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Adaptive PID-like Fuzzy Variable Structure Control for Uncertain MIMO Systems

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Abstract: In this study, the stability analysis of the PID-like adaptive fuzzy Sliding Model Controller (SMC) for a class of nonlinear uncertain MIMO systems is presented. The upper bounds of structured uncertainties and external disturbance are not needed to be known and approximated by a fuzzy inference system. To overcome the chattering problem in the conventional SMC scheme, an adaptive Proportional Integral Derivative (PID) controller is designed to replace the switching part of the SMC scheme. The approximation and estimation errors are also assumed to be unknown and estimated online by using adaptive laws. The global stability and robustness of the closed-loop system is ensured by the derivation of the stability criterion based upon Lyapunov's direct method. Finally, numerical simulations for an two-link rigid robot under different controllers are provided and the results show that the proposed approach achieves satisfactory performance from the viewpoint of chattering removement and tracking accuracy.

Key words: Multi-input multi-output (MIMO) systems, fuzzy control, sliding mode control (SMC), proportional integral derivative (PID) control, chattering removement

INTRODUCTION

Research on Variable Structure Control (VSC) was originated in early 1950s for single input systems with high order differential equations (Utkin, 1977). Recently, the VSC control with Sliding Mode Control (SMC) was widely researched for uncertain Multi-Input Multi-Output (MIMO) systems in the continuous domain and discrete time domain (Chen *et al*., 2008). SMC is an efficient tool to control complex high-order dynamic plants with structured or/and unstructured uncertainties due to its order reduction property and low sensitivity to external disturbances and plant parameter variations. In SMC, the states of the controlled system are first forced to slide to a designed surface (i.e., the sliding surface) with a equivalent control law in state space and then keeping them there with a switching law (Perruquetti and Barbot, 2002). There has been a wide variety of applications of SMC in classical MIMO systems such as induction machines, power control, aerospace and process control (Lasaad *et al*., 2007; Zribi and Al-Rifai, 2006). However, its major drawback in practical applications is the chattering problem. Numerous techniques have been proposed to eliminate this phenomenon in SMC.

Conventional SMC methods used to eliminate the chattering are to replace the relay control by the neural network based SMC (Kang and Jin, 2010), integral sliding control (Choi, 2007) and boundary layer technique (Chen *et al*., 2002). The boundary layer method was introduced to eliminate the chattering around the switching surface and the control discontinuity within this thin boundary layer. If systems uncertainties are large, the sliding-mode controller would require a high switching gain with a thicker boundary layer to eliminate the higher resulting chattering effect. However, if we continuously increase the boundary layer thickness, we are actually reducing the MIMO system to a system without sliding mode.

To tackle these difficulties, Fuzzy Logic Controllers (FLC) are often used to deal with the discontinuous sign function in the reaching phase of SMC (Feng, 2006). Recently, Adaptive Fuzzy SMC (AFSMC) methods are also used for this purpose which is shown to be quite effective (Wai *et al*., 2008). Designing adaptive fuzzy controllers by the integration of fuzzy logic and the SMC for ensuring stability and consistent performance has been widely researched. Many new algorithms have been proposed based on the integration of these control methods (Yufeng *et al*., 2011; Zhang *et al*., 2011). These

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approaches are similar in the aspect that they directly approximate the sliding mode control law by fuzzy approximator. To overcome these limitations, Hamzaoui *et al.* (2003) and Essounbouli and Hamzaoui (2006) applied an adaptive Takagi-Sugeno fuzzy system. Aloui *et al.* (2008) and Ho *et al.* (2009) proposed a method to eliminate the chattering phenomenon by using an adaptive Proportional Integral controller for a SISO nonlinear system. However, the discontinuity function is still there and the chattering is not completely eliminated.

In this study, for the purpose of simultaneously reducing the chattering phenomenon as well as to ensure a faster convergence to zero of the tracking errors, a novel PID-like AFSMC control algorithm is developed by combining the fuzzy SMC approach with the Proportional Integrate Derivative (PID) control method. To the best of our knowledge, it was the first time for the idea to appear in the literatures. Thus, we obtain a faster convergence to the sliding surfaces. Moreover, in the controller design, we do not need to know the upper bounds of both of external disturbances and the structured uncertainties. The robustness of the closed loop system is ensured by Lyapunov arguments and simulation results verify the correctness and effectiveness of the proposed method.

PROBLEM FORMULATION

Consider the following square MIMO system with p inputs and q outputs:

$$\begin{cases} \dot{x}_1 = x_2 \\ \vdots \\ \dot{x}_{(n_1-1)} = x_{n_1} \\ \dot{x}_{n_1} = f_1(x) + g_{11}(x)u_1 + \cdots + g_{1p}(x)u_p + d_1(x) \\ \dot{x}_{(n_1+1)} = x_{(n_1+2)} \\ \\ \dot{x}_{(n_1+n_2-1)} = x_{(n_1+n_2)} \\ \dot{x}_{(n_1+n_2)} = f_2(x) + g_{21}(x)u_1 + \cdots + g_{2p}(x)u_p + d_2(x) \\ \vdots \\ \dot{x}_m = f_p(x) + g_{p1}(x)u_1 + \cdots + g_{pp}(x)u_p + d_p(x) \\ y_1 = x_1 \\ y_2 = x_{(n_1+1)} \\ \vdots \\ y_q = x_{(m-n_q+1)} \end{cases} \tag{1}$$

where, the number of states $m = n_1+n_2+...+n_p$ and the state vector $x\epsilon R^m$ is assumed to be measurable, $u(t) = [u_1, ..., u_p]^T$ is control input, $f_i(x)$ is smooth system function, $g_{ij}(x)$ is unknown but bounded nonlinear function, $i = l, ..., p$, $j = 1, ..., q$, the external disturbance $d(t) = (d_1, ..., d_p)^T$ and $|d_i| \le D_i$, where $D_i>0$, $\forall i = 1, ..., p$.

