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Locating the Pollution Sources in Sensornets with a Partial Differential Equation

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Abstract: This study proposed a location model for information sources in sensornets and it can be applied in environment monitoring. A Partial Differential Equation (PDE) is introduced to describe the pollution evolution in a certain region. With the observation data collected by the sensor nodes, several parameters can be determined by an optimization problem. With solving the related partial differential equation, the evolution of the pollution can be represented and the sources can be located easily. It is useful for government and managers to reduce pollution. The efficiency and the accuracy of the proposed model are shown in presented numerical examples.

Key words: Location, partial differential equation, environmental monitoring, evolution, optimization

INTRODUCTION

Recently, with the rapidly development of sensornets, a lot of new problems have been involved in many applicable scenarios (Akyildiz *et al*., 2007, 2002). The information collected by the nodes composes a discrete distribution and most applications will be processing on it, such as navigation in parking lots, environment monitoring and so on (Atluri and Zhu, 1998; Belytschko, 1996).

Over the past few years, factories in developing countries exhibit great variety in environmental performance despite the widely acknowledged weaknesses of the regulatory framework. Similarly, a great variety in environmental performance is observed in underdeveloped countries and developed countries (Chau, 2005).

These facts create problems for conventional thinking about locating the pollution sources and reducing the pollution. The location of pollution sources is the basis of next works for reducing the pollution. Because of the introduction of law concerning pollution monitoring, local authorities are looking for automatic models which should be able to locate pollution sources. Precise representation of pollution evolution is beneficial to environmental management since it allows them to have more float time to take appropriate precautionary measures. However, the extremely complex dynamics of pollution evolution are related to various pertinent physical and biochemical factors and are not well-comprehended (Nath and Patil, 2006).

The involving processes are highly complex and uncertain which may consume enormous computing cost and time. Artificial Neural Networks (ANN), have been applied in pollution controlling (Chau and Cheng, 2002). However, slow training convergence speed and easy entrapment in a local minimum are inherent drawbacks of the commonly used these algorithm (Rumelhart *et al*., 1994). Swarm intelligence is another recent SC technique that is developing quickly (Clerc and Kennedy, 2002; Kennedy and Eberhart, 1995). This technique has been applied in hydrological problems and accomplished satisfactory results (Chau, 2004a, b). In some other numerical modeling, the physical problem is represented by highly coupled, non-linear, PDEs.

In this study, a PDE-based model with several parameters is introduced to locate the pollution sources in environment monitoring. With the collection of previous observation data, the parameters can be solved by an optimization problem. Then the pollution evolution can be represented and the pollution sources can be located. It is helpful to take efficient measures for reduction of the pollution. Some numerical methods can be applied to solve the related PDE. The accuracy and efficiency are shown in the numerical examples.

A PDE-BASED MODEL FOR POLLUTION EVOLUTION IN SENSORNETS

As shown in Fig. 1, the distribution and evolution of the pollution in a region $\Omega = [-10, 10]\times[-10, 10]$ (kilometer) will be considered. For precise representation of the pollution evolution, some observation data should be collected in sensornets. As we can see, about forty observation nodes have been set in the region as shown in Fig. 1.

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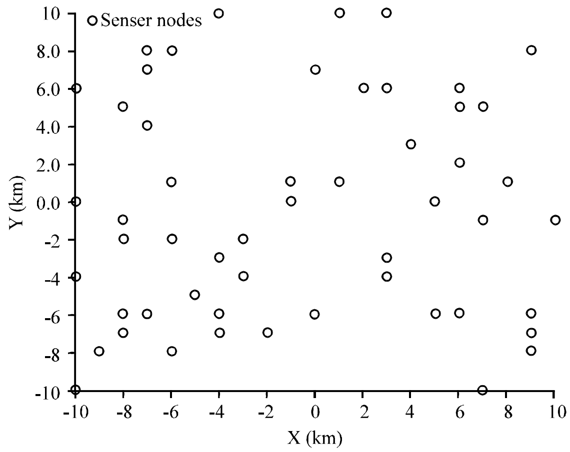

