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Using Technology of Short Time Fourier Transform and Filtering to Improve the Performance of Prony Algorithm

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Abstract: This study has proposed a data preprocessing scheme to overcome the negative effects of non-linearity and signal-to-noise ratio level of multiple input signals on the application of Prony algorithm. Technology including short time fourier transform and filtering are utilized to mitigate the negative effects. One case with simulated data and one case with practical measured data are investigated. The results show that the proposed scheme can mitigate the negative effects of input signal on Prony algorithm and should be able to benefit electromechanical mode identification.

Key words: Power system stability, Prony algorithm, signal-to-noise ratio (SNR), short time fourier transformer (STFT), fast fourier transformer (FFT)

INTRODUCTION

Small signal stability problems have been a primary concern in power system stability and control (Kuridur, 1994; Pal and Chaudhuri, 2005). An unstable oscillatory mode is one of the major threats to power grid stability and reliability. Moreover, growing modes with negatively damped modes can result in widespread outages such as the August 10, 1996 western outage in North America (Kosterev *et al.*, 1999). Thus, it is of great importance that accurate electromechanical mode information can be obtained in time to guide the operation of dispatch and control.

Generally, there are two basic methods for identifying electromechanical modes: model-based methods and measurement-based methods (Prony, 1795). With regard to the model-based methods, the differential equations are determined and formulated by state variables, input variables and output variables. Then the equations are linearized around an operating point. The number of equations depends on system scale. Modes can be obtained by calculating eigenvalue of the state matrix. This method is dependent on the system model which is a bit difficult to be accurately determined because of the system topology changes frequently. The measurement-based methods are kinds of signal processing tools which only depend on the measurement

data and can perform well no matter how large the system scale is.

The Prony method is one of the most representative methods based on measurements. It was originally developed by Baron de Prony in 1795 to explain the expansion of various gases (Prony, 1795). Prony analysis has become a popular approach in analyzing small signal stability issues in power systems (Zhou *et al.*, 2010; Hauer *et al.*, 1990; Trudnowski *et al.*, 1999; Pierre *et al.*, 1997).

Various road blocks limit the practical application of Prony algorithm. The limiting factors include the data set containing significant effect of non-linearity (Trudnowski *et al.*, 1997; Palmer, 2009), a multiple signal data set including some channel signals with high noise level. The main objective of this paper is to address these issues.

BRIEF OVERVIEW OF PRONY ALGORITHM

Common mathematical description: Each method estimates a signal $y(t)$ as a weighted sum of exponential terms of the form:

$$y(t) = \sum_{i=1}^n R_i \exp(\lambda_i t) \quad (1)$$

When $y(t)$ is sampled at a constant sampling period Δt , the following discrete form is obtained:

$$y(k) = \sum_{i=1}^n R_i z_i^k, k = 0, 1, \dots, N-1 \quad (2)$$

where, $z_i = \exp(\lambda_i \Delta t)$, $t = k\Delta t$. N is the number of data samples and R_i is the signal residue associated with the mode λ_i . n is called the model order which is not known for real power system measurements.

The objective of modal analysis is to find the value of R_i , λ_i from the measurement data $y(k)$.

Single channel Prony algorithm: Equation 2 can be written in the following form:

$$\begin{bmatrix} y(0) \\ y(1) \\ \dots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ z_1 & z_2 & \dots & z_n \\ \dots & \dots & \dots & \dots \\ z_1^{N-1} & z_2^{N-1} & \dots & z_n^{N-1} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ \dots \\ R_n \end{bmatrix} \quad (3)$$

Constructing a discrete Linear Prediction Model (LPM) and z_i can be obtained by finding the roots of the characteristic polynomial associated with the LPM:

$$z^n - (a_1 z^{n-1} + a_2 z^{n-2} + \dots + a_n z^0) = 0 \quad (4)$$

Left-multiplying $\underbrace{[-a_n, -a_{n-1}, \dots, -a_1, 1, 0, \dots, 0]}_N$ to the

both sides of Eq. (3), then the following equation can be acquired using (4):

$$\underbrace{[-a_n, -a_{n-1}, \dots, -a_1, 1, 0, \dots, 0]}_N \begin{bmatrix} y(0) \\ y(1) \\ \dots \\ y(N-1) \end{bmatrix} = 0 \quad (5)$$

Further left-multiplying $\underbrace{[0, -a_n, -a_{n-1}, \dots, -a_1, 1, 0, \dots, 0]}_N$

to the both sides of Eq. 3 and the result on the right hand side is also zero. Eq. 6 can be obtained by repeating the same left-multiply operation:

$$\begin{bmatrix} y(n) \\ y(n+1) \\ \dots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} y(n-1) & y(n-2) & \dots & y(0) \\ y(n) & y(n-1) & \dots & y(1) \\ \dots & \dots & \dots & \dots \\ y(N-2) & y(N-3) & \dots & y(N-n-1) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{bmatrix} \quad (6)$$

a_i can be calculated by solving Eq. 6 in the least-square sense, then calculate the roots of (4) to get z_i , solve (3) for residue R_i .