For the desired tracking trajectory $y_i(t)$, $i = 1, ..., q$, if its $1\sim n_i$ order difference exists, define the tracking error vector as:

$$e = x - y_d \tag{2}$$

Where:

$$e = (e_1, \cdots, e_q)^T,$$
$$e_i = (e_i, \dot{e}_i, \cdots, e_i^{(n_i-1)}),$$
$$e_i = x_i - y_{d_i},$$
$$y_d = (y_{d_1}, \dot{y}_{d_1}, \cdots, y_{d_1}^{(n_1-1)}, \cdots, y_{d_q}, \dot{y}_{d_q}, \cdots, y_{d_q}^{(n_p-1)})^T.$$

For the square nonlinear system, we have $q = p$. Differentiating $y_1, y_2, \dots, y_p$ in (1) with respect to time for $n_1, n_2\dots, n_p$ times, respectively, until the inputs appear, one obtains the input/output form of (1) as:

$$\begin{cases} y_1^{(n_1)} = f_1(x) + \sum_{j=1}^{p} g_{1j}(x)u_j(t) + d_1 \\ y_2^{(n_2)} = f_2(x) + \sum_{j=1}^{p} g_{2j}(x)u_j(t) + d_2 \\ \vdots \\ y_p^{(n_p)} = f_p(x) + \sum_{j=1}^{p} g_{pj}(x)u_j(t) + d_p \end{cases} \tag{3}$$

Let:

$$F(x) = [f_1(x), f_2(x), \cdots, f_p(x)]^T,$$
$$y^{(n)} = [y_1^{(n_1)}, \cdots, y_q^{(n_q)}]^T,$$
$$x = [y_1, y_1^{(n_1-1)}, \cdots, y_p, \cdots, y_p^{(n_p-1)}]:$$

$$G(x) = \begin{bmatrix} g_{11}(x) & \cdots & g_{1p}(x) \\ \vdots & \ddots & \vdots \\ g_{p1}(x) & \cdots & g_{pp}(x) \end{bmatrix}$$

then the equality (3) can be represented as:

$$y^{(n)} = F(x) + G(x)u + d(x, t) \tag{4}$$

Consider the parameters in the nominal condition without all the uncertainty deviation, the nominal model of the nonlinear dynamical system given by Eq. 4 can be written as follows:

$$y^{(n)} = F_0(x) + G_0(x)u$$

where, $F_0(x)$ and $G_0(x)$ are known system functions. Equation 4 can be rewritten as:

$$y^{(n)} = F_0(x) + \Delta F(x) + (G_0(x) + \Delta G(x))u + d(x,t) \tag{5}$$

where, $\Delta F(x)$ and $\Delta G(x)$ are the unknown uncertainties. Thus, the term $\Delta F(x) + \Delta G(x)u$ is not only unknown, but also depends on the value of the control input $u(t)$.

Assumption 1: G (x) is bounded away from singularity, thus G^{-1} (x) exists and has a bounded norm over a compact set $\Omega \subset R^n$.

Assumption 2: Let D (x, t) = ΔF (x)+ΔG (x) u+d (x, t), it means that D (x, t) contains the whole uncertainties and ‖D (x, t)‖≤ δ (x), where δ (x) is unknown positive function.

CONVERTIONAL SLIDING MODE CONTROL

Design of the conventional SMC controller involves two important phases. The first phase is to design a suitable sliding surface function s so that once the system enters the hyper-plane s, the desired dynamic characteristics can be realized. The second is to design a proper controller u (t) so that it can drive the system's dynamics into the designed hyper-plane and stay thereafter.

We first define the linear sliding surfaces as follows. The proposed sliding mode function is:

$$s_i = e_i^{(n_i-1)} + \alpha_{i(n_i-1)} e_i^{(n_i-2)} + \cdots + \alpha_{i2} \dot{e}_i + \alpha_{i1} e_i(t) \tag{6}$$

where, α_{ij} is the designed SMC coefficient and α_{ij} should be properly choiced so that the polynomial $s^{n_i-1} + \alpha_{i(n_i-2)} s^{n_i-2} + \cdots + \alpha_{i1} s$ is Hurwitz. The sliding mode vector $S = [S_1, ..., S_p]^T$.

