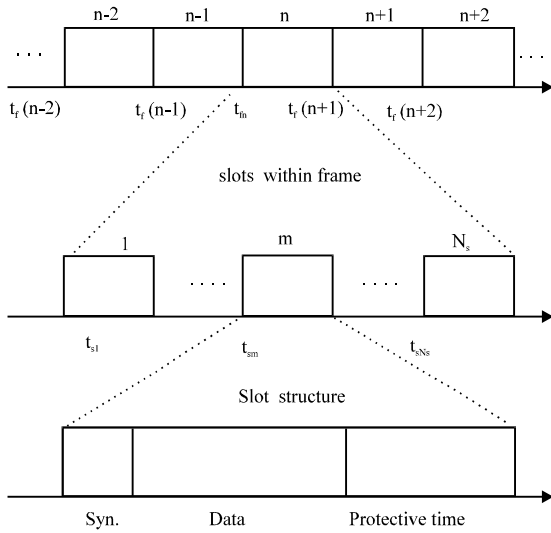


Fig. 1: Frames and slots in TDMA system

$$t_{fn} = (n-1)T_f + t_{r1} \quad (1)$$

One frame is divided into N_s slots and the slot is the minimum allocation unit for every terminal. Each slot comprises synchronization header, data and protective time. Suppose that the slot duration is not changed.

$s(n, m)$ is used for denoting the No. m slot within No. n frame, the sending time is t_{nm} , the slot duration time is $T_s = T_f/N_s$ there by:

$$t_{nm} = t_{n(m-1)} + T_s = (m-1)T_s + t_{fn} = (m-1)T_s + (n-1)T_f + t_{r1} \quad (2)$$

It can be known from Eq. 1 and 2 that if the sending time of any one frame is known, the sending time of any slot in subsequent frame can be calculated.

Suppose that the sensor receives the signals of No. m slot in No. n frame, the relationship among the receiving time t_{nm} , sending time t_{nm} and range r between sensor and TDMA terminal is as follows:

$$t_{nm} = t_{nm} + r/c \quad (3)$$

The sending time of any slot can be confirmed if the TDMA system synchronization time is known. For this synchronized TDMA system, the range r can be estimated according to the receiving time of slot signal. If the range r is known, the slot sending time can be confirmed through the slot receiving time, then the system synchronization time can be calculated through the Fig. 1 and 2, namely, the sending time of any slot can be determined.

Suppose the sensor receives two slot signals, their sending times, receiving times and ranges are,

respectively represented as $t_{m_1m_1}$ and $t_{m_2m_2}$, $r_{n_1m_1}$ and $r_{n_2m_2}$, n_1, m_1, n_2 and m_2 are unknown, the $t_{m_1m_1}$ and $t_{m_2m_2}$ and the difference between the two times can not be confirmed. Since the sending time difference of any two slots is the integer multiple (represented with Δm) of the slot period, there by:

$$t_{m_1m_1} - t_{m_2m_2} = \Delta m T_s \quad (4)$$

The following relationship of receiving times can be obtained by Eq. 3:

$$t_{m_1m_1} - t_{m_2m_2} = \Delta m T_s + (r_{n_1m_1} - r_{n_2m_2}) / c \quad (5)$$

The electromagnetic wave propagation time is taken into account during designing the TDMA system and protection time is preserved for each slot. In the above equation, $r_{n_1m_1} - r_{n_2m_2}$ represents the range difference of two slot signals to the sensor that is generally far smaller than the diffusion distance of the electromagnetic wave within the signal cycle T_s . If the following condition is met:

$$T_s > 2|r_{n_1m_1} - r_{n_2m_2}| / c \quad (6)$$

Δm can be calculated by:

$$\Delta m = \text{round}((t_{m_1m_1} - t_{m_2m_2}) / T_s) \quad (7)$$

where, $\text{round}()$ is a mathematic round function. From (5), the following is provided:

$$r_{n_1m_1} - r_{n_2m_2} = c(t_{m_1m_1} - t_{m_2m_2} - \Delta m T_s) \quad (8)$$

Therefore, the range difference between the different slots to sensor can be calculated according to the times of arrival (TOA) of slot signals.

