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A Preprocessing Algorithm for Blind Estimation of DS-SS Signals in Multipath

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Abstract: In this study, a criterion is derived to determine appropriate starting point of receiving data as a preprocessing algorithm of subspace method because inherent channel order uncertain and ill-conditional identification induced by random transmission delay may significantly deteriorate the performance of blind estimation of DS-SS signals in multipath. The proposed algorithm avoids channel order estimation and shows more robust blind estimation performance by simulations.

Key words: DS-SS signals, SIMO, blind channel identification, subspace method

INTRODUCTION

In non-cooperative applications such as eavesdropping and spectrum surveillance, blind estimation of Direct Sequence Spread Spectrum (DS-SS) signals is challenging to obtain the information since the spreading code is unknown to the receiver. Most existing approaches of blind estimation of DS-SS signals (Bouder *et al.*, 2004; Zhang *et al.*, 2005; Zhang and Zhang, 2006) do not consider multipath channel which may deteriorate severely in some practical scenarios if multipath effect can not be neglected. Tsatsanis and Giannakis (1997) show the blind estimation of DS-SS signals in multipath is equivalent to the blind identification/equalization of a Single-Input Multi-Output (SIMO) and Finite Impulse Response (FIR) channel. They exploits the subspace method (SSM) (Moulines *et al.*, 1995) which is a popular blind SIMO and FIR channel identification method and usually has better performance in the presence of noise (Tong and Perreau, 1998). However, SSM requires the channel order which is usually unknown in prior. Furthermore when a channel impulse response contains "Small" Leading and/or Trailing Terms (SLTT) which is ill-conditioned and leads to effective overmodeling (Liavas *et al.*, 1999a), the estimation quality of SSM degrades dramatically in the noisy cases. Therefore, effective channel order is suggested by Liavas *et al.* (1999b) to substitute true channel order, which only considers the significant part of the true channel to avoid SLTT. As to our problem, we observe that for the same signal, different transmission delay presents different blind estimation performance. Some delays may increase the channel order and/or introduce SLTT which both may lead to significant

performance loss. Unfortunately, transmission delay is unknown and is difficult to estimate without known spreading code.

In this study, an optimal criterion in the sense of the separation of signal and noise subspaces is derived to determine appropriate data starting point. And a corresponding preprocessing algorithm is proposed which makes the following SSM more robust and avoids the channel order estimation.

Notations: Uppercase and lowercase boldface letters denote matrices and vectors, respectively. M_i , M_{ij} and $\|M\|$ denote the i th column, the (i, j) entry and the norm of the matrix M .

PROBLEM STATEMENT

Assume the received baseband short-code DS-SS signal with spreading factor of L is sampled at the chip rate:

$$y(n) = \sum_{m=-\infty}^{\infty} b(m)h(n - mL - \tau) + w(n) \triangleq x(n) + w(n) \quad (1)$$

where, $b(m)$, $w(m)$ and the integer τ ($0 \leq \tau \leq L$) are an independent identically distributed (i.i.d.) and equiprobable symbol sequence, additive white Gaussian noise with variance σ^2 and transmission delay in chip period T_c , respectively. The combined Impulse Response (IR):

$$h(n) = \sum_{l=0}^{L-1} c(l)g(n-l)$$

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where, $\{c_i\}_{i=0}^{L-1}$ is spreading code and $g(n) = g_c(nT_c)$ is sampled overall channel IR with channel length L_1 , i.e., $g(n) = 0$ ($n > L_1$, $n < 0$) (in addition to $g(0) \dots g(L_1-1) \neq 0$). Assuming $0 < L_1 < L$ which is reasonable in many multi-path scenarios, even for some high-speed DS-SS systems. $g_c(t)$ is the convolution of the multipath channel IR $g_{\text{mult}}(t)$ and the transmitter/receiver filter IR. Suppose the observation samples length is N . Stack observation samples:

$$\{y(n)\}_{n=0}^{N-1}$$

into an $L \times M$ matrix Y with $N = ML$ and M is the sample size in terms of the number of spreading code length:

$$Y_m = \begin{bmatrix} y((m-1)L) \\ y((m-1)L+1) \\ \vdots \\ y(mL-1) \end{bmatrix} = H \begin{bmatrix} b(m) \\ b(m-1) \\ \vdots \\ b(m-Q) \end{bmatrix} + W_m \quad (2)$$

$$\triangleq X_m + W_m$$

where, W is the noise matrix and $L \times (Q+1)$ matrix H is the SIMO IR with the channel order Q :

$$H = \begin{bmatrix} 0 & h(L-\tau) & \dots & h(QL-\tau) \\ \vdots & \vdots & \dots & \vdots \\ 0 & \vdots & \dots & h(L+L_1-1) \\ h(0) & \vdots & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ h(L-\tau-1) & h(2L-\tau-1) & \dots & 0 \end{bmatrix}$$