In the case that the system functions and the parameters are all known and invariant, the conventional SMC controller can be designed as:

$$u = G^{-1}(x)\,[-F(x)+v-u_S] \tag{7}$$

where, the switching control part of SMC is:

$$u_s = \begin{bmatrix} \eta_1 \operatorname{sgn}(s_1) \\ \vdots \\ \eta_p \operatorname{sgn}(s_p) \end{bmatrix}$$

and the switching gain $\eta_i > |d_i|$, $\forall i = 1, ..., p$. v is the feedback linearization control law and defined as:

$$v = \bar{y}_d^{(n)} - \Theta_{n-2} e^{(n-1)} - \cdots - \Theta_1 \dot{e} \tag{8}$$

where, $\bar{y}_d^{(n)} = (y_{d1}^{(n_1)}, y_{d2}^{(n_2)}, \cdots, y_{dp}^{(n_p)})^T$, the diagonal matrix $\Theta_1 = \text{diag}(\alpha_{11}, \alpha_{21}, \cdots, \alpha_{p1}), \ldots, \Theta_1 = \text{diag}(\alpha_{11}, \alpha_{21}, \cdots, \alpha_{p1})$, $e^{(i)} = (e_1^{(i)}, e_2^{(i)}, \cdots, e_p^{(i)})^T$, i = 1, 2, ..., n-2.

If F (x) and G (x) are exactly known and the switching gain η_i satisfies the sliding condition, the system trajectories will enter the sliding mode and the tracking error converges to zero, thus the control objective can be achieved by the control law designed as Eq. 7 and 8. However, there always exist the unknown functions ΔF (x) and ΔG (x), the control law Eq. 7 is not applicable generally and not robust to the uncertainties D (x, t).

ADAPTIVE PID-LIKE FUZZY SMC CONTROL

Approximation of the upper bound: Since the uncertainties D (x, t) imposes adverse impact on the control performance of SMC controller and cannot be exactly known, a fuzzy system is designed in this section to onlinely approximate D (x, t). The inputs of the designed fuzzy system are chosen as $x = [x_1, x_2, ..., x_n]^T$, by using the singleton fuzzifier, product inference and weighted average defuzifier (Feng, 2006), then the outputs of the fuzzy model can be expressed as:

$$\hat{\delta}(x) = y = \theta_\delta^T \xi(x) \tag{9}$$

where, θ_δ^T is the adjustable parameters vector and $\xi T(x) = [\xi_1, \xi_2, ..., \xi_N]^T$ is the vector of Fuzzy Basis Functions (FBF) defined as:

$$\xi_k(x) = \frac{\prod_{i=1}^{n} \mu_{A_i^k}(x_i)}{\sum_{k=1}^{N} \prod_{i=1}^{n} \mu_{A_i^k}(x_i)} \tag{10}$$

where, k = 1,2,...,N is the index of the rule, $\mu_{A_i^k}(x_i)$ denotes the fuzzy sets assigned to x_i (i = 1,...,5).

The parameter θ_δ^T belongs to the compact set Ω_δ that is defined as $\Omega_\delta = \{\theta_\delta \in \Re^N \mid \|\theta_\delta\| \le m_\delta\}$, where m_δ is finite positive constants. According the fuzzy sets theory, there exists the optimal parameter θ_δ^T to deduce a minimal approximation error as following:

$$\theta_\delta^* = \arg\min_{\theta_\delta \in \Omega_\delta} \left[\sup_{x \in \Omega_x} \left| \delta(x) - \hat{\delta}(x) \right| \right] \tag{11}$$

$$\tilde{\theta}_\delta = \theta_\delta^* - \theta_\delta, \; e_\delta = \delta(x) - \hat{\delta}(x) \tag{12}$$

where, $\tilde{\theta}_\delta$ and e_δ are the approximation error of the parameter and the upper bound, respectively. Obviously, $\tilde{\theta}_\delta$ and e_δ will be arbitrary small and e_δ cannot be directly computed. In the controller design, e_δ should be replaced by the estimation $\hat{e}_\delta$.

Adaptive PID-like SMC control: In the conventional SMC design, the presence of the signum function in the switching term leads to the chattering phenomenon which can excite the high frequency dynamics. To avoid this problem and to achieve the previous control objectives, an adaptive PID term is added to the control law and replaces the discontinuous term u_S. In fact, the derivative action, by compensating the inertia due to dead time, accelerates the response of the system and improves the stability of the closed loop by allowing fast oscillations due to the appearance of a disturbance or a sudden change of the reference signal. Thus, we want a faster convergence to the sliding surfaces.

To replace the term u_S, after used the sliding function S_i, its derivative $\dot{s}_i$ and integration $Is(t)=\int_0^t s_i(\tau)d\tau$ as the variables, the PID controller is designed as:

$$u_{PID}=\begin{bmatrix} k_{p_1}s_1(t)+k_{d_1}\dot{s}_1(t)+k_{i_1}Is_1(t) \\ \vdots \\ k_{p_p}s_p(t)+k_{d_p}\dot{s}_p(t)+k_{i_p}Is_p(t) \end{bmatrix} \qquad (13)$$

where, k_{pi}, k_{di} and k_{i_i}, i = 1, ..., p, are adaptive proportional, differential and integral coefficients, respectively.