Multi channel Prony algorithm: Assuming there is a number of m signals expressing as $y_1(k), y_2(k), \dots, y_m(k)$ and (5) can be rewritten as:

$$\begin{bmatrix} y_1(n) \\ y_1(n+1) \\ \dots \\ y_1(N-1) \\ \vdots \\ y_m(n) \\ y_m(n+1) \\ \dots \\ y_m(N-1) \end{bmatrix} = \begin{bmatrix} y_1(n-1) & y_1(n-2) & \dots & y_1(0) \\ y_1(n) & y_1(n-1) & \dots & y_1(1) \\ \dots & \dots & \dots & \dots \\ y_1(N-2) & y_1(N-3) & \dots & y_1(N-n-1) \\ \vdots & \vdots & \vdots & \vdots \\ y_m(n-1) & y_m(n-2) & \dots & y_m(0) \\ y_m(n) & y_m(n-1) & \dots & y_m(1) \\ \dots & \dots & \dots & \dots \\ y_m(N-2) & y_m(N-3) & \dots & y_m(N-n-1) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{bmatrix} \quad (7)$$

There are a total of $(N-n) \times m$ equations with n unknown coefficients a_i in Eq. 7. a_i can be solved in the least-square sense. With the same method, z_i is solved by (4). Finally, a total of $m \times n$ residues of m signals can be obtained by solving Eq. 8:

$$\begin{bmatrix} y_1(0) & y_2(0) & \dots & y_m(0) \\ y_1(1) & y_2(1) & \dots & y_m(1) \\ \dots & \dots & \dots & \dots \\ y_1(N-1) & y_2(N-1) & \dots & y_m(N-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ z_1 & z_2 & \dots & z_n \\ \dots & \dots & \dots & \dots \\ z_1^{N-1} & z_2^{N-1} & \dots & z_n^{N-1} \end{bmatrix} \begin{bmatrix} R_{11} & R_{21} & \dots & R_{m1} \\ R_{21} & R_{22} & \dots & R_{m2} \\ \dots & \dots & \dots & \dots \\ R_{1n} & R_{2n} & \dots & R_{mn} \end{bmatrix} \quad (8)$$

SELECT THE PROPER PORTION OF RINGDOWN DATA CONSIDERING NON-LINEAR EFFECTS

Prony analysis is known to be applicable to ringdown data which is generated after some major disturbance, such as a line tripping and results in observable oscillations (Zhou *et al.*, 2010).

The use of eigenvalues to describe modes is derived from the concept of a linear model for a power system. A power system however, like many systems is actually non-linear and the linear model is only a best approximation when the system is not subjected to large disturbances. Trudnowski *et al.* (1997) reported a detailed investigation into the effects on system eigenvalues of four kinds of non-linear. More recently the work reported by Palmer (2009) and Ledwich *et al.* (2009) described how such mitigation of nonlinear effects may be effected.

In general mode identification algorithms can reach a good performance for the response of the system provided non-linear effects are minimal when a spectral

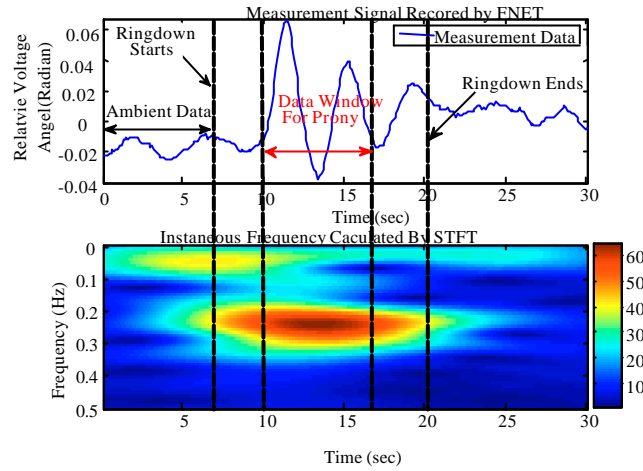


Fig. 1: An example on selecting proper portion of ringdown data

analysis of the ringdown waveform should show constant frequency over the portion of ringdown data. Conversely, if the assumption on constant frequency cannot be guaranteed, techniques such as Prony analysis, AR, ARMA and FFT based techniques, etc. cannot be used without incurring errors.