By stacking $K+1$ consecutive column vectors of Y , Eq. 2 may be rewritten as:

$$Z_m = \begin{bmatrix} Y_m^T & \dots & Y_{m-K}^T \end{bmatrix}^T \triangleq T(h)B_m + N_m$$

$$= \begin{bmatrix} H_1 & H_2 & \dots & H_{Q+1} & 0 & \dots & 0 \\ 0 & H_1 & H_2 & \dots & H_{Q+1} & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & H_1 & H_2 & \dots & H_{Q+1} \end{bmatrix} \begin{bmatrix} b(m) \\ \vdots \\ b(m-Q-K) \end{bmatrix} + \begin{bmatrix} W_m \\ \vdots \\ W_{m-K} \end{bmatrix} \quad (3)$$

where, $K \geq Q$ is the smoothing factor. The case $K = Q$ is considered in this study. Block Toeplitz matrix $T(h)$ is the filtering matrix and $h = [H_1^T \dots H_{Q+1}^T]^T$. In a noncooperation context, only L is known. By enforcing the signal subspace to have the block Toeplitz form, the SSM method is based on the orthogonality of the noise and

signal subspaces under some identification conditions. A common assumption of the SSM is that the channel order Q is known. Even for the same signals, different transmission delay possibly presents different Q and causes the SLTT effect as well. For instance, suppose that the spreading factor $L = 15$ and the transmitter/receiver filter IR is the unit rectangular pulse, the multipath channel A has three distinct paths, i.e.:

$$g_{\text{mult}}(t) = a_1 \delta(t - \bar{\tau}_1) + a_2 \delta(t - \bar{\tau}_2) + a_3 \delta(t - \bar{\tau}_3) \quad (4)$$

where, the path delays $\bar{\tau}_{1,2,3} = (0.9 \ 4.8 \ 11.3)T_c$ and a_i ($i = 1, 2, 3$) are the complex gains. If the transmission delay $\tau = 2$, then $Q = 1$. However, if $\tau = 11$, then $Q = 2$ which implies relatively worse blind estimation performance for higher dimensions of parameters to be estimated. On the other hand, blind estimation may suffer from the SLTT effect for $\tau = 11$ since there are many zero entries in H_1 and H_3 (at least eleven zeros in H_1 and seven zeros in H_3).

A PREPROCESSING ALGORITHM FOR BLIND CHANNEL IDENTIFICATION

Denote by σ_i the i th largest singular value of $T(h)$. The smallest nonzero singular value σ_{2Q+1} affects the distance between the signal and the noise subspace. The bigger σ_{2Q+1} is, the more insensitive the signal/noise subspace is to perturbations of the noise/signal subspace, namely, more well-conditioned. Otherwise, the SLTT effect may lead to ill-conditional identification. Therefore, it is shown that when the SSM is used, modeling only the significant parts is better than modeling all terms of the IR to avoid the SLTT effect. Moreover, the effective channel order, Q' , is proposed to substitute the actual channel order Q , where $Q' \leq Q$. As to this problem, since the transmission delay plays an important role to cause the SLTT effect and determines the channel order, appropriate transmission delay is chosen besides using the effective channel order to improve the performance of the SSM. In fact, fixing $Q' = 1$ so long as the significant parts of the IR can be contained for $Q = 2$ which may also be accomplished by choosing appropriate transmission delay. In the following, the optimal criterion of choosing the transmission delay in the sense of maximizing σ_3 with respect to $Q = 1$ will be derived and then it is extended to the case of $Q = 2$.