Single-sensor location model: The two-dimensional target is taken as an example to illustrate the position principle. Assume that a TDMA target makes constant-speed movement in two-dimensional space with the velocity of (v_x, v_y) and the sensor is on coordinate origin as shown in Fig. 2. If the sensor receives $N+1$ signals provided with corresponding receiving time $t_{r(k-i)}$ and azimuth β_{k-i} ($i = 0, 1, \dots, N$) which are, respectively sent by the moving target on the positions T_k, T_{k-1}, \dots and T_{k-N} . The single-sensor location algorithm aims at realizing the estimation of two-dimensional coordinate (x, y) according

to the receiving time $t_{r(k-i)}$ and azimuth β_{k-i} ($i = 0, 1, \dots, N$) of the $N+1$ signals.

An actual target cannot make straight constant-speed movement always but its trajectory can be considered as a piecewise straight constant-speed movement one. For positioning a target of specific TDMA system, suppose that the cycle T_s of slot signal is known.

The target moving time between any two positions is the sending time difference of corresponding signals, namely:

$$\Delta t_{ij} = t_{r(k-i)} - t_{r(k-j)} = \Delta t_{0j} - \Delta t_{0i} \quad (9)$$

According to Eq. 4,

$$\Delta t_{ij} = \Delta m_{ij} T_s \quad (10)$$

Where:

$$\Delta m_{ij} = \text{round}((t_{r(k-i)} - t_{r(k-j)}) / T_s) \quad (11)$$

According to Eq. 8, the range difference between the target positions at T_{k-i} and T_{k-j} to the sensor is:

$$\Delta r_{ij} = r_{k-i} - r_{k-j} = c[(t_{r(k-i)} - t_{r(k-j)}) - \Delta t_{ij}] \quad (12)$$

Where the range equation is given by:

$$r_{k-i} = ((x - v_x \Delta t_{0i})^2 + (y - v_y \Delta t_{0i})^2)^{1/2} \quad (13)$$

TARGET LOCATION ALGORITHM

As shown in Fig. 2, the following relationship exists for the target position T_{k-i} :

$$y - v_y \Delta t_{0i} = r_{k-i} \cos \beta_{k-i} \quad (14)$$

According to Eq. 12, use r_k to express r_{k-i} , then:

$$r_{k-i} = r_k - \Delta r_{0i} \quad (15)$$

Equation 14 is deformed as:

$$y - v_y \Delta t_{0i} - \cos \beta_{k-i} r_k = -\Delta r_{0i} \cos \beta_{k-i} \quad (16)$$

In the same way, the following formula is derived for x-axis:

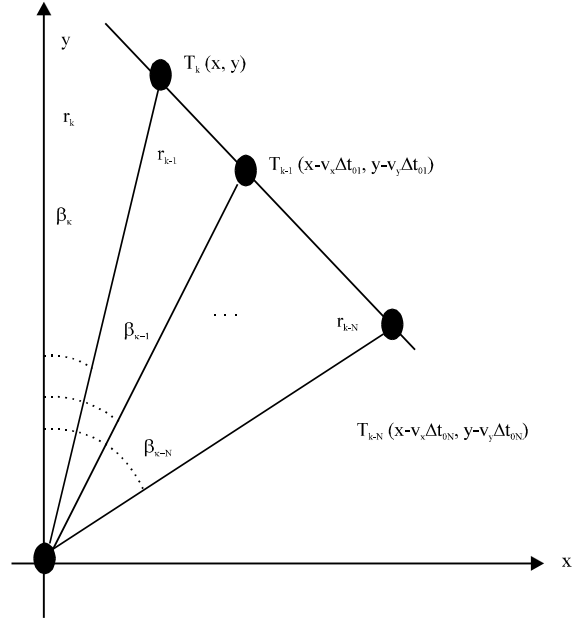


Fig. 2: Trajectory chart

$$x - v_x \Delta t_{0i} - \sin \beta_{k-i} r_k = -\Delta r_{0i} \sin \beta_{k-i} \quad (17)$$