When mode frequencies cannot be assumed to be constant, techniques are needed to describe variation in Instantaneous Frequency (IF). If f_i is defined as follows:

$$f_i = \frac{1}{2\pi} \frac{d\phi(t)}{dt} \quad (9)$$

where, $z(t) = a(t) e^{j\phi(t)}$ is the analytic version of the measured signal $y(t)$ computed by taking its Hilbert transform and $\phi(t)$ is known as the instantaneous phase function. If the input signal is a damped sinusoid, $\phi(t) = 2\pi f$.

Techniques such as the Short-Time Fourier Transform (STFT), the Wigner-Ville Distributions (WVD) and Wavelets are well known and can be used to determine the IF. However, these techniques have different characteristics. For example, WVD is not suitable for IF analysis of multi mode signals because of the cross terms issue (Boashash, 2003). Because of the simplicity and robustness, STFT is employed to calculate the IF and can help to select the proper portion of ringdown data.

Figure 1 shows an example on selecting proper portion of ringdown data. The upper Figure is a relative voltage angle data before and after a generator trip disturbance recorded by Wide Area Frequency Monitoring Network (FNET) (Zhong *et al.*, 2005). The bottom Figure is IF plot calculated by STFT with the same period of recorded data. The Figure represents that IF varies with time and different color represents the varying magnitude of frequency component. A warmer color

denotes a greater magnitude of frequency component. The starting point of ringdown data can be detected when a significant mode with relatively great magnitude appears according to the color variation. It is represented by the first black dash straight line in Fig. 1. Prony analysis can not start until the IF keep constant (corresponding to the consistent warmest color plot). The proper portion data for Prony analysis is the data window between the two red arrows. When the significant mode starts to disappear the ringdown ends accordingly, as shown with the last black dash straight line.

DISCUSSING THE EFFECTS OF VARIED QUALITY OF CHANNEL DATA ON MULTI CHANNEL PRONY ANALYSIS

Individual signals are analyzed independently often resulting in conflicting frequency and damping estimates (due to noise effects). The multi channel Prony analysis can solve this problem and also allows for more accurate estimation of electromechanical oscillation modes under noisy conditions (Trudnowski *et al.*, 1999). Nevertheless, multi channel data is sampled by different measurement device and the data quality may be varied. Some of them may have a low signal-to-noise ratio (SNR) or outliers resulting from the temporary communication problem, measurement device failure, etc.

A simulation case is developed to investigate the effects of varied quality of channel data on multi channel Prony analysis. Two channel simulated signal, S1 and S2 which shares a common eigenvalues, $-0.0311 + j1.5538$, is generated as shown in Eq. 10:

$$\begin{aligned} S1 &= 10e^{-0.0311t} \cos(2\pi \times 0.2473t + 20^\circ) \\ S2 &= 5e^{-0.0311t} \cos(2\pi \times 0.2473t + 40^\circ) \end{aligned} \quad (10)$$

