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A Biased Rule for Data Fusion

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Abstract: In this study, a new data fusion method, called the Biased State-Vector Fusion Estimator (BSFE), was presented. It differed from the existed Minimum Variance Unbiased (MVU) fusion estimator. The mathematical analysis results showed that the proposed BSFE reduced the estimation Mean Squared Error (MSE) of the traditional unbiased fusion estimator. The simulation results verified the theoretical results that the BSFE can outperform the traditional estimator in terms of MSE.

Key words: Data fusion, biased estimation, least squares, minimum variance unbiased, simulation

INTRODUCTION

Data fusion or information fusion, including state-vector fusion and measurement fusion, have been widely studied (Wang, 2011; Cheng, 2008; Li and Wang, 2011; Cheng *et al*., 2006; Fan and Li, 2006; Duan and Li, 2011; Ahn and Kim, 2008). Measurement (or centralized) fusion methods directly fuse the sensor measurements to obtain a weighted or combined measurement and then use a single estimator to obtain the final state estimate based upon the fused observation. Measurement fusion methods generally involves minimal information loss, however, it may result in severe computational overhead due to overloading of the filter with more data than it can handle, thus it may suffer from poor accuracy and stability when there is severe data fault. State-vector (or distributed) fusion methods use a group of estimators to obtain individual sensor-based estimates which are then fused to obtain an improved joint estimate. In state-vector fusion, the requirements of communication and memory space at the fusion center are broadened and the parallel structures would lead to increase in the input data rates. Furthermore, it leads to easy fault detection and isolation (Gan and Harris, 2001; Sun and Deng, 2004). It is also well known that under linear-Gaussian assumption (i.e., linear measurements with jointly Gaussian noise), optimal distributed fusion is algebraically equivalent to centralized fusion if measurement noises are uncorrelated across sensors. Various distributed fusion algorithms have been reported over the last decades. Carlson (1990) developed a federated architecture applicable to decentralized sensor systems with parallel processing capabilities (Carlson, 1990). Kim (1994) proposed an optimal fusion filter under the assumption of normal distribution based on the maximum likelihood sense for systems with multiple sensors and assumes the process noise to be independent of measurement noises (Kim, 1994). Li and Zang (2001) discussed the optimality and efficiency of distributed Fusion with Best Linear Unbiased Estimation (BLUE), also known as Linear Minimum Mean-Square Error (LMMSE) estimation and optimal Weighted Least Squares (WLS) estimation (Li and Zhang, 2001).

It is known that the existed methods are concerned with unbiased estimation. However, in some cases an unbiased estimator may not exist or the unbiasedness requirement can produce nonsensical results. Furthermore, perhaps the most important objection to the constraint of unbiasedness is that it reduces estimates whose optimality is based on the difference between estimate and its average value, not estimate and the true value, as measured by the MSE which is actually of prime importance, the biased estimates may produce smaller MSE than unbiased estimates do (Kay and Eldar, 2008). In this study, a new state-vector fusion method, called the Biased State-Vector Fusion Estimator (BSFE) is proposed. It differs from the existed Minimum Variance Unbiased (MVU) fusion estimator. This study mostly aims for providing better fusion results than that of the existed unbiased fusion methods in terms of MSE. The results of mathematic analysis and simulation show that the proposed BSFE exhibits smaller MSE than the existed unbiased fusion method.

THE BIASED STATE-VECTOR DATA FUSION

Suppose there are two local unbiased scalar estimates, $\hat{x}_1$ and $\hat{x}_2$, with variance P_1 and P_2. The optimal unbiased global estimate is as follows:

$$\hat{X}_g = W_1\hat{X}_1 + W_2\hat{X}_2 \tag{1}$$

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Where:

$$W_1 = P_g P_1^{-1}$$
$$W_2 = P_g P_2^{-1} \quad (2)$$
$$P_g = \left(P_1^{-1} + P_2^{-1}\right)^{-1}$$

W_1 and W_2 are the weights of the local estimates $\hat{x}_1$ and $\hat{x}_2$, respectively, P_g is the variance of the global estimate $\hat{X}_g$. The MSE can be written as the sum of the squared bias and the variance, i.e.:

$$\mathrm{MSE}\left[\hat{X}_g^b\right] = E\left[\left(\hat{X}_g^b - X\right)^2\right] = \mathrm{var}\left[\hat{X}_g^b\right] + \left(E\left[\left(\hat{X}_g^b - X\right)\right]\right)^2 \quad (3)$$

since $\hat{X}_g$ is unbiased, P_g is also the MSE of $\hat{X}_g$.