According to Eq. 16 and 17, 2(N+1) equations can be established for N+1 positions. The matrix form of equations is:

$$AX = B \quad (18)$$

where, $\Delta t_{00} = 0, \Delta r_{00} = 0,$

$$A = \begin{pmatrix} 1 & 0 & -\Delta t_{00} & 0 & -\sin \beta_k \\ 0 & 1 & 0 & -\Delta t_{00} & -\cos \beta_k \\ 1 & 0 & -\Delta t_{0i} & 0 & -\sin \beta_{k-i} \\ 0 & 1 & 0 & -\Delta t_{0i} & -\cos \beta_{k-i} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & -\Delta t_{0N} & 0 & -\sin \beta_{k-N} \\ 0 & 1 & 0 & -\Delta t_{0N} & -\cos \beta_{k-N} \end{pmatrix}$$

$$X = \begin{pmatrix} x \\ y \\ v_x \\ v_y \\ r_k \end{pmatrix}, B = \begin{pmatrix} -\Delta r_{00} \sin \beta_k \\ -\Delta r_{00} \cos \beta_k \\ -\Delta r_{0i} \sin \beta_{k-i} \\ -\Delta r_{0i} \cos \beta_{k-i} \\ \vdots \\ -\Delta r_{0N} \sin \beta_{k-N} \\ -\Delta r_{0N} \cos \beta_{k-N} \end{pmatrix}$$

The solvable necessary and sufficient condition of Eq. 18 is A to be a column full rank matrix, i.e.:

$$\text{rank}\{A\} = 5 \quad (19)$$

Suppose that A_i expresses the column vector of column i of matrix A . When the target makes the radial motion, the azimuth of target maintains invariable expressed as β_0 . According to Eq. 18, the following relationship exists:

$$-\sin\beta_0 A_1 - \cos\beta_0 A_2 = A_5 \quad (20)$$

Equation 20 explains that the 5th column vector is linearly correlation with the 1st and 2nd columns, i.e., Eq. 19 is untenable. So, Eq. 18 has no solution when the target makes the radial motion.

Equation 18 is get under the condition of straight constant-speed movement. If the target makes a circle movement, the range difference Δr_{0i} along trajectory for any subscript i is equal to zero and the target position coordinates cannot be obtained because B in (18) is a zero-vector.

Above observability condition of target position is the same as literatures of Li *et al.* (2004) and Xie *et al.* (2007).

The position coordinates velocity components and range can be estimated simultaneously through Eq. 18.

THEORETIC ACCURACY ANALYSIS OF TARGET LOCATION

The differential of Eq. 16 is:

$$\begin{aligned} dy - \Delta t_{0i}(dv_y) - \cos\beta_{k-i}(dr_k) &= (r_k - \Delta r_{0i}) \sin\beta_{k-i}(d\beta_{k-i}) \\ + v_y(d\Delta t_{0i}) - \cos\beta_{k-i}(d\Delta r_{0i}) \end{aligned} \quad (21)$$

where, dy denotes the differential of variable y .

According to Eq. 10:

$$d\Delta t_{0i} = \Delta m_{0i}(dT_s) \quad (22)$$

According to Eq. 12:

$$d(\Delta r_{0i}) = r_k - r_{k-i} = c(dt_{ik} - dt_{r(k-i)} - \Delta m_{0i}dT_s) \quad (23)$$

Eq. 21 can be expressed as:

$$\begin{aligned} dy - \Delta t_{0i}(dv_y) - \cos\beta_{k-i}(dr_k) &= (r_k - \Delta r_{0i})\sin\beta_{k-i}(d\beta_{k-i}) \\ - c \cdot \cos\beta_{k-i}(dt_{ik}) + c \cdot \cos\beta_{k-i}(dt_{r(k-i)}) &+ (v_y + c \cdot \cos\beta_{k-i}) \Delta m_{0i}(dT_s) \end{aligned} \quad (24)$$