Change the transmission delay by changing the starting point of observation. Stack observation samples $y(n)_{n=k}^{L(M-1)+k-1}$ into a $L \times (M-1)$ $Y^{(k)}$, where $k = 0, 1, \dots, L-1$. First only noise-free receiving data $Y^{(k)} = X^{(k)}$ is

considered. Without losing any generality, assuming $\tau = 0$. When $Q' = Q = 1$, $Z_m^{(k)} = \begin{bmatrix} X_m^{(k)T} & X_{m-1}^{(k)T} \end{bmatrix}^T = T(h^{(k)})B_m$ where $k = L_1, L_1+1, \dots, L-1$ and $h^{(k)} = [0 \dots h(0) \dots h(L_1-1)]$ whose $L-k+1$ th element is $h(0)$. In order to provide the optimal k among $k = L_1, L_1+1, \dots, L-1$ in the sense of maximizing σ_3 , $\|H(k)\|^2 = C_1$ is first defined for it is positive constant independent of k if M is large enough:

$$\Delta(k) = \frac{H_1^{(k)H} H_1^{(k)}}{C_1} - \frac{1}{2}$$

and:

$$r(k) = \frac{H_1^{(k)H} H_2^{(k)}}{C_1}$$

also are defined. Now proceeds to the following theorem and corollary.

Theorem 1: For block Toeplitz matrix:

$$T(h^{(k)}) = \begin{bmatrix} H_1^{(k)} & H_2^{(k)} & 0 \\ 0 & H_1^{(k)} & H_2^{(k)} \end{bmatrix},$$

$\sigma_3^{(k)}$ monotonically increases with decreasing $|\Delta(k)|$ for $k = L_1, L_1+1, \dots, L-1$.

Proof: Since $T^H(h^{(k)})T(h^{(k)})$ has the same non-zero eigenvalues with $T^H(h^{(k)})T^H(h^{(k)})$, it is easy to see that:

$$T^H(h^{(k)})T(h^{(k)}) = \begin{bmatrix} H_1^{(k)H} H_1^{(k)} & H_1^{(k)H} H_2^{(k)} & 0 \\ (H_1^{(k)H} H_2^{(k)})^* & C_1 & H_1^{(k)H} H_2^{(k)} \\ 0 & (H_1^{(k)H} H_2^{(k)})^* & H_2^{(k)H} H_2^{(k)} \end{bmatrix} \quad (5)$$

then:

$$\frac{T^H(h^{(k)})T(h^{(k)})}{C_1} \triangleq \Omega = \begin{bmatrix} \frac{1}{2} + \Delta(k) & r(k) & 0 \\ r(k)^* & 1 & r(k) \\ 0 & r(k)^* & \frac{1}{2} - \Delta(k) \end{bmatrix}$$

Where:

$$\Delta(k)^2 \leq \frac{1}{4} - |r(k)|^2$$

by the Schwarz inequality. Without lost of generality, only considers the case $\Delta(k) \geq 0$.

$$|\lambda I - \Omega| = \lambda^3 - 2\lambda^2 + \left(\frac{5}{4}\Delta(k)^2 - 2|r(k)|^2 \right)\lambda + \Delta(k)^2 + |r(k)|^2 - \frac{1}{4} = 0 \quad (6)$$

With the cubic function of Shelby (1975), the three roots of Eq. 6 are the eigenvalues of Ω and they are:

$$\begin{aligned} \lambda_1 &= \frac{2}{3} + \frac{2}{3}\sqrt{A} \cos \frac{\theta}{3} \\ \lambda_2 &= \frac{2}{3} + \frac{2}{3}\sqrt{A} \cos \left(\frac{\theta - 2\pi}{3} \right) \lambda_3 = \frac{2}{3} + \frac{2}{3}\sqrt{A} \cos \left(\frac{\theta + 2\pi}{3} \right) \end{aligned} \quad (7)$$

Where:

$$A = 3\Delta(k)^2 + 6|r(k)|^2 + \frac{1}{4},$$

$$\theta = \arccos \left(\frac{T}{-2A^{3/2}} \right),$$

$$T = 9\Delta(k)^2 - 9|r(k)|^2 - \frac{1}{4}$$

Since $A > 0$, $\theta \in [0, \pi]$, $\lambda_1 \geq \lambda_2 \geq \lambda_3$ satisfies. Derivate λ_3 to $\Delta(k)$:

$$\begin{aligned} \lambda_3' &= -\frac{2T'A - 3A'T}{18\sqrt{A^4 - T^2A/4}} \sin \frac{\theta + 2\pi}{3} \\ &+ \frac{A'}{3\sqrt{A}} \cos \frac{\theta + 2\pi}{3} \\ &\triangleq \alpha \sin \left(\frac{\theta + 2\pi}{3} \right) + \beta \cos \left(\frac{\theta + 2\pi}{3} \right) \end{aligned}$$