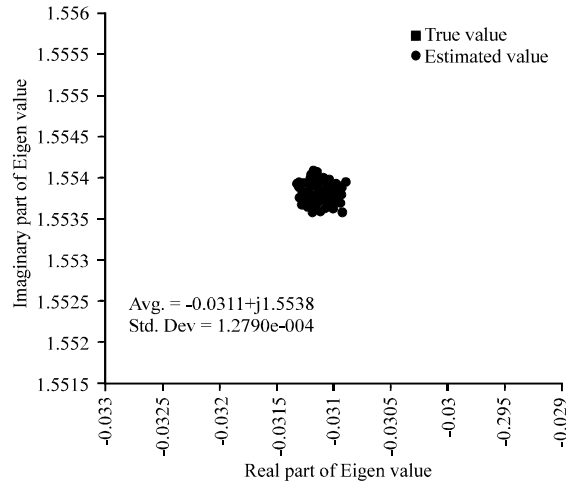


Fig. 2: Mode identification results for 40dB SNR of S2

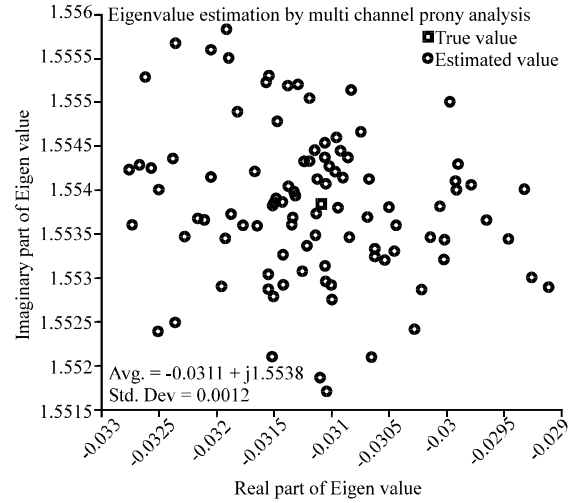


Fig. 4: Mode identification results for 20dB SNR of S2

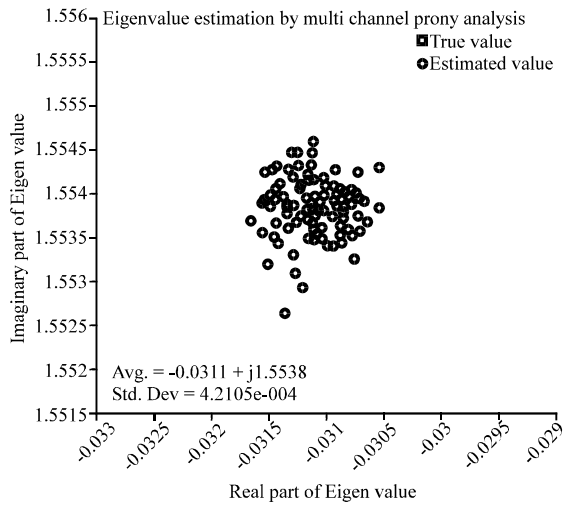


Fig. 3: Mode identification results for 30dB SNR of S2

One channel signal S1 is pure without any noise and noise with a certain SNR ranged from 20 to 40 dB is added into another channel signal S2. Then, multi channel Prony is employed to estimate the eigenvalue. The simulation is run 100 times independently for each SNR case. Average value and standard deviation value of estimated eigenvalue are calculated. Returning to the first step, another SNR model is injected into S2 and the global process is repeated. The repetition continues until all SNR candidates have been investigated. The estimated eigenvalue, average value and standard deviation value corresponding to different SNR of S2 are shown in Fig. 2, 3 and 4, respectively.

From Fig. 2 to 4, it can easily observe that multi channel Prony analysis performs worse with higher standard deviation as the SNR of S2 decreases. Thus, the signal

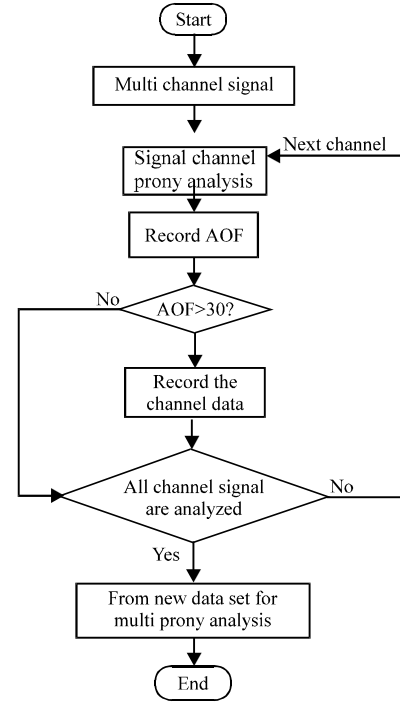


Fig. 5: Flow chart for filtering channel data

filtering process before multi channel data go through multi channel Prony module is indispensable.

Accuracy of Fitting (AOF) as shown in Eq. 11 is employed to evaluate the fitting performance between reconstructed signal and original signal:

$$AOF = 20 \log_{10} \frac{\|x(k)\|}{\|x(k) - \hat{x}(k)\|} \quad (11)$$

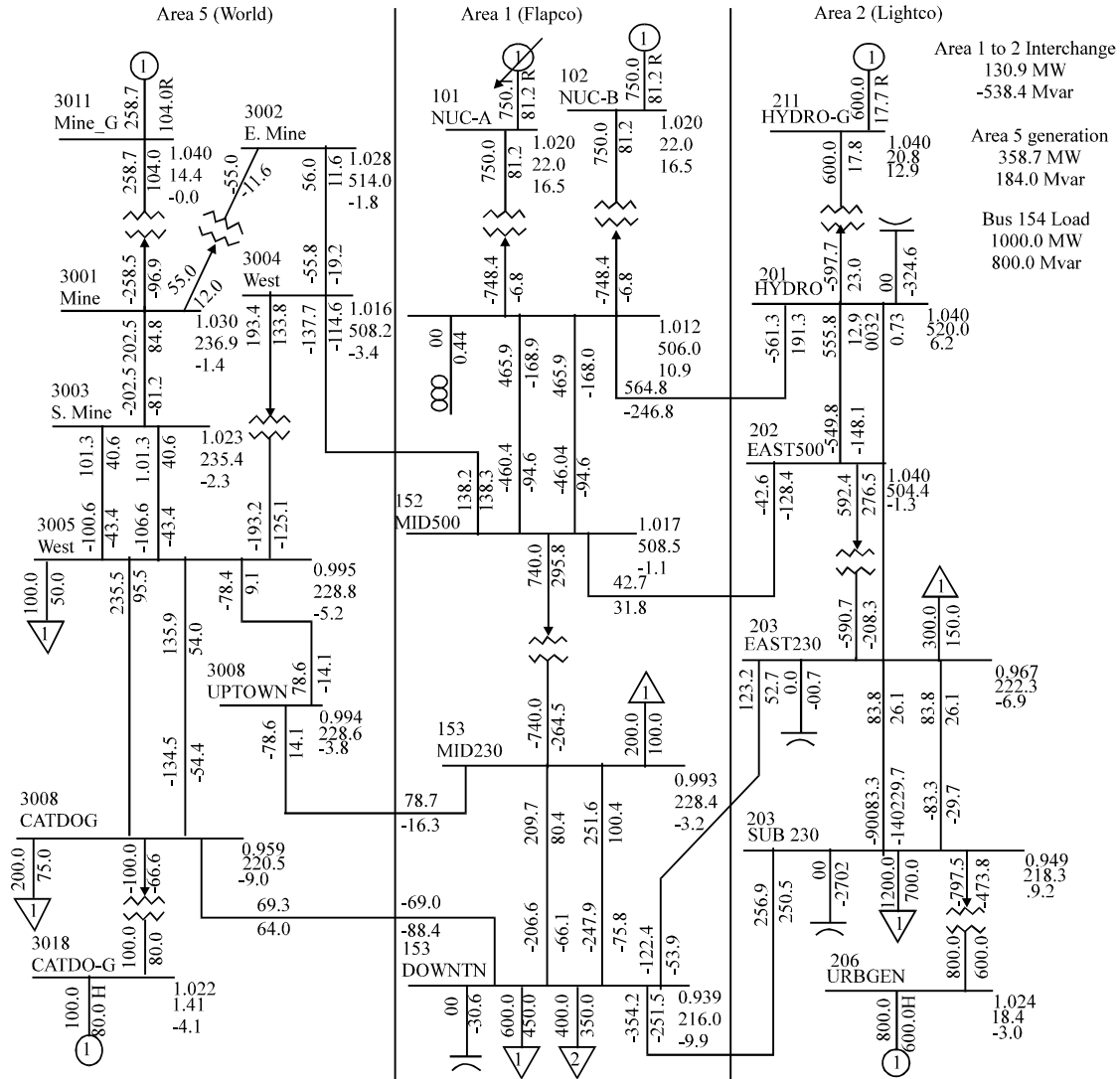


Fig. 6: 23-bus system used for dynamic simulation

where, $x(k)$ and $\hat{x}(k)$ are criterion signal and reconstructed signal, respectively. $||\cdot||$ denotes the usual root-mean-square norm and the units of the AOF metric are decibels (dB). From Equation 11, it can be concluded that the higher AOF, the better the fitting performance. Thus, more accurate modal analysis results can be obtained.

Initially, AOF for every channel data can be obtained by single channel Prony analysis. The channel data with its AOF greater than a threshold is regarded as a qualified data prepared for multi channel Prony analysis. Conversely, the channel data with its AOF less than the threshold is thought as the inappropriate data and should be discarded. Figure 5 shows the flow chart of this filtering process.

VALIDATION of DATA PREPROCESSING SCHEME

In order to evaluate the effectiveness of the proposed data preprocessing scheme, two cases in which multi channel data is obtained from dynamic simulation and Wide Area Measurement System (WAMS) are investigated.

