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A Non-Monotone Trust Region Algorithm with Memory Model for Unconstrained Optimization

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Abstract: In this study, we developed a non-monotone trust region algorithm for unconstrained optimization. Different from the tradition non-monotone trust region algorithm, this algorithm includes memory model which make the algorithm more farsighted in the sense that its behavior is not completely dominated by the local nature of the objective function. We presented a non-monotone trust region algorithm that has this feature and prove its global convergence under suitable conditions.

Key words: Unconstrained optimization, memory model, non-monotonic trust region algorithm, global convergence

INTRODUCTION

Consider an unconstrained optimization problem:

$$\min f(x),\ x\in R^n \tag{1}$$

f: R^n,→R is twice continuously differentiable. The line search and trust region method are used to solve the problem. The first method is novel and has reliable algorithms and strong convergence which is good not only to quickly solve the well-conditioned problem but also to solve the optimization problem of ill-conditioned. So the trust region method is important research field of nonlinear optimization. The method has become mature with the studies of these years. In order to ensure the global convergence of the algorithm, the method requires each step of the iterations is successful. In other words, the new iteration point need guarantee the value of the objective function or merit function than the current value must be strictly monotone decreasing. It is found in the actual calculation for certain problems, the method does not guarantee algorithm is effective. Deng *et al*. (1993) Naiyang of mathematicians, who first proposed the strong convergence of a class of non-monotone trust region algorithm, solved this problem in 1993, Yao *et al*. (2003), Zhao (1997) and Li and Deng (1999) are the further study of this work.

Although convergence of non-monotone trust region algorithm is better than the traditional trust region algorithm, trust region subproblem contains only information on the current iteration point and abandoned the previous like the traditional trust region algorithm. So, it is called no memory model. When the objective functions of a relatively high degree of nonlinearity, what is the second-order Taylor series as a function of x changes rapidly when the pure trust region iteration is harmful to local character. The no memory iteration may be deceived by the local properties and lose some global properties which is critical on the selection of the search direction. It is helpful in selection of search directions that remembering the previous iterative information.

Gould *et al*. (1998) proposed a line search algorithm with memory model of an unconstrained optimization problem and proved its global convergence in. To a convex constrained optimization problem, Yu and Wang (2004) proposed a trust region method with memory model and proved its global convergent.

In this study, unconstrained optimization problem, we will use the skills of the memory model in non-monotonic trust region algorithm and prove that the algorithm is globally convergent under certain conditions.

ALGORITHM

Let the iteration point x and the trust region subproblem corresponding to Eq. 1:

$$\phi_k(d) = g_k^T d + d^T B_k d/2 \tag{2}$$

where, $g_k\in R^n$ is the approximation of gradient $\nabla f(x_k)$, $B_k\in R^{n\times n}$ is symmetric matrix, $d = x - x_k\in R^n$. In Gould *et al*. (1998):

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$$\phi_k^M (d) = (1-\mu_k)\, \phi_k (d)+\mu_k\, \phi_{k-1}^M (d) \tag{3}$$

is the memory model trust region, where $\mu_k \leq \min\{\bar{\mu}, \delta\|d_{k-1}\|^\tau\}$ (*), $\bar{\mu}\in[0,1), \delta>0, \tau>0$ are constants. We can get the corresponding trust region sub-problem:

$$\phi_k^M(d) = < g_k^M, d > + \frac{1}{2} < d, B_k^M d > \tag{4}$$

$$\text{s.t. } \|d_k\| \leq \Delta_k \tag{5}$$

Apply the memory model to non-monotone trust region algorithm, we can get the following specific algorithm.

Algorithm 1:

Step 1: Choose the initial point $x_0 \in R^n$, symmetric matrix $B_0 \in R^{n\times n}$, $0<\bar{\Delta}\in R^1, \Delta_0<\bar{\Delta}, 0<\eta_1<\eta_2<1, \gamma_1<1<\gamma_2$ and the integral M>0, δ>0, τ>0, $\mu_0 = 0$, k = 0

Step 2: Compute $g_k = \nabla f(x_k)$, stop when $\|g_k\| = 0$

Step 3: (4) get the approximate solution d_k

Step 4: Compute:

$$f_{l(k)} = f(x_{l(k)}) = \max_{0\leq j\leq m(k)} \{f(x_{k-j})\} \tag{6}$$

$$ared_k = f_{l(k)} - f(x_k+d_k) \tag{7}$$

$$pred_k = f_{l(k)} - f(x_k) - \phi_k^M (d_k) \tag{8}$$

$$\theta_k = \frac{ared_k}{pred_k} = \frac{f_{l(k)} - f(x_k+d_k)}{f_{l(k)} - f(x_k) - \phi_k^M(d_k)} \tag{9}$$

Step 5: If $\theta_k<\eta_1$, 则 $\Delta_k = \gamma_1\Delta_k$ compute μ_{k+1} to satisfy (*), k = k+1, we should switch to step 3

Step 6: $x_{k+1} = x_k+d_k$:

$$\Delta_{k+1} = \begin{cases} \min\{\gamma_2\Delta_k, \bar{\Delta}\} & \theta_k\geq\eta_2, \|dk\| = \Delta_k \\ \Delta_k, & \text{其他} \end{cases} \tag{10}$$

$$m(k+1) = \min\{(k)+1, M\} \tag{11}$$

Compute μ_{k+1} to satisfy (*), B_{k+1}; k = k+1 We switch to Step 2.