Fig. 1: Topology of observation stations

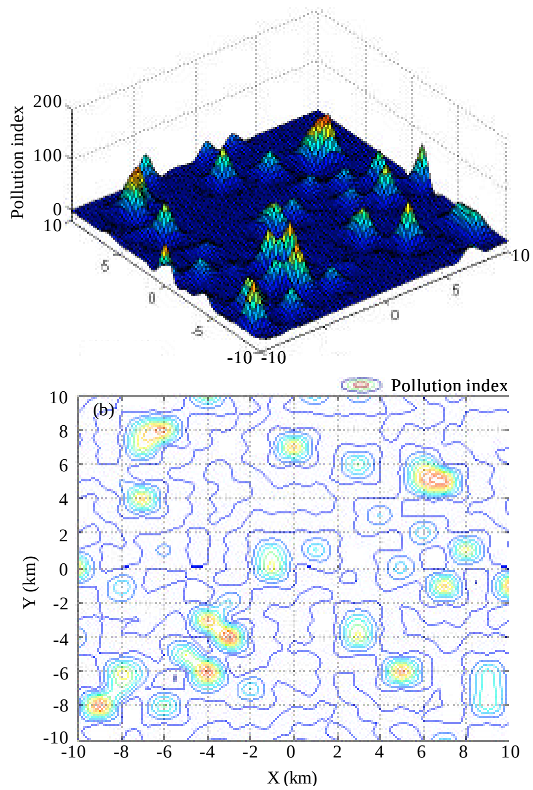

Fig. 2 (a-b): A certain observation data of pollution. (a) Distribution of pollution index (b) Level sets of pollution index

Then the observation data of pollution evolution will be collected periodically. Using some mathematical methods, a pollution surface on Ω (Fig. 2a) can be constructed based on a certain pollution distribution. The level sets of the pollution are shown in Fig. 2b.

Many PDE-based models have been applied in presented researches, such as constructing potential field (Qiao *et al*., 2010; Wei *et al*., 2010; Zhou *et al*., 2010a), image segmentation (Zhou *et al*., 2010b; Zhou and Mu, 2010) and so on. Diffusion behavior is a common and important phenomenon in real world and it is also rising in the evolution of pollution.

If there is no other factors such as pollution sources and measures for controlling, diffusion of the pollution will arise and last for some time until the stable distribution is reached. In mathematical sense, the diffusion behavior is governed by Heat Equation denoted as following:

$$\begin{cases} \frac{\partial u}{\partial t} = c \cdot \nabla^2 u, & (x, y, t) \in \Omega \times (0, +\infty), \\ u(x, y, 0) = u_0(x, y), & (x, y) \in \Omega. \end{cases} \quad (1)$$

Here u (x, y, t) denotes the pollution surface at time t, u_0 (x, y) is the initial surface, c is a constant means the velocity of diffusion.

Figure 3 shows such a natural evolution process with a certain initial surface. As shown in the Fig. 3 the pollution surface becomes smoother and smoother. More exactly, it will be a plane after enough time.

However, many factors will affect the natural diffusion process in fact. The actual diffusion behavior is extremely complex and highly nonlinear. Then a novel nonlinear partial differential equation model is proposed to describe the pollution evolution:

$$\begin{cases} \frac{\partial u}{\partial t} = a \cdot \nabla^2 u + b, & (x, y, t) \in \Omega \times (0, +\infty), \\ u(x, y, 0) = u_0(x, y), & (x, y) \in \Omega, \end{cases} \quad (2)$$

where, a and b are functions to be determined.

With given parameters a and b, Eq. 2 can be used to represent the previous evolution and to represent the future evolution.

OPTIMIZING THE PARAMETERS AND LOCATING THE POLLUTION SOURCES

In above section, a PDE-based model with several parameters is proposed to simulate the pollution evolution. Now there are collected observation data u (I = 0, 1, ···, m) and period time τ. It is natural that following equations should be satisfied:

$$\int_{(i-1)\tau}^{i\tau} \frac{\partial u}{\partial t} dt = u_i - u_{i-1}, (x, y) \in \Omega, \quad (3)$$

where, I = 1, 2, ···, m.