23-bus system dynamic simulation: A 23-bus system shown in Fig. 6 is employed for dynamic simulation by power system simulation software PSSE. At 1st second, the 101-generator with 750 MW marked as black arrow is tripped and the simulation lasts for 30 seconds. All 23 buses voltage data are recorded and the DC component

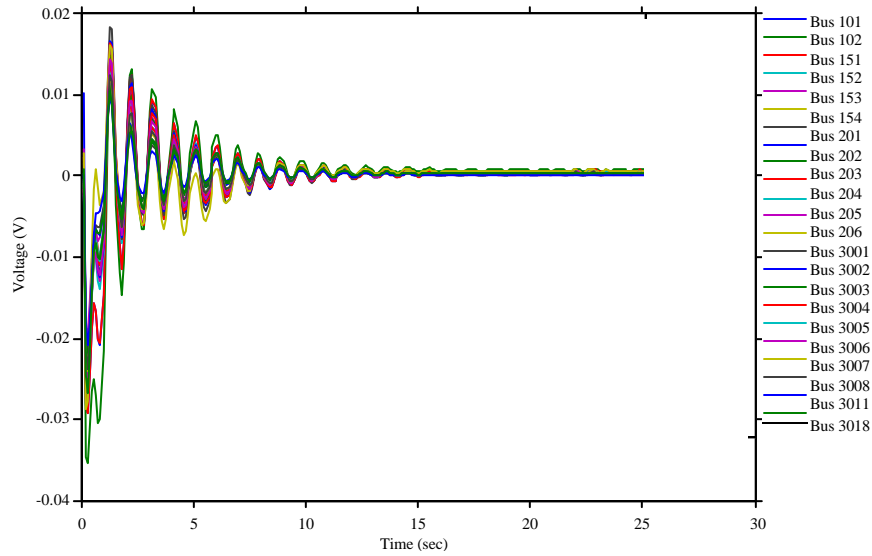


Fig. 7: The 23 buses voltage data with DC removed for case 1

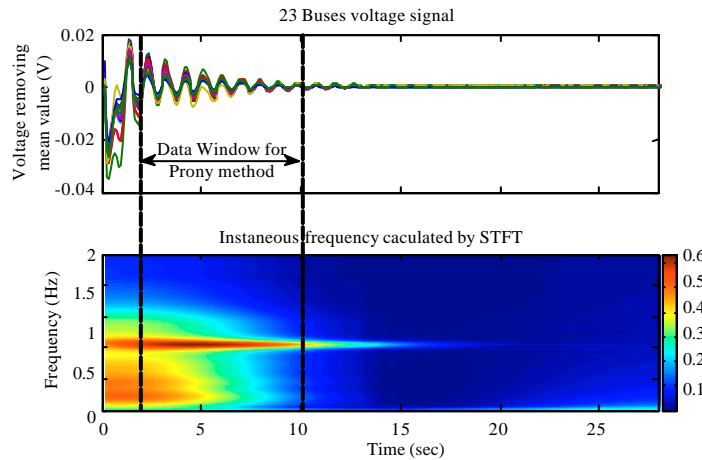


Fig. 8: Time frequency analysis plot for 23 channels data in case 1

is removed before the signal enters the Prony analysis module by subtracting average value for all 23 buses, respectively, as shown in Fig. 7.

The small signal stability software NEVA is performed on the system and mode identification results are obtained as listed in Table 1. Ten modes are determined and the corresponding frequency and damping ratio information are also provided. Frequency of ten modes is ranged from 0.11 to 2.45 Hz. These results can be used as criteria to evaluate accuracy of the proposed algorithm.

Since the simulated 23 channels data is pure without any noise, the module of filtering channel data is not necessary to perform. Then, the module of selecting proper portion of data is enabled and the time frequency analysis is illustrated in Fig. 8. Apparently, the data time

Table 1: Mode identification results of 23-bus system by eigenvalue analysis

Mode	Frequency (Hz)	Damping Ratio (%)	Mode	Frequency (Hz)	Damping ratio (%)
1	0.11	55.6	6	1.33	30.5
2	0.12	63.6	7	1.54	13.6
3	1.09	15.3	8	1.64	17.0
4	1.18	7.0	9	2.29	68.3
5	1.27	74.2	10	2.45	65.7

Table 2: Dominant mode identification results of 23-bus system by three methods

Method	Dominant mode frequency (Hz)	Dominant mode damping ratio (%)
Eigenvalue analysis	1.09	15.3
Prony with preprocessed data	1.09	15.6
Prony with original data	1.06	4.42

window suitable for Prony method is between 2 and 10 sec because of constant frequency. Dominant mode frequency around 1 Hz can be estimated directly.