Approximate u_S by using Eq. 13 and rewrite it in matrix form, one can obtain:

$$u_s \approx u_{PID}=\left(\zeta_1^T\theta_k(s_1),\cdots,\zeta_p^T\theta_k(s_p)\right)^T=\zeta(s)\theta_k \qquad (14)$$

where, the adjustable parameter vector $\theta_{kj}=\theta_k(S_j)=(k_{pj},k_{ij},k_{di})^T$, the regressive vector $\zeta_k^T(s_j)=(s_j,Is,\dot{s}_j)$, $j=1,2,\cdots,p$ and $\theta_k=\left(\theta_{k1}^T,\cdots,\theta_{kp}^T\right)^T$:

$$\zeta(S)=\begin{pmatrix}\zeta_k^T(s_1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \zeta_k^T(s_p)\end{pmatrix}$$

Define the optimal parameter:

$$\theta_{k_i}^*=\underset{\theta_{k_i}\in\Omega_k}{\operatorname{argmin}}\left[\sup_{s\in\Omega_s}\left|\zeta(s)\theta_k-u_s\right|\right]$$

where, $\Omega_{k_i},\Omega_{s_i}$ belongs to the compact sets for θ_{k_i} and S_i, respectively. Let $\tilde{\theta}_{k_i}=\theta_{k_i}^*-\theta_{k_i}$ denote the parameter approximation error, thus $e_{PID}=\zeta(s)\theta_k-u_S$ is the minimal approximation error for the switching control term. Obviously, the value of e_{PID} is directly affected by the adjustable parameters θ_k and should be replaced by its estimation $\hat{e}_{PID}$.

The global PID SMC control law is designed as:

$$\begin{cases} u=G_0^{-1}(-F_0+v-u_{PID}+u_r+u_n) \\ u_{PID}=\zeta(s)\theta_k \\ u_r=-\dfrac{S}{\|S^T\|}\hat{\delta}(x\,|\,\theta_\delta) \\ u_n=-\dfrac{S}{\|S^T\|}\hat{e}_\delta+\hat{e}_{PID} \end{cases} \qquad (15)$$

with the following adaptive laws:

$$\begin{cases} \dot{\theta}_\delta=\gamma_\delta\,\xi(x)\|S^T\| \\ \dot{\theta}_k=\gamma_k\,\zeta(S)S \\ \dot{\hat{e}}_\delta=\gamma_e\|S^T\| \\ \dot{\hat{e}}_{PID}=-\gamma_{PID}\,S \end{cases} \qquad (16)$$

where, the adaptive rates $\gamma_\delta>0$, $\gamma_k>0$, $\gamma_{PID}>0$.

In the control law Eq. 15, the global controlled composes four terms, i.e., the first term $(-F_0+v)$ is the feedback linearization one to deal with the nominal system (1); the second term is the fuzzy PID control to approximate the switching part of SMC; the third term is the fuzzy robust control to counteract the model uncertainties; the last term is the compensation for the approximation error of PID controller and robust controller.

Stability analysis: The following theory gives the stability condition for the control system (5).

Theorem 1: Consider the nonlinear system (5), suppose that the upper bound is approximated by the fuzzy system, the sliding surface S_i is given by Eq. 6 and the control law is designed as Eq. 15 with the adaptive laws given in Eq. 16, then the resulted closed-loop system is asymptotically stable and the trajectories will enter the sliding mode motion, thus the tracking error will converge to zero.

Proof: Consider the following Lyapunov candidate:

$$V(x)=\frac{1}{2}S^T S+\frac{1}{2\gamma_\delta}\tilde{\theta}_\delta^T\tilde{\theta}_\delta+\frac{1}{2\gamma_e}\tilde{e}_\delta^2+\frac{1}{2\gamma_k}\tilde{\theta}_k^T\tilde{\theta}_k+\frac{1}{2\gamma_{PID}}\tilde{e}_{PID}^T\tilde{e}_{PID}>0 \qquad (17)$$

The derivative of V(x) can be obtained as:

$$\dot{V}(x)=S^T\dot{S}+\frac{1}{\gamma_\delta}\tilde{\theta}_\delta^T\dot{\tilde{\theta}}_\delta+\frac{1}{\gamma_w}\tilde{e}_\delta\dot{\tilde{e}}_\delta+\frac{1}{\gamma_k}\tilde{\theta}_k^T\dot{\tilde{\theta}}_k+\frac{1}{\gamma_{PID}}\tilde{e}_{PID}^T\dot{\tilde{e}}_{PID} \qquad (18)$$

Also, the derivative of the sliding function can be computed as:

$$\dot{S}=y^{(n)}-v=F_0(x)+G_0(x)u+D(x,t)-v=u_r+u_n-u_{PID}+D(x,t) \qquad (19)$$

Since $\dot{\tilde{\theta}}_\delta=-\dot{\theta}_\delta,\dot{\tilde{\theta}}_k=-\dot{\theta}_k,\dot{\tilde{e}}_\delta=-\dot{\hat{e}}_\delta,\dot{\tilde{e}}_{PID}=-\dot{\hat{e}}_{PID}$ hold, Eq. 18 can be rewritten as follows:

$$
\begin{aligned}
\dot{V}(\mathbf{x}) &= S^T\left(u_r + u_n - u_{PID} + D(\mathbf{x},t)\right) + \frac{1}{\gamma_\delta}\tilde{\theta}_\delta^T \dot{\tilde{\theta}}_\delta + \frac{1}{\gamma_e}\tilde{e}_\delta \dot{\tilde{e}}_\delta + \frac{1}{\gamma_k}\tilde{\theta}_k^T \dot{\tilde{\theta}}_k + \frac{1}{\gamma_{PID}}\tilde{e}_{PID}^T \dot{\tilde{e}}_{PID} \\
&= S^T\left(u_r + u_n - u_{PID} + D(\mathbf{x},t)\right) - \frac{1}{\gamma_\delta}\tilde{\theta}_\delta^T \dot{\theta}_\delta - \frac{1}{\gamma_e}\tilde{e}_\delta \dot{\hat{e}}_\delta - \frac{1}{\gamma_k}\tilde{\theta}_k^T \dot{\theta}_k - \frac{1}{\gamma_{PID}}\tilde{e}_{PID}^T \dot{\hat{e}}_{PID} \\
&= S^T\left(u_r + u_n + D(\mathbf{x},t)\right) - S^T \theta_k^T \zeta(S) - \frac{1}{\gamma_\delta}\tilde{\theta}_\delta^T \dot{\theta}_\delta - \frac{1}{\gamma_e}\tilde{e}_\delta \dot{\hat{e}}_\delta - \frac{1}{\gamma_k}\tilde{\theta}_k^T \dot{\theta}_k - \frac{1}{\gamma_{PID}}\tilde{e}_{PID}^T \dot{\hat{e}}_{PID} \\
&= S^T\left(u_r + u_n + D(\mathbf{x},t)\right) - S^T \hat{\theta}_k^T \zeta(S) - S^T \tilde{\theta}_k^T \zeta(S) - \frac{1}{\gamma_\delta}\tilde{\theta}_\delta^T \dot{\theta}_\delta - \frac{1}{\gamma_e}\tilde{e}_\delta \dot{\hat{e}}_\delta - \frac{1}{\gamma_k}\tilde{\theta}_k^T \dot{\theta}_k \\
&\quad - \frac{1}{\gamma_{PID}}\tilde{e}_{PID}^T \dot{\hat{e}}_{PID}
\end{aligned} \tag{20}
$$

Substitute the adaptive laws Eq. 16 into 20, one can obtain:

$$
\dot{V}(\mathbf{x}) = S^T\left(u_r + u_n + D(\mathbf{x},t)\right) - S^T \tilde{\theta}_k^T \zeta(s) - \frac{1}{\gamma_\delta}\tilde{\theta}_\delta^T \dot{\theta}_\delta - \frac{1}{\gamma_e}\tilde{e}_\delta \dot{\hat{e}}_\delta - \frac{1}{\gamma_{PID}}\tilde{e}_{PID}^T \dot{\hat{e}}_{PID} \tag{21}
$$

Notice the Assumption 2 that $\|D(x, t)\| \le \delta(x)$ holds, Eq. 21 can be further transformed as following:

$$
\begin{aligned}
\dot{V}(\mathbf{x}) &\le S^T(u_r + u_n) + \left\|S^T\right\| \cdot \delta(\mathbf{x}) - S^T \tilde{\theta}_k^T \zeta(S) - \frac{1}{\gamma_\delta}\tilde{\theta}_\delta^T \dot{\theta}_\delta - \frac{1}{\gamma_e}\tilde{e}_\delta \dot{\hat{e}}_\delta - \frac{1}{\gamma_{PID}}\tilde{e}_{PID}^T \dot{\hat{e}}_{PID} \\
&\le S^T(u_r + u_n) + \left\|S^T\right\| \cdot \left(e_\delta + \hat{\delta}(\mathbf{x})\right) - S^T \tilde{\theta}_k^T \zeta(S) - \frac{1}{\gamma_\delta}\tilde{\theta}_\delta^T \dot{\theta}_\delta - \frac{1}{\gamma_e}\tilde{e}_\delta \dot{\hat{e}}_\delta - \frac{1}{\gamma_{PID}}\tilde{e}_{PID}^T \dot{\hat{e}}_{PID}
\end{aligned} \tag{22}
$$

Substitute the control laws u_r (t) and into Eq. 22, one can have:

$$
\begin{aligned}
\dot{V}(x) &\le \left\|S^T\right\|\left(e_\delta - \hat{e}_\delta\right) + \left\|S^T\right\| \cdot \left(\hat{\delta}(x \mid \theta_\delta^*) - \hat{\delta}(x \mid \theta_\delta)\right) - S^T \tilde{\theta}_k^T \zeta(S) - \frac{1}{\gamma_\delta}\tilde{\theta}_\delta^T \dot{\theta}_\delta - \frac{1}{\gamma_e}\tilde{e}_\delta \dot{\hat{e}}_\delta \\
&\quad - \frac{1}{\gamma_{PID}}\tilde{e}_{PID}^T \dot{\hat{e}}_{PID} + S^T \hat{e}_{PID} \\
&\le \left\|S^T\right\|\tilde{e}_\delta + \left\|S^T\right\| \cdot \tilde{\theta}_\delta^T \xi(x) - S^T \tilde{\theta}_k^T \zeta(S) - \frac{1}{\gamma_\delta}\tilde{\theta}_\delta^T \dot{\theta}_\delta - \frac{1}{\gamma_w}\tilde{e}_\delta \dot{\hat{e}}_\delta - \frac{1}{\gamma_{PID}}\tilde{e}_{PID}^T \dot{\hat{e}}_{PID} + S^T \hat{e}_{PID} \\
&\le \left[\left\|S^T\right\|\tilde{e}_\delta - \frac{1}{\gamma_w}\tilde{e}_\delta \dot{\hat{e}}_\delta\right] - \eta S^T \operatorname{sgn}(S) - S^T e_{PID} + \left[\left\|S^T\right\| \cdot \tilde{\theta}_\delta^T \xi(x) - \frac{1}{\gamma_\delta}\tilde{\theta}_\delta^T \dot{\theta}_\delta\right] + S^T \hat{e}_{PID} \\
&\quad - \frac{1}{\gamma_{PID}}\tilde{e}_{PID}^T \dot{\hat{e}}_{PID} \\
&\le \left[\left\|S^T\right\|\tilde{e}_\delta - \frac{1}{\gamma_w}\tilde{e}_\delta \dot{\hat{e}}_\delta\right] + \left[\left\|S^T\right\| \cdot \tilde{\theta}_\delta^T \xi(x) - \frac{1}{\gamma_\delta}\tilde{\theta}_\delta^T \dot{\theta}_\delta\right] - \left[S^T \tilde{e}_{PID} - \frac{1}{\gamma_{PID}}\tilde{e}_{PID}^T \dot{\hat{e}}_{PID}\right] - \eta S^T \operatorname{sgn}(S)
\end{aligned} \tag{23}
$$

Notice the adaptive laws Eq. 16 again, we have $\dot{V}(x) \le -\eta S^T \operatorname{sgn}(S) = -\eta|S| < 0$. Thus, the sliding function ultimately converges to zero with finite time and the tracking error will also converge to zero.

Based the above analysis, the design procedure of the PID-like fuzzy SMC can be summarized as following:

Step 1: Obtain the measured feedback states x, then compute the tracking error e

Step 2: Choose proper coefficients α_{ij} and design the sliding surface as Eq. 6

Step 3: Choose the fuzzy basis functions ξ (x) and construct the fuzzy system Eq. 9 with specified initial values to approximate the upper bound $\hat{\delta}$; Compute the regressive vector ζ (s) and approximate the PID control term u_{PID} according to Eq. 14

Step 4: Give the adaptive rates and compute the parameters θ_δ, θ_k, the estimated errors $\hat{e}_\delta, \hat{e}_{PID}$

Step 5: Update the weights θ_δ, θ_k and the switching gains η as Eq. 16

Step 6: Apply the PID-like fuzzy SMC controller as given by Eq. 15 to control the nonlinear system

SIMULATION RESULTS

In this section, a nonlinear system is applied to verify the effectiveness of the proposed approach. The dynamics of a two-link rigid robot with rotational joints can be described as the following:

$$
M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + \tau_d = \tau
$$

where, q is the 2×1 vector of the joint coordinates; $M(q) \in R^{2\times2}$ is the inertia matrix which is symmetric and positive definite; $C(q,\dot{q}) \in R^{2\times2}$ takes into account the Coriolis and centrifugal forces; $G(q) \in R^{2\times1}$ is the vector of the gravity forces; $F(\dot{q})$ is the friction vector; τ is the vector of the applied torques; τ_d is the external torque disturbance.

We first list the properties and the physical parameters owned by the simulated robot model as:

$$
M(q) = \begin{bmatrix} p_1 + p_2 + 2p_3\cos q_2 & p_2 + p_3\cos q_2 \\ p_2 + p_3\cos q_2 & p_2 \end{bmatrix}
$$

$$
C(q,\dot{q}) = \begin{bmatrix} -p_3\dot{q}_2\sin q_2 & -p_3(\dot{q}_1 + \dot{q}_2)\sin q_2 \\ p_3\dot{q}_1\sin q_2 & 0 \end{bmatrix},
$$

$$
G(q) = \begin{bmatrix} p_4 g\cos q_1 + p_5 g\cos(q_1 + q_2) \\ p_5 g\cos(q_1 + q_2) \end{bmatrix},
$$

$$
F(\dot{q}) = 0.02\operatorname{sgn}(\dot{q}),\ t_d = \begin{bmatrix} 0.2\sin(t) & 0.2\sin(t) \end{bmatrix}^T,
$$

$$
p = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 & p_5 \end{bmatrix} = \begin{bmatrix} 2.9 & 0.76 & 0.87 & 3.04 & 0.87 \end{bmatrix}
$$

The initial states of the robot is x = $[0.1, 0, 0.1, 0]^T$ and the desired trajectories for the two links are $q_{1d} = \sin(\pi t)$, $q_{2d} = \sin(\pi t)$. In simulation, the adaptive rates are set as $\gamma_\delta = 0.5$, $\gamma_w = 0.25$, $\gamma_k = 0.02$ and $\gamma_{PD} = 0.1$. The initial values for the adjustable vector θ_k are set to zero.