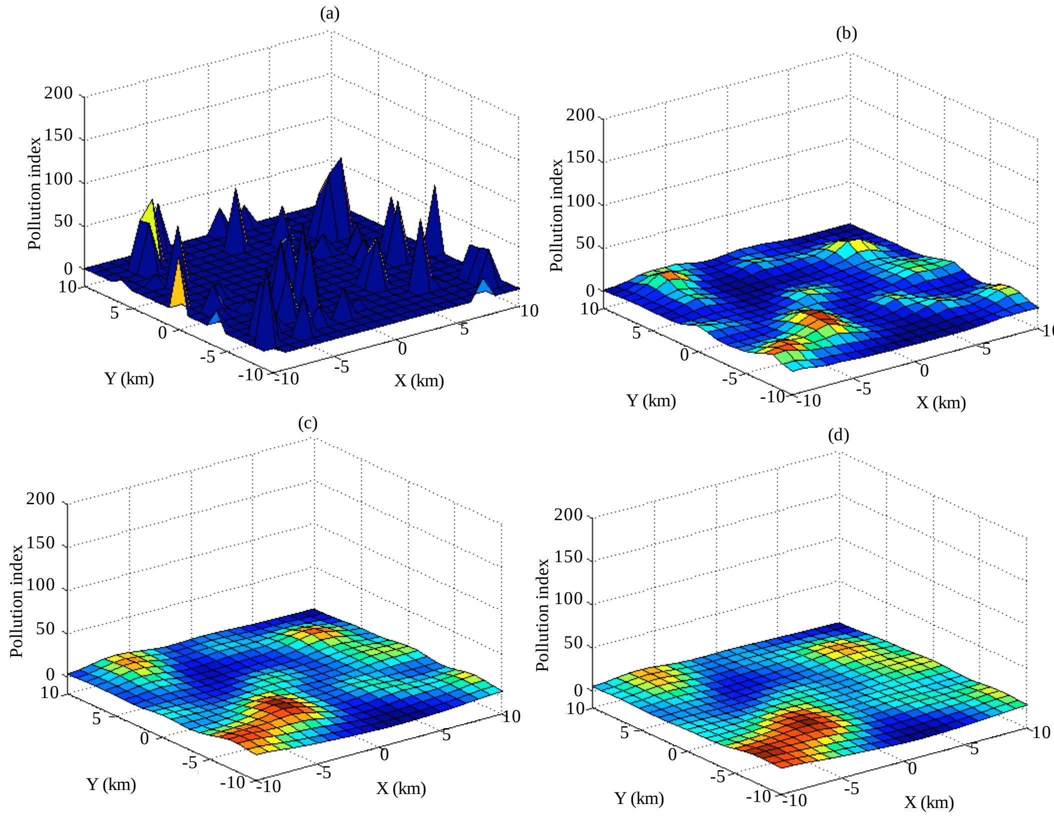

Fig. 3 (a-d): Natural evolution of the pollution. (a) T = 0, (b) T = 3, (c) T = 6 and (d) T = 9

Because of the error in proposed model, existence of the theoretical solution cannot be hold. To determine the optimization values of parameters a and b following energy functionals are introduced to characterize the distance between the evolution described by Eq. 2 and the actual one:

$$E(a,b)=\sum_{i=1}^{m} e_i(a,b), \tag{4}$$

$$e_i(a,b)=\left[\int_{(i-1)\tau}^{i\tau}\frac{\partial u}{\partial t}\,dt-(u_i-u_{i-1})\right]^2 \tag{5}$$

Then the parameters a and b can be determined by:

$$\begin{cases}(\sum_{i=1}^{m} P_i^2)a+(\sum_{i=1}^{m} P_iQ_i)b=\sum_{i=1}^{m} P_iR_i,\\ (\sum_{i=1}^{m} P_iQ_i)a+(\sum_{i=1}^{m} Q_i^2)b=\sum_{i=1}^{m} Q_iR_i,\end{cases} \tag{6}$$

where, $P_i=\int_{(i-1)\tau}^{i\tau}\nabla^2 u\,dt, Q_i=\tau, R_i=u_i-u_{i-1}$.

NUMERICAL EXPERIMENTS

In this section, the proposed model will be used to represent the evolution of pollution surface. Efficiency and applicability of proposed model are illustrated by following steps. Assume that the pollution evolution is governed by Eq. 2 embed with two parameters as shown in Fig. 4.

Then some observation data are collected and a part of them is shown in Fig. 5, P_i, Q_i, R_i (I = 1,2,3,4) are calculated as illustrated in previous section.

Second, $\sum_{i=1}^{4} P_i^2, \sum_{i=1}^{4} P_iQ_i, \sum_{i=1}^{4} P_iR_i, \sum_{i=1}^{4} P_iQ_i, \sum_{i=1}^{4} Q_i^2, \sum_{i=1}^{4} Q_iR_i$ can be obtained. Then, a and b are get by Eq. 6 as shown in Fig. 6.

The error surface of the two parameters are shown in Fig. 7.

For solving the above Eq. 2, it is necessary to adopt some numerical methods and algorithms such as Finite Differential Method (FDM), Finite Element Method (FEM), iterative algorithm and so on.