In the same way, the differential of Eq. 17 is:

$$\begin{aligned} dx - \Delta t_{0i}(dv_x) - \sin\beta_{k-i}(dr_k) &= -(r_k - \Delta r_{0i}) \cos\beta_{k-i}(d\beta_{k-i}) \\ - c \cdot \sin\beta_{k-i}(dt_{ik}) + c \cdot \sin\beta_{k-i}(dt_{r(k-i)}) &+ (v_x + c \cdot \sin\beta_{k-i}) \Delta m_{0i}(dT_s) \end{aligned} \quad (25)$$

The 2 (N+1) equations can be established for N+1 positions. The matrix form of equations is:

$$AdX = Cd\beta + DdT_r + EdT_s \quad (26)$$

Where:

$$C = \begin{pmatrix} -r_k \cos\beta_k & 0 & 0 \\ r_k \sin\beta_k & 0 & 0 \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ 0 & 0 & -(r_k - \Delta r_{0N})\cos\beta_{k-N} \\ 0 & 0 & (r_k - \Delta r_{0N})\sin\beta_{k-N} \end{pmatrix}$$

$$D = c \cdot \begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ -\sin\beta_{k-1} & \sin\beta_{k-1} & \dots & 0 \\ -\cos\beta_{k-1} & \cos\beta_{k-1} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ -\sin\beta_{k-N} & 0 & \dots & \sin\beta_{k-N} \\ -\cos\beta_{k-N} & 0 & \dots & \cos\beta_{k-N} \end{pmatrix}$$

$$E = \begin{pmatrix} (v_x + c \cdot \sin\beta_k) \Delta m_{00} \\ (v_y + c \cdot \cos\beta_k) \Delta m_{00} \\ (v_x + c \cdot \sin\beta_{k-1}) \Delta m_{01} \\ (v_y + c \cdot \cos\beta_{k-1}) \Delta m_{01} \\ \vdots \\ (v_x + c \cdot \sin\beta_{k-N}) \Delta m_{0N} \\ (v_y + c \cdot \cos\beta_{k-N}) \Delta m_{0N} \end{pmatrix}$$

$$d\beta = (d\beta_k \quad d\beta_{k-1} \quad \dots \quad d\beta_{k-N})^T$$

$$dT_r = (dt_{ik} \quad dt_{r(k-1)} \quad \dots \quad dt_{r(k-N)})^T$$

The solution of Eq. 26 is given by:

$$dX = C_1 d\beta + D_1 dT_r + E_1 dT_s \quad (27)$$

Where:

$$C_1 = (A^T A)^{-1} A^T C$$

$$D_1 = (A^T A)^{-1} A^T D$$

$$E_1 = (A^T A)^{-1} A^T E$$

Equation 27 illuminates that the target location estimation error is related with the measurement errors of location system, synchronization error of TDMA system and target states in two-dimensional space. The

measurement errors of location system include ones of azimuth ($d\beta$) and TOA measurements (dT_i) of $N+1$ signals. The synchronization error of TDMA system is a jitter one of the slot signal period (dT_s).

The target states related to location error include range, velocity, azimuth and navigation angle. Matrix C contains range $r_{k,i}$ and azimuth $\beta_{k,i}$. Vector E contains velocity components v_x and v_y . The navigation angle θ of target movement is related to the components v_x and v_y which relation is as follows:

$$\text{tg}\theta = \frac{v_y}{v_x} \quad (28)$$

Suppose that the errors of azimuth, TOA and synchronization are mutually independent white noises with zero mean and respective variances of σ_β^2 , σ_r^2 and σ_s^2 . The covariance matrix of dX is:

$$P_{dx} = E\{dX \times dX^T\} = C_1 P_\beta C_1^T + D_1 P_r D_1^T + E_1 P_s E_1^T \quad (29)$$

Where:

$$P_\beta = E\{d\beta \times d\beta^T\} = \sigma_\beta^2 I_{(N+1) \times (N+1)}$$

$$P_r = E\{dT_i \times dT_i^T\} = \sigma_r^2 I_{(N+1) \times (N+1)}$$

$$P_s = E\{dT_s^2\} = \sigma_s^2$$

$I_{(N+1) \times (N+1)}$ is a unit matrix.