Theorem 1: Let $\hat{x}_1$ and $\hat{x}_2$ be two uncorrelated local unbiased estimates of the scalar parameter X, with variance P_1 and P_2, respectively. Then one biased global estimate may be obtained as follows:

$$\hat{X}_g^b = W_1^b \hat{X}_1 + W_2^b \hat{X}_2 \quad (4)$$

Where:

$$W_1^b = \frac{P_g X^2}{X^2 + P_g} P_1^{-1}$$
$$W_2^b = \frac{P_g X^2}{X^2 + P_g} P_2^{-1} \quad (5)$$
$$P_g = \left(P_1^{-1} + P_2^{-1}\right)^{-1}$$

And the relationship of the MSEs of the biased global estimate $\hat{X}_g^b$ and the unbiased global estimate $\hat{X}_g$ is given as:

$$\mathrm{MSE}\left[\hat{X}_g^b\right] = \frac{X^2}{P_g + X^2}\mathrm{MSE}\left[\hat{X}_g\right] < \mathrm{MSE}\left[\hat{X}_g\right] \quad (6)$$

Proof: The biased global estimate is get by scaling the unbiased estimate as follows:

$$\hat{X}_g^b = \alpha \hat{X}_g \quad (7)$$

where, α is chosen to minimize the MSE of $\hat{X}_g^b$. Then:

$$\begin{aligned}\mathrm{MSE}\left[\hat{X}_g^b\right] &= E\left[\left(\hat{X}_g^b - X\right)^2\right] = \mathrm{var}\left[\hat{X}_g^b\right] + \left(E\left[\hat{X}_g^b\right] - X\right)^2 \\ &= \mathrm{var}\left[\alpha\hat{X}_g\right] + \left(E\left[\alpha\hat{X}_g\right] - X\right)^2 = \alpha^2\,\mathrm{var}\left[\hat{X}_g\right] + \left(\alpha E\left[\hat{X}_g\right] - X\right)^2 \\ &= \alpha^2\,\mathrm{var}\left[\hat{X}_g\right] + (\alpha - 1)^2 X^2 = \alpha^2 P_g + (\alpha - 1)^2 X^2\end{aligned} \quad (8)$$

The last equation holds because $\hat{X}_g$ is unbiased. In order to minimize $\mathrm{MSE}\left[\hat{X}_g^b\right]$, then differentiate the MSE and set the result equal to zero as follows:

$$\frac{d\mathrm{MSE}[\hat{X}_g^b]}{d\alpha} = 2\alpha P_g + 2(\alpha - 1)X^2 = 0 \quad (9)$$

And:

$$\alpha = \frac{X^2}{P_g + X^2} = \frac{1}{1 + \frac{P_g}{X^2}} \quad (10)$$

By substituting Eq. 10 to Eq. 8:

$$\mathrm{MSE}\left[\hat{X}_g^b\right] = \left(\frac{X^2}{P_g + X^2}\right)^2 P_g + \left(\frac{X^2}{P_g + X^2} - 1\right)^2 X^2 = \frac{X^2}{P_g + X^2} P_g \quad (11)$$

Since:

$$\mathrm{MSE}\left[\hat{X}_g\right] = E\left[\left(\hat{X}_g - X\right)^2\right] = \mathrm{var}\left[\left(\hat{X}_g\right)\right] + \left(E\left[\left(\hat{X}_g - X\right)\right]\right)^2 = \mathrm{var}\left[\hat{X}_g\right] = P_g \quad (12)$$

One has that:

$$\mathrm{MSE}\left[\hat{X}_g^b\right] = \frac{X^2}{P_g + X^2} P_g = \frac{X^2}{P_g + X^2}\mathrm{MSE}\left[\hat{X}_g\right] < \mathrm{MSE}\left[\hat{X}_g\right] \quad (13)$$

Equation 6 is thus proved. By combining Eq. 2, 7 and Eq. 10, 5 is obtained.