Finite differential method and iterative algorithm are simple and easy to be applied in solving it.

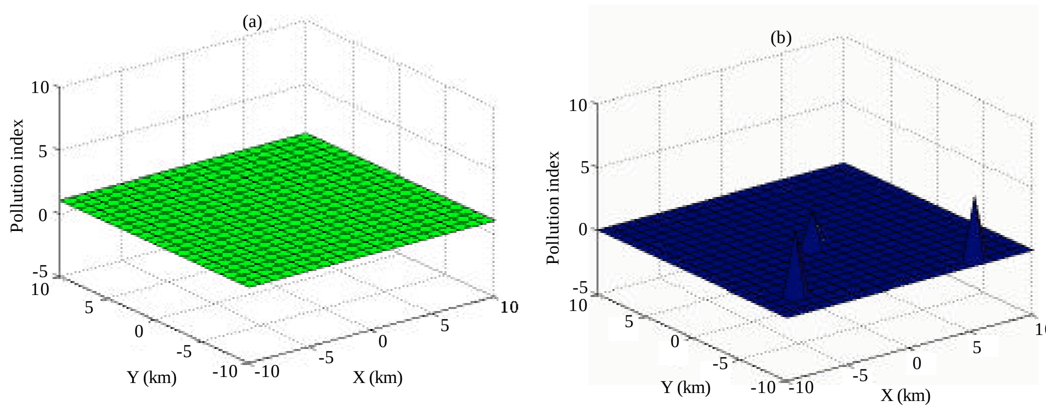

Fig. 4 (a-b): Exact parameters in Eq. 2. (a) Exact a and (b) Exact b

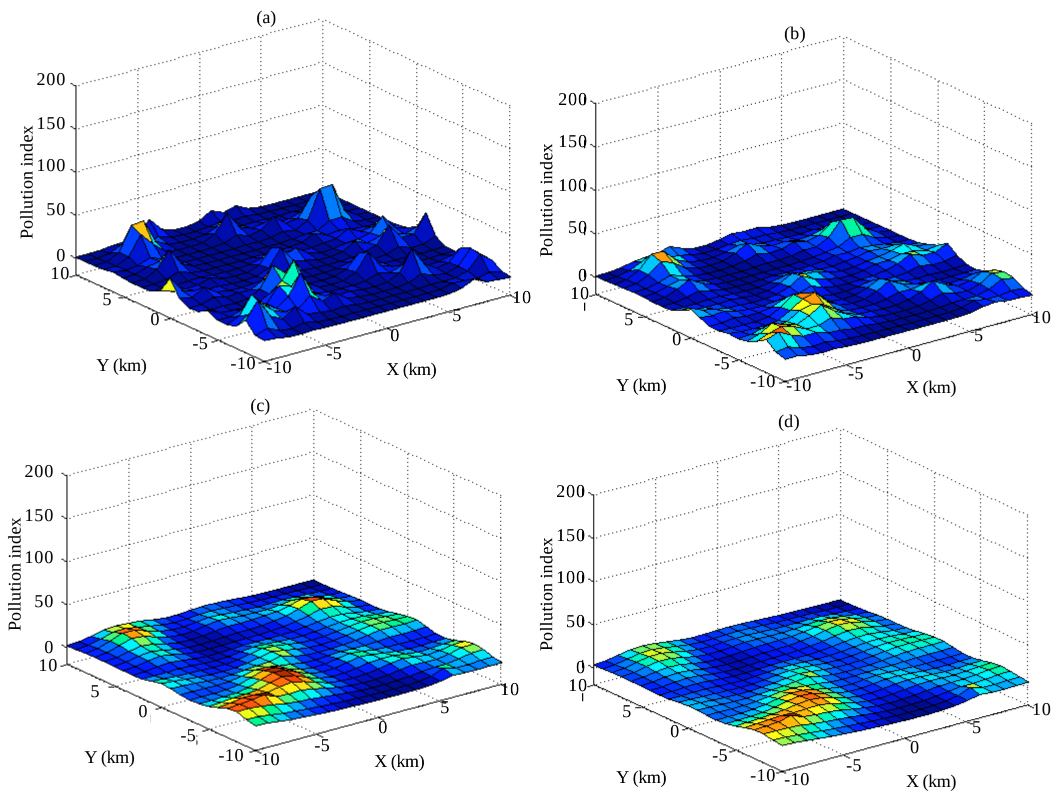

Fig. 5 (a-d): A part of collected observation data. (a) T = 1, (b) T = 2, (c) T = 4 and (d) T = 6