To formulate the fuzzy basis functions, six Gaussian membership functions are chosen for each of the robot states and the fuzzy memberships are selected as:

$$
\mu_{A_i^1}(x_i) = 1/(1 + \exp(5(x+2))),\ \mu_{A_i^2}(x_i) = \exp(-(x+1.5)^2),
$$

$$
\mu_{A_i^3}(x_i) = \exp(-(x+0.5)^2),\ \mu_{A_i^4}(x_i) = \exp(-(x-0.5)^2),
$$

$$
\mu_{A_i^5}(x_i) = \exp(-(x-1.5)^2),\ \mu_{A_i^6}(x_i) = 1/(1 + \exp(-5(x-2)))
$$

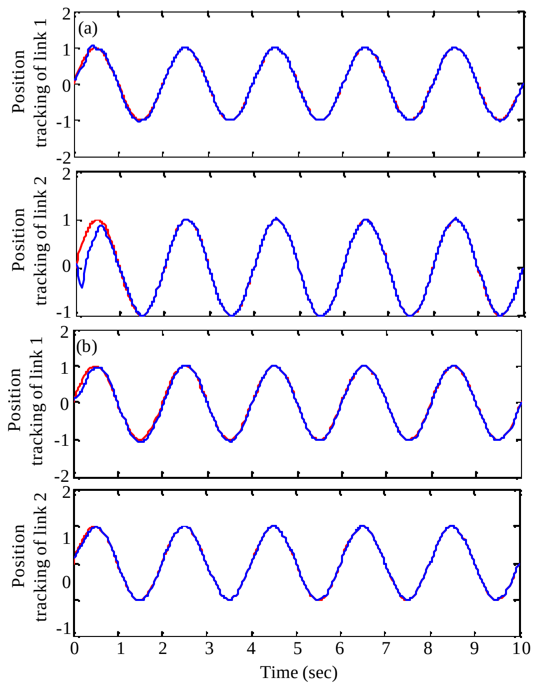

Fig. 1: The position tracking for joint 1 and 3 the proposed method without (a) and with (b) the compensation term u_n

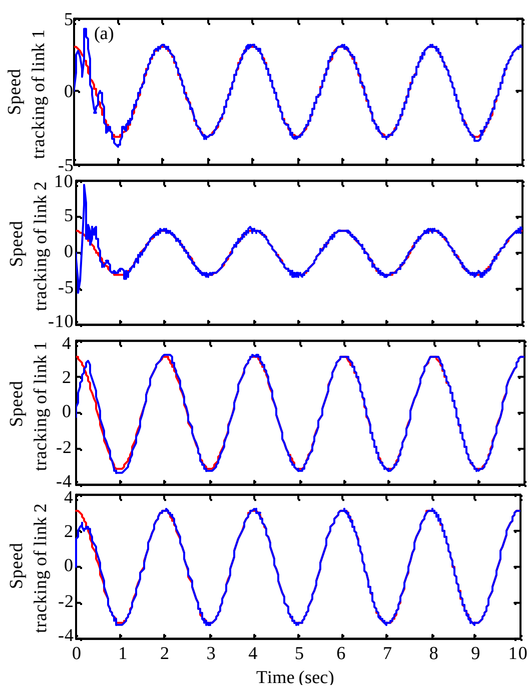

Fig. 2: The speed tracking for joint 2 and 4 the proposed method without (a) and with (b) the compensation term u_n

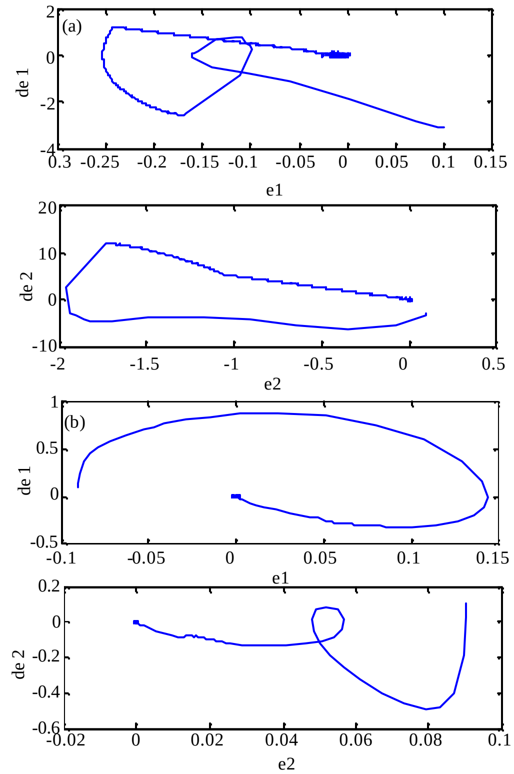

Fig. 3: The phase diagram of the tracking errors e1 and e2: the proposed method without (a) and with (b) the compensation term u_n

Figures 1-6 show the simulation results under the proposed adaptive PID-like fuzzy SMC algorithm. For comparison, two methods, the proposed control with and without the compensation term u_n, are simulated, respectively. Figure 1a and b show the position tracking curves of joint 1 and joint 2 for the proposed method and the proposed method without the compensation term u_n, respectively. Figure 2a and b show the speed tracking results of joint 1 and joint 2 for the two methods. Since the compensation term is added, it can be seen that the proposed method has better tracking performance. The phase plane trajectories are also shown in Figure 3a and b and we can see the convergence to zero of the system and the attractiveness of the sliding surfaces.