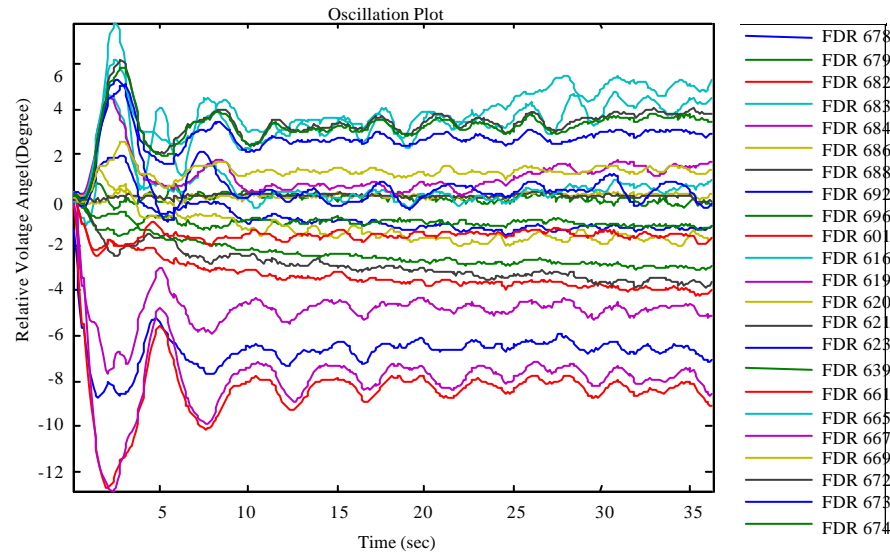


Fig. 9: Relative voltage angle recorded by 23 FDR units for case 2

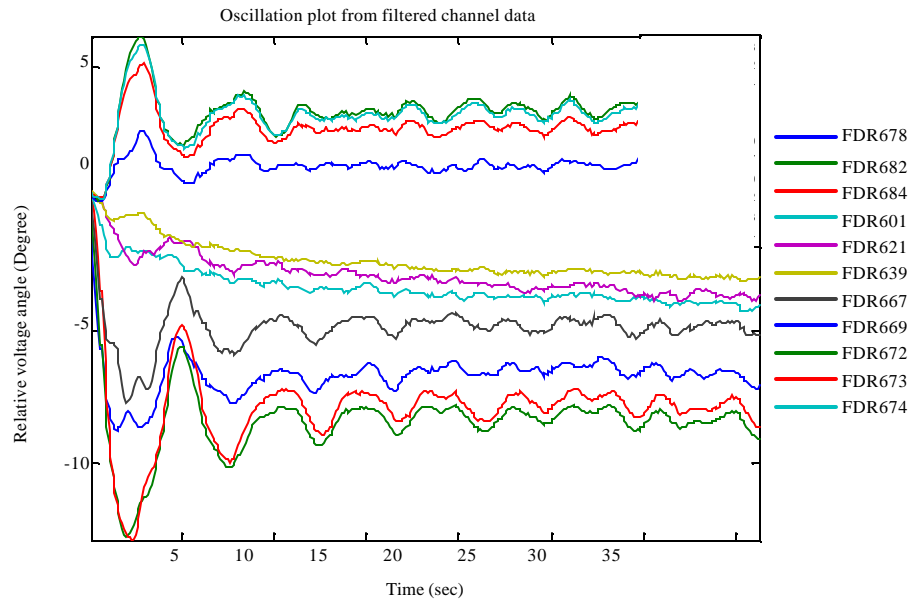


Fig. 10: Relative voltage angle recorded by 23 FDR units for case 2

Mode identification results by multi channel Prony algorithm from the processed data and original data are listed in Table 2. Only mode information of dominant mode with most concern is introduced. The criterion of mode information by eigenvalue analysis is also listed in order to compare conveniently.

Table 2 shows that the multi channel Prony algorithm with the preprocessed data performs better than it with the original data. It can be concluded that Prony algorithm combined with the data preprocessing is more accurate and effective on mode identification for dynamic

simulation data. Actually, estimation of dominant mode frequency from original data is reasonably with small error but damping ratio estimation has totally opposite situation which may be resulted from the non-linear effects of input data.