The numerical formula is denoted as following and the evolution process of the pollution can be obtained:

$$u_{i,j}^{n+1} = u_{i,j}^{n} + \tau \cdot \left[a_{i,j}\left(\frac{u_{i+1,j}^{n} + u_{i-1,j}^{n} - 2 \cdot u_{i,j}^{n}}{h_x^2} + \frac{u_{i+1,j}^{n} + u_{i-1,j}^{n} - 2 \cdot u_{i,j}^{n}}{h_y^2}\right) + b_{i,j} \right] \quad (7)$$

Finally, with the applying of Eq. 7, the evolution of the pollution can be presented and the prediction of pollution distribution can be implemented. It will help to evaluate the seriousness of the pollution sources and the importance of some measurements.

From the results of present experiments, the pollution sources can be located by our model. The presented algorithm is successful in accuracy, convergence speed and insensitivity to initial observation nodes.

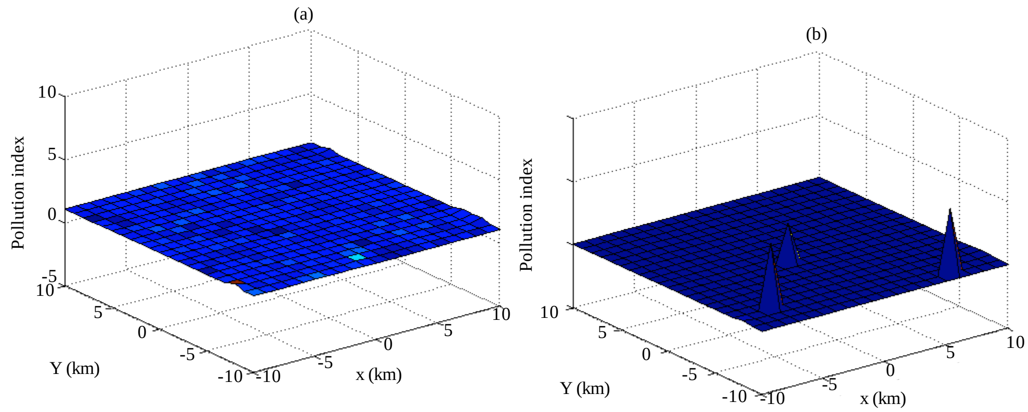

Fig. 6 (a-b): Representation of the parameters. (a) Representation of a and (b) Representation of b

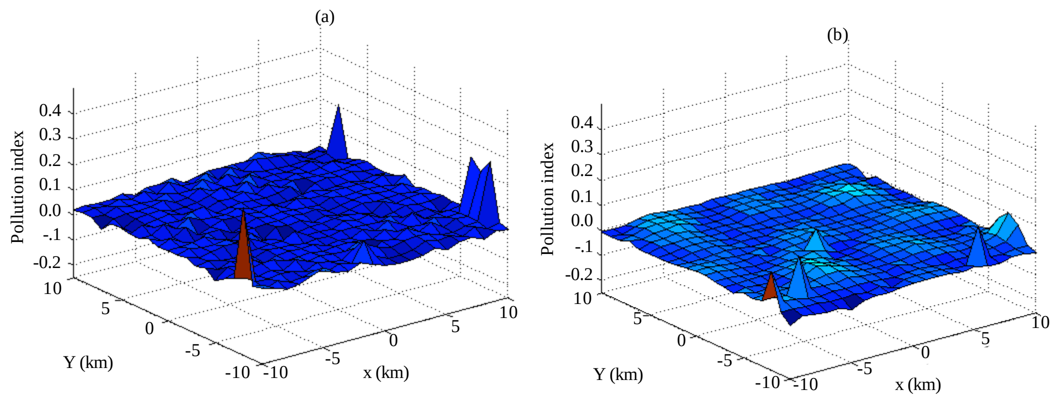

Fig. 7 (a-b): Error surfaces of the parameters. (a) Error surface of a and (b) Error surface of b

CONCLUSIONS

This study proposed a PDE-based model for the location of information sources in sensornets and it can be applied to locate pollution sources. With the collected observation data of the pollution evolution in a certain region, several parameters can be optimization and determined. With solving the partial differential equation, the pollution evolution can be represented and the pollution sources can be determined. Then some efficient measures can be adopted to reduce the pollution. The efficiency and the accuracy of the proposed model are shown in presented numerical examples.

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