As:

$$\frac{2T'A - 3A'T}{18\sqrt{A^4 - T^2A/4}} = 54\Delta(k) \left(-\Delta(k)^2 + 7|r(k)|^2 + \frac{1}{4} \right) \geq 0,$$

the above equation can be written as:

$$\lambda_3' = -\sqrt{\alpha^2 + \beta^2} \sin \left(\frac{\theta + 2\pi}{3} + \phi \right) \quad (8)$$

Where:

$$\alpha = \frac{2T'A - 3A'T}{18\sqrt{A^4 - T^2A/4}},$$

$$\beta = \frac{A'}{3\sqrt{A}},$$

$$\phi = \arctan \frac{\beta}{\alpha}$$

Obviously, there is:

$$\frac{\beta}{\alpha} = \frac{6A'\sqrt{A^3 - T^2/4}}{3TA' - 2T'A} < 0,$$

then $\phi \in [-\pi/2, 0]$. So, when $\Delta(k) \geq 0$, $\lambda_3' \leq 0$. Similarly, $\Delta(k) \leq 0$, $\lambda_3' \geq 0$. Moreover $\sigma_3^2 = C_1 \lambda_3$ which implies that σ_3 monotonically increases with decreasing $|\Delta(k)|$. This completes the proof.

It is obvious that if $|\Delta(k)|$ is minimized, H_1 and H_2 usually have the approximately same number of zero entries, i.e., the SLTT effect may be eliminated. The following corollary provides a criterion to minimize $|\Delta(k)|$.

Corollary 1: Denoting $\Lambda_i^{(k)}$ as the i th largest singular value of $H^{(k)}$, $T_1^{(k)} = \Lambda_1^{(k)} + \Lambda_2^{(k)}$ and $T_2^{(k)} = \Lambda_1^{(k)} \Lambda_2^{(k)}$, if:

$$k_1 = \arg \max_{k=L_1, \dots, L-1} (T_1^{(k)}),$$

$$k_2 = \arg \max_{k=L_1, \dots, L-1} (T_2^{(k)}),$$

then $k_1 = k_2$, $T_1^{(k)}$ and $T_2^{(k)}$ are unimodal functions and $|\sigma_3^{(k_1)}| \geq |\sigma_3^{(k)}|$ for $k = L_1, \dots, L-1$.

Proof: Since:

$$\begin{aligned} \frac{H^{(k)H} H^{(k)}}{C_1} &= \begin{bmatrix} \frac{1}{2} + \Delta(k) & r(k) \\ r(k)^* & \frac{1}{2} - \Delta(k) \end{bmatrix} \\ \frac{(\Lambda_{1,2}^{(k)})^2}{C_1} &= \frac{1}{2} \pm \sqrt{\Delta(k)^2 + r(k)^2} \end{aligned} \quad (9)$$

Therefore,

$$\begin{aligned} k_1 &= \arg \max_{k=L_1, \dots, L-1} (\Lambda_1^{(k)} + \Lambda_2^{(k)}) = \arg \max_{k=L_1, \dots, L-1} (\Lambda_1^{(k)} + \Lambda_2^{(k)})^2 \\ &= \arg \max_{k=L_1, \dots, L-1} (\Lambda_1^{(k)} \Lambda_2^{(k)}) = k_2 = \arg \min_{k=L_1, \dots, L-1} (|\Delta(k)|) \end{aligned}$$

The last equation implies that $|\sigma_3^{(k_1)}| \geq |\sigma_3^{(k)}|$ for $k = L_1, \dots, L-1$ which completes the proof.

Theorem 1 and Corollary 1 show that by maximizing $T_1^{(k)}$ or $T_2^{(k)}$ when $Q = 1$, the optimal starting point can be chosen.