Remark 1: In theorem 1, X should be known in order to have the optimal result. But this is usually impossible. However, since $\hat{x}_1$ and $\hat{x}_2$ are unbiased estimates, the mean of them may be used to approximate X which can be termed as the blind estimate, it is reasonable as in the study of Ben-Haim and Eldar (2007). And this approximation will become better as show in remark 2. Also the following method may be applied. Suppose that $|X| \le X_0$, then from Eq. 8 one has:

$$\mathrm{MSE}\left[\hat{X}_g^b\right] = \alpha^2 P_g + (\alpha - 1)^2 X^2 \le \alpha^2 P_g + (\alpha - 1)^2 X_0^2 \quad (14)$$

And α will be choose so that:

$$\mathrm{MSE}\left[\hat{X}_g^b\right] \le \mathrm{MSE}\left[\hat{X}_g\right] \quad (15)$$

i.e.,:

$$\alpha^2 P_g + (\alpha - 1)^2 X_0^2 - P_g \le 0 \tag{16}$$

This produces:

$$\alpha \ge \frac{X_0^2 - P_g}{X_0^2 + P_g} \tag{17}$$

Since the MSE is expected to be reduced as much as possible, α is choosed to minimize $\alpha^2 P_g + (\alpha-1)^2 X_0^2 - P_g$ by differentiating it and setting it equal to zero, this produces:

$$\alpha_0 = \frac{X_0^2}{P_g + X_0^2} \tag{18}$$

As Eq. 18 satisfies Eq. 17, X_0 may be used instead of X in Eq. 5 to get the biased estimate if a priori knowledge that $|X| \le X_0$ is given.

Corollary 1: Let $\hat{x}_i$, i = 1, ..., N, be N uncorrelated local unbiased estimates of the scalar parameter X, with variance P_i, i = 1, ..., N, respectively. Then one biased global estimate may be obtained as follows:

$$\hat{x}_g^b = \sum_{i=1}^{N} W_i^b \hat{x}_i \tag{19}$$

Where:

$$W_i^b = \frac{P_g X^2}{X^2 + P_g} P_i^{-1}, \quad P_g = \left(\sum_{i=1}^{N} P_i^{-1}\right)^{-1} \tag{20}$$

And the relationship of the MSEs of the biased global estimate $\hat{x}_g^b$ and the unbiased global estimate $\hat{x}_g$ is given as:

$$MSE[\hat{x}_g^b] = \frac{X^2}{P_g + X^2} MSE[\hat{x}_g] < MSE[\hat{x}_g] \tag{21}$$

Proof: This follows directly from Theorem 1 and its proof.

Remark 2: As in remark 1,

$$\frac{1}{N}\sum_{i=1}^{N} \hat{x}_i$$

can be used to approximate X and the approximation becomes better as N goes bigger because of the unbiasedness of $\hat{x}_i$.

Remark 3: All the results presented so far may be naturally extended to the estimation of a vector parameter X.

SIMULATION EXAMPLE

Suppose that it is desired to estimate an unknown parameter X from n local sensors, each of which has a series of linear observations (Bar-Shalom *et al.*, 2001):

$$Z_i = H_i X + w_i, \; i = 1,2,...,n$$

where, i denotes the sensor ID, Z_i is a n_z vector, H_i is the observation matrix, w_i is the uncorrelated Gaussian observation noise with mean zero and covariance σ_i. Suppose that the least squares estimates $\hat{x}_i$ of the local sensors are given and one wants to have the global fusion estimate. The existed unbiased method in Eq. 1 and 2 and the proposed biased one in theorem 1 are both used. All the results are based on 100 Monte Carlo simulations.