Algorithm is over.

Note: To solve the trust region sub-problem in algorithm 2, we use the optimal line method in Ge and Chen (2001) and the approximate solution d_k should satisfy:

$$-\phi_k^M(d) \geq \frac{1}{2}\|g_k^M\| \min\left\{\min\{\|g_k^M\|, \Delta_k\}, \frac{\|g_k^M\|}{\|B_k^M\|}\right\} \tag{12}$$

CONVERGENCES

In this study we assume that Algorithm 1 produces the infinite sequences which satisfy:

Hypothesis 1: f (x) level set $L \doteq \{x : | f(x) \leq f(x_0)\}$ is continuously differentiable. And bounded

Hypothesis 2: Sequence $\{B_k\}$ is uniformly bounded

Hypothesis 3: ∇f (x) level set L uniform continuity

First we analyst the memory model:

Lemma 1: If ϕ_k^M (d) is defined here, we have:

$$\phi_k^M(d) = \sum_{i=0}^{k}(1-\mu_i)\left(\prod_{j=i+1}^{k}\mu_j\right)\phi_i(d) \tag{13}$$

$$\text{and} \sum_{i=0}^{k}(1-\mu_i)\left(\prod_{j=i+1}^{k}\mu_j\right) \leq \frac{1}{1-\bar{\mu}}. \tag{14}$$

Proof: See (Gould *et al*., 1998) Lemma 1, we can get the proof.

Lemma 2: From the Lemma 1, we can get:

$$g_k^M = \sum_{i=0}^{k}(1-\mu_i)\left(\prod_{j=i+1}^{k}\mu_j\right)g_i \tag{15}$$

$$B_k^M = \sum_{i=0}^{k}(1-\mu_i)\left(\prod_{j=i+1}^{k}\mu_j\right)B_i \tag{16}$$

Proof: From the Lemma 1, we get the proof.

Lemma 2 under the assumption 1, if:

$$\lim_{k\to\infty}\|x_{k+1} - x_k\| = 0 \tag{17}$$

we can get:

$$\lim_{k\to\infty}\|g_k^M - g_k\| = 0. \tag{18}$$

Proof: From the Lemma 1, we get:

$$\begin{aligned} g_k^M - g_k &= \sum_{i=0}^{k}(1-\mu_i)\left(\prod_{j=i+1}^{k}\mu_j\right)g_i - g_k \\ &= \sum_{i=0}^{k-1}(1-\mu_i)\left(\prod_{j=i+1}^{k}\mu_j\right)g_i - \mu_k g_k \end{aligned} \tag{19}$$

Let:

$$L_k \doteq \sum_{i=0}^{k-1}(1-\mu_i)\left(\prod_{j=i+1}^{k-1}\mu_j\right),$$

from Lemma 1, we get:

$$L_k \le \frac{1}{1-\bar{\mu}}.$$

From the 1, there exists a constant m>0 and $\|g_i\| \le m$ (i = 0, 1, 2, ..., k), from the definition of μ_k:

$$\|g_k^M - g_k\| \le \mu_k m(L_k+1) \le \delta m(L_k+1)\|d_{k-1}\|^\tau \le \frac{\delta m(2-\bar{\mu})}{1-\bar{\mu}}\|d_{k-1}\|^\tau,$$

From Eq. 12 we get $\|d_k\| \to 0$, $(k \to \infty)$ and $\|g_k^M - g_k\| \to 0$, $(k \to \infty)$

Lemma 3 Under the assumptions 1-2, any $\eta \in (0, 1)$, there exists Δ_η which satisfies $\theta_k > \eta$ when $\Delta_k \le \Delta_\eta$.

Proof: See the Lemma 3 in Zhao (1997).

Theory 1: Under the assumptions 1-3, the algorithm 1 is available, i.e. the Step3-Step4-Step5-Step3 must stop within a finite number of steps.

Proof: (Reduction to absurdity) If the conclusion does not work, there exists a $k \in N$ to satisfy $\|g_k\| \ne 0$ and $x_{k+1} = x_k$, $\theta_{k+1} < \eta_1$ (i = 0, 1, 2, ...), $\lim_{i \to \infty} \Delta_{k+i} = 0$. This obviously conflicts with Lemma. So we get the conclusion.

Lemma 4: Under the assumptions 1-3: $\lim_{k \to \infty} \|g_k^M\| = 0$

Proof: From the assumptions 1-2, $\{B_k^M\}$ is uniform bound, algorithm 1 and the Theory 2, we get $\lim_{k \to \infty} \|g_k^M\| = 0$.

Theory 2: Under the assumptions 1-4, $\lim_{k \to \infty} \|g_k^M\| = 0$.

Proof: From the Lemma 2 and 4, we get the conclusion.

CONCLUSION

Theorem above shows that the trust region algorithm with non-memory model is global convergence and avoiding the localization difficulty of the traditional method of non-monotone trust region depending on the current point totally.

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