The main contribution of our proposed method comparing to the other one is that not only the asymptotical stability of the system is guaranteed but also the chattering phenomenon is eliminated as well. Figure 4a and b give the control inputs of the proposed method and the classical SMC method, respectively. Due to the PID approximation to the switching term, it is obvious that our proposed method has low chattering

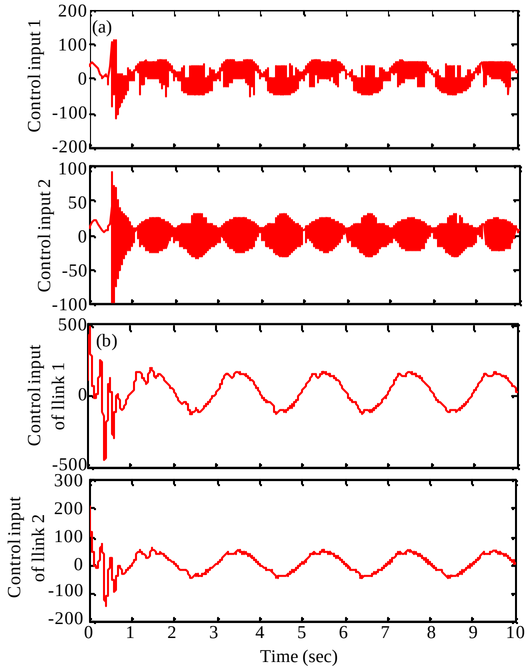

Fig. 4: Control inputs: the classical SMC method (a) and the proposed method (b)

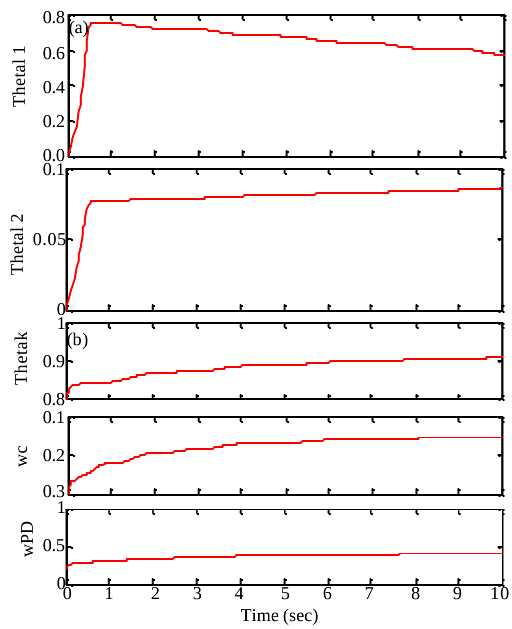

Fig. 5: The adaptive parameters of the proposed method

while serious highly chattering exists in the classical SMC method.

Figure 5 illustrates the behavior of adaptation parameters. After a short-time adaptation process, all the parameters will be bounded and converge to some constants. In addition, the application of the control scheme developed (Zhang *et al.*, 2011) is more difficult than our proposed approach. In fact, the robustness of the closed loop system (Zhang *et al.*, 2011) is ensured by an H8 supervisor and an off-line approach based Riccati equation, thus, these lead to a complicated and high computation of the control algorithm. Figure 6 gives the estimation of the upper bound and it can be that the inequality $\hat{\delta}(x) \geq \|E(x,t)\|$ always holds.

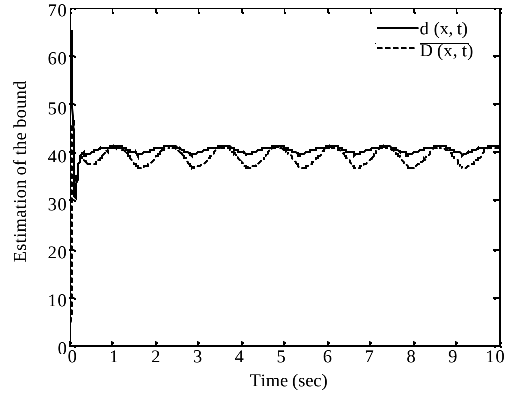

Fig. 6: The whole uncertainties D (x, t) and the estimated upper bound $\hat{\delta}(x,t)$

Therefore, compared with other existing fuzzy methods (Chen *et al.*, 2008; Zhang *et al.*, 2011), our sachem has a faster speed response and high control performance.

DISCUSSION AND CONCLUSIONS

The main contribution of this study is to propose the PID-like fuzzy sliding mode controller for a class of uncertain and nonlinear MIMO system. The unknown uncertainties are estimated by using a fuzzy logic system. In order to eliminate the chattering phenomenon brought by the conventional variable structure control, the signum function is replaced by an adaptive PID term in the proposed approach. By added the compensation control term, the resulted errors from the estimation and the approximation are deduced. A stability criterion as well as the adaptive laws is derived from Lyapunov's direct method to ensure stability of the nonlinear system. Finally, we discuss an example by providing a numerical simulation. The results demonstrate that the control methodology can rapidly and efficiently control a complex and nonlinear MIMO system.

Compared with existed control scheme, the proposed PID-like AFSMC controller has the following merits and novelties:

- Unlike the traditional model-based controller, the proposed approach does not need the upper bounds of structured uncertainties and external disturbance which are approximated by a fuzzy inference system in this study
- In order to alleviate the chattering of the conventional VSC, for the first time, the PID control is introduced to replace the switching part of the SMC scheme
- Also, the approximation and estimation errors are also assumed to be unknown and estimated online by using adaptive laws

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