When $Q = 2$, i.e., $k = 0, \dots, L-1$ and substitutes Q with $Q' = 1$, $X^{(k)}$ has three non-zero singular values and maximizing $T_1^{(k)}$ or $T_2^{(k)}$ can not always guarantee to maximize σ_3 . However, since:

$$\sum_{i=1}^3 (\Lambda_i^{(k)})^2 = C_1$$

independent of k for large enough M , it is expected if $T_1^{(k)}$ or $T_2^{(k)}$ is large then $\Lambda_3^{(k)}$ is small, that is, the significant parts of the IR is reserved for $Q' = 1$. When $\Lambda_3^{(k)}$ is small, theorem 1 and corollary 1 should approximately hold under small perturbation. Thus, it should be expected $\sigma_3^{(k_1)}$ and $\sigma_3^{(k_2)}$ are large (although maybe not the largest) among all possible values of $\sigma_3^{(k)}$ for $k = 0, \dots, L-1$ and $|k_1 - k_2| \approx 0$ or L .

Now considers noisy observation. Denote by $\hat{\gamma}_i^{(k)}$ the i th largest eigenvalue of:

$$R^{(k)} = \frac{Y^{(k)} (Y^{(k)})^H}{M-1}$$

When $M \rightarrow \infty$, $\hat{\gamma}_i^{(k)} = (\Lambda_i^{(k)})^2 + \sigma^2$. To avoid estimating the noise variance σ^2 , the following criterion is proposed:

$$k_3 = \arg \max_{k=L_1, \dots, L-1} (\hat{\gamma}_1^{(k)} \hat{\gamma}_2^{(k)}). \quad (10)$$

Since $\hat{\gamma}_1^{(k)} \hat{\gamma}_2^{(k)} = \sigma^2 (T_1^{(k)})^2 + (T_2^{(k)} - 2\sigma^2) T_2^{(k)} + \sigma^4$ and $T_2^{(k_3)} \gg 2\sigma^2$ (otherwise the signal is too weak to identify), this criterion is identical to corollary 1 for noise-free case and can tolerate the noise, that is, $|k_1 - k_2| \approx 0$ or L and $\sigma_3^{(k_3)}$ is large for noisy cases. Using the criterion Eq. 10, the proposed preprocessing algorithm substitutes the data matrix Y with $Y^{(k_3)}$, then $Y^{(k_3)}$ is used with the blind estimation of DS-SS signals based on the SSM method.

SIMULATIONS

Here, the following simulation results are presented: the validity of criterion in corollary 1 extended for $\forall k \in \{0, \dots, L-1\}$ and the performance improvement for the SSM provided by proposed preprocessing algorithm. Linear Minimum Mean Square Error (MMSE) receiver is used to estimate the symbol sequence and take the Bit Error Rate (BER) as the measure of blind estimation performance.

Test 1: The validity of corollary 1 when it is extended for $\forall k \in \{0, \dots, L-1\}$.

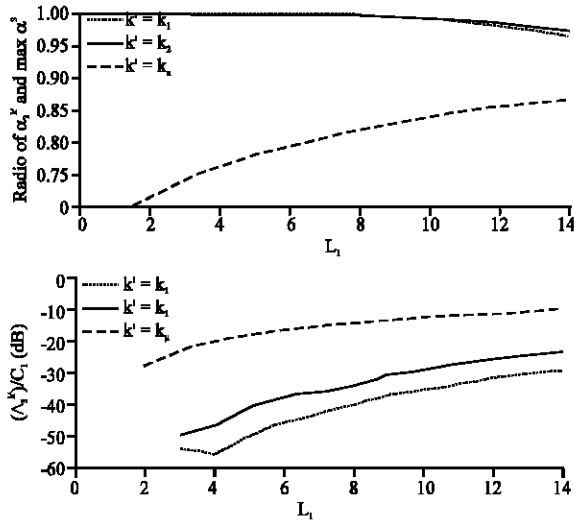


Fig. 1: The validity of extended criteria in corollary 1

Suppose $L = 15$ and:

$$\{h(l)\}_{l=0}^{L+L_1-1}$$

is an i.i.d. complex Gaussian channel IR. 10^4 Monte Carlo runs are used. Figure 1 plots:

$$\sigma_3^{(k_{1,2})} / \max_{k=0, \dots, L-1} (\sigma_3^{(k)}) \text{ and } (\Lambda_3^{(k_{1,2})})^2 / C_1$$

under different channel length L_1 with the criteria in corollary 1 extended for $k \in \{0, \dots, L-1\}$. The cases with uniformly distributed $k_e \in \{0, \dots, L-1\}$ are also plotted to represent no preprocessing used. Compared with non-preprocessing method, both k_1 and k_2 criteria can equivalently and approximately obtain the largest σ_3 , i.e., well-conditional identification and they lead to little marginal model error substituting the effective channel order $Q' = 1$ with the actual one Q . Figure 1 also shows that:

$$(\Lambda_3^{(k_{1,2})})^2 / C_1$$

increases as L_1 increases which implies the performance improvement would become relatively insignificant.