The GDOP (Geometric Dilution of Precision) of location precision analysis is:

$$\text{GDOP} = \sqrt{P_{dx}(1,1) + P_{dx}(2,2)} \quad (30)$$

SIMULATION ANALYSIS OF TARGET LOCATION

In the simulation, the single-sensor location accuracy of TDMA target is analyzed along the target trajectory and with different amount of measurements.

Assume that a target makes constant-speed movement from left to right with the constant flight velocity of 200 m sec^{-1} and sends a signal every 5 sec. The trajectory is parallel to the X-axis in two-dimensional space.

Suppose that the RMS (Root Mean Square) error of range difference is 30 m which corresponds to the TOA measurement error, that of target system synchronization 1 microsecond and that of azimuth 0.5 degree.

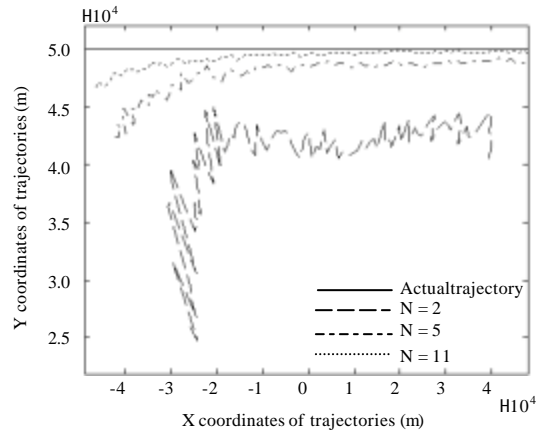


Fig. 3: Relationship of actual and estimated trajectory

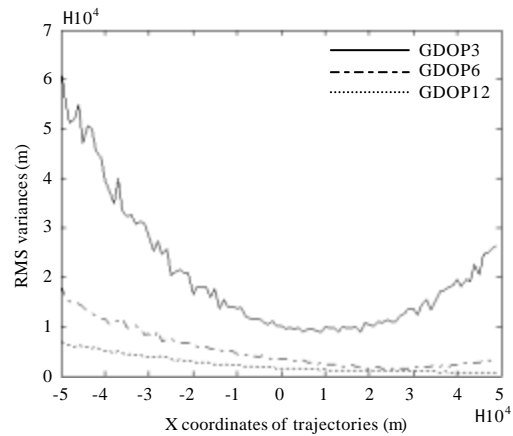


Fig. 4: Location precision analysis of different amount of data

The TOA and DOA measurements of 3 positions ($N = 2$), 6 positions ($N = 5$) and 12 positions ($N = 11$) are used in analyzing the location accuracy.

Figure 3 show the Actual and estimated trajectories. The more the amount of data is used, the closer the estimated trajectory approaches to the actual one.

Figure 4 show the simulation results of location accuracy. GDOP $_i$ is used in indicating the location accuracy using the measurements of i positions. According to Fig.4, the following conclusions could be obtained:

- The more the amount of data is used, the better the location accuracy will be. The location accuracy

using the measurements of 12 positions is obviously better than that of 6 and 3 positions

- The closer the target approaches to sensor, the better the location accuracy will be
- The positioning accuracy is asymmetric at the same distance on the left and right of sensor during the entire trajectory. This is because the current target position is estimated using previous measurements. The positioning accuracy on the right is better than that on the left at the same distance

CONCLUSION

According to the quasi-periodic characteristic of slot signal of TDMA target, this paper makes the parametric target motion analysis to realize single-sensor target location based on the measurements of TOA and DOA. The main conclusions are as follows:

- The TDMA target can be positioned using the TOA and DOA measurements of more than two target positions. The more the amount of data is used, the better the location accuracy will be
- The target location estimation error is related with the measurement errors of location system, synchronization error of TDMA system and target states in two-dimensional space

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