Field measurement data from FNET: As mentioned above, multi channel Prony method with preprocessed data has a good mode identification performance on simulated dynamics data. However, the practical application circumstance could be more complicated since

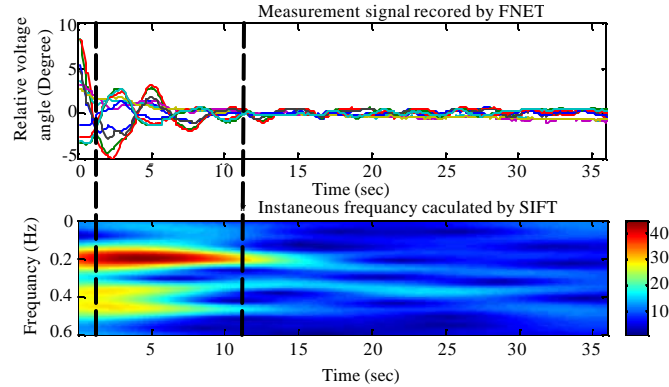


Fig. 11: Time frequency analysis plot for 23 channels data in case 2

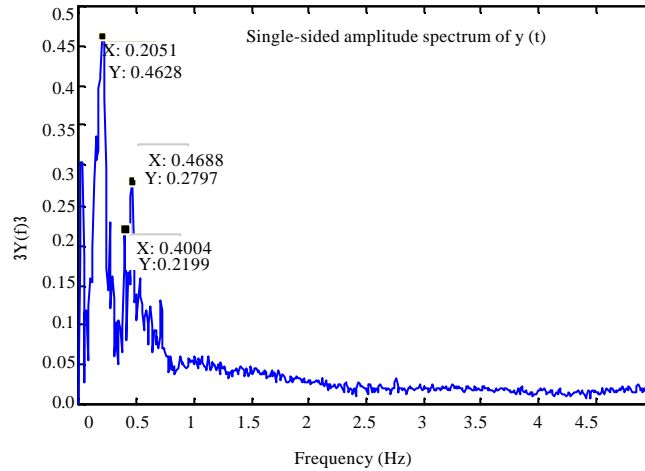


Fig. 12: Results of FFT for case 2

field measurement data are subjected to the effect of noise, measurement device failure and temporary communication problems. Therefore, it is indispensable to validate the proposed scheme by field measurement data. One random oscillation case occurred on November 7, 2010 in the Eastern Interconnection (EI) in North America was recorded by FNET and is shown in Fig. 9. A 32-sec relative voltage angle data (referenced to one FDR's measurement) is captured by 23 FDR units for Prony analysis.

Let the original 23 channel ringdown data go through the module of filtering channel data shown in Fig. 5, the filtered dataset with 11 channel data is shown in Fig. 10.

Then time frequency analysis plot is illustrated by Fig. 11. Obviously, the portion ringdown data suitable for Prony analysis is between 1 and 11 sec. Obviously, three significant modes with frequency around 0.2 and 0.4 Hz can be observed.

Table 3: Mode identification results for case 2 by three methods

Mode	Methods	Frequency (Hz)	Damping ratio (%)
1	FFT	0.2051	
	Prony with preprocessed data	0.1922	17.54
	Prony with original data	0.1924	21.36
2	FFT	0.4004	
	Prony with preprocessed data	0.4017	8.36
	Prony with original data	0.7456	12.83
3	FFT	0.4688	
	Prony with preprocessed data	0.4828	9.68
	Prony with original data	0.9130	16.36

The Prony algorithm is performed with preprocessed data and original data and respective mode identification results are listed in Table 3. Besides, the estimated frequency result of an FFT analysis is used as the criteria, shown in Fig. 12.

From Table 3, it can be concluded that the Prony algorithm with preprocessed data performs accurately on dominant mode frequency estimation compared to FFT and has a better performance than Prony algorithm with original data. Specifically, only one of three dominant

modes can be revealed by Prony algorithm with original data. Probably the reason is because of the effects of unfiltered channel data and non-linearity of input signal.

CONCLUSION

This study discussed practical issues occurred when the Prony algorithm is employed for mode identification on practical ringdown data. Varied noise level of multi channel data could downgrade the performance of mode identification. Hence, a data filtering method on multi channel data is provided. Time frequency analysis by STFT is applied to ringdown data to select the proper portion data for Prony analysis to eliminate the effect of power system non-linearity on mode identification. Two cases including simulated dynamic case and practical measurement case recorded by FNET are investigated and results show that the proposed data preprocessing scheme can overcome the practical issues and definitely improve the robustness and accuracy of Prony algorithm.

Additional cases should also be studied in depth to validate the proposed scheme. Future research will focus on the on-line implementation and more effort on algorithm will be made to accommodate on-line application.

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