Test 2: BER versus L_1 : This test compares under different channel length L_1 , the BER of proposed preprocessing algorithm, original SSM with known actual channel order and improved SSM with estimated effective channel order. Setting $M = 102$, $\text{SNR} = -6$ dB and generate transmission delay τ uniformly. The other parameters are the same as test 1. Figure 2 shows the proposed

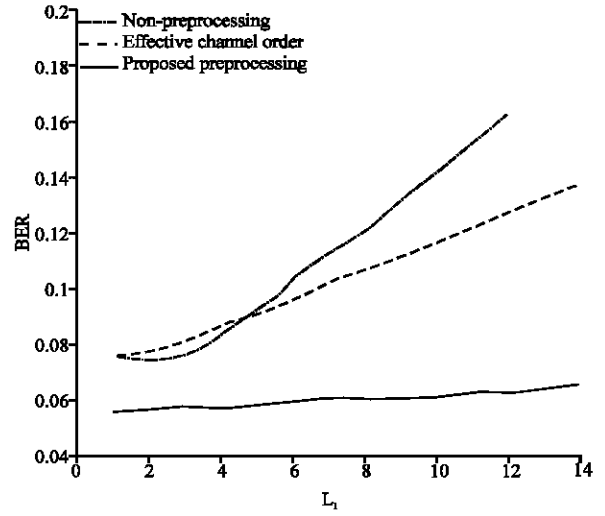
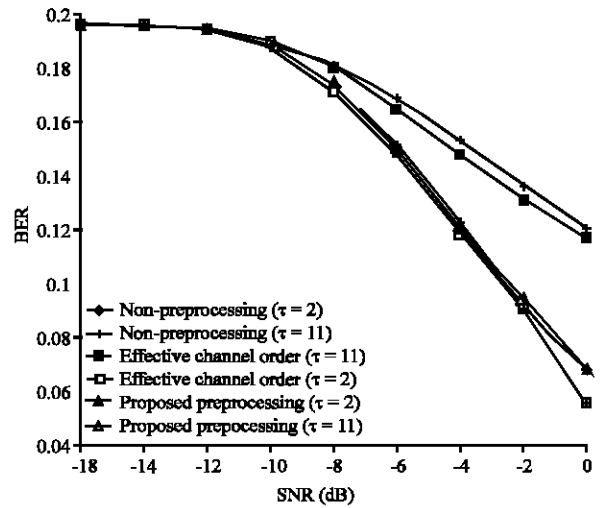

 Fig. 2: BER with different L_1


Fig. 3: BER with different SNR

preprocessing method behaves better than the other methods because it not only exploits the effective channel order but also selects the appropriate starting point of receiving data. On the other hand, with L_1 increasing, the performance of the proposed only deteriorates marginally while the others deteriorate significantly.

Test 3: BER versus SNR: As compared with test 2, considers the multipath channel A in section 3 with complex Gaussian i.i.d. gains. Two cases $\tau = 2$ and $\tau = 11$ are considered. Figure 3 shows that all methods can obtain better performance as SNR increases and when $\tau = 2$, they all behave almost identically. However, both SSM and its improvement with the effective channel order present significant worse performance for $\tau = 11$. On the contrary, the proposed method obtains

identical good performance for both cases which implies the proposed algorithm is robust for different transmission delay.

CONCLUSIONS

Starting point would significantly affect blind estimation performance of DS-SS signals in multipath for the channel order uncertain and the SLLT effect. A preprocessing algorithm is derived to determine appropriate starting point of receiving data and avoid the channel order estimation. Simulation results show that the preprocessing algorithm significantly outperforms conventional methods and increase the robustness of blind estimation of DS-SS signals based on the subspace method.

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