Simulation 1: In this simulation, i = 6, X = 1, $H_1 = H_2 = [1, 1, 1]^T$, $H_3 = H_4 = [2, 2, 2]^T$, $H_5 = H_6 = [3, 3, 3]^T$, $\sigma_1 = \sigma_2 = \text{diag}([0.03, 0, 03, 0.03])$, $\sigma_3 = \sigma_4 = \text{diag}([0.05, 0, 05, 0.05])$, $\sigma_5 = \sigma_6 = \text{diag}([0.06, 0, 06, 0.06])$. The mean of $\hat{x}_i$ are used to approximate X in Eq. 5 as in remark 1. The results are given in Table 1. Where the estimation means are at the same level, however, the MSE of the existed unbiased method is 0.935 which is more larger than that of the proposed BSFE, 0.662. From Table 1 one may conclude that the proposed BSFE can effectively reduce the MSE as compared with the existed unbiased method.

Simulation 2: In this simulation, i = 6, X = 1, H_i are generated randomly as in the study of Kay and Eldar (2008). The MSE of the proposed biased method is compared to that of the existed unbiased one as a function of the SNR, defined by 101 g (X/σ). As can be seen from Fig. 1 and 2 (note that Fig. 2 is the enlarged form of Fig. 1 near 15 dB SNR), the proposed BSFE really improves the performance within all the simulation SNR area. And the improvements become more evident as the SNR decreases. For example, the MSE difference between the existed unbiased method and the

Table 1: Simulation results

Fusion method	Estimation mean ($\times 10^0$)	MSE ($\times 10^{-3}$)
Unbiased	0.998	0.935
Biased	0.997	0.662

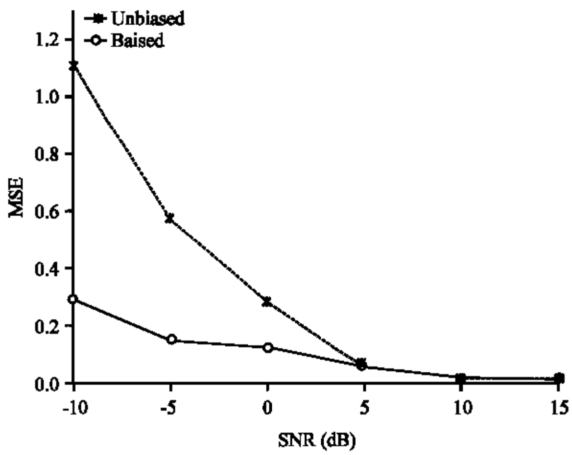

Fig. 1: MSE as a function of SNR

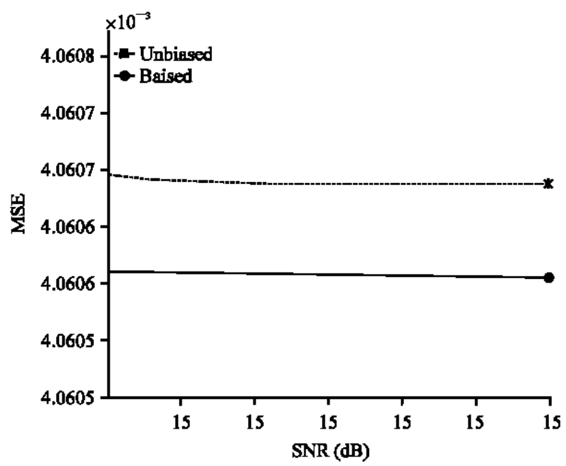

Fig. 2: Enlarged form of Fig. 1 near 15 dB

proposed BSFE is about 0.2 at 0 dB SNR, while, this difference becomes 0.8 or so at -10 dB SNR.

CONCLUSIONS

In this study, a new state-vector fusion method, called the Biased State-Vector Fusion Estimator (BSFE), is proposed. It can outperform the existed unbiased estimator in terms of MSE. The simulation results show that the proposed BSFE may really reduce the MSE effectively as compared with the unbiased fusion method.

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