http://ansinet.com/itj

ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 10 (11): 2182-2188, 2011
ISSN 1812-5638 / DOI: 10.3923/itj.2011.2182.2188

TOF Estimation of Ultrasonic Echo Signal for Object Location

[1,2]Zhenkun Lu, [1]Gang Wei and [1]Fangjiong Chen
[1]School of Electronic and Information Engineering, South China University of Technology, Guangzhou 510641, People's Republic of China
[2]School of Mathematics and Physics, Wuzhou University, Wuzhou 543002, People's Republic of China

Abstract: The accurate estimation of ultrasonic Time-of-flight (TOF) is essential in ultrasonic object loaction system. In this study, a new method for TOF estimation through envelope is proposed. Firstly, the Hilbert Transform (HT) is used in ultrasonic signal processing in order to extract the envelope of the echo and to reduce the computational burden. Then, the wavelet denoising technique is applied to the extracted noisy envelope to improve the estimation accuracy. Finally, the echo parameters are estimated by using a Modified Gauss Newton (MGN) based nonlinear Least Squares (LS) estimation method. Numerical simulation has been carried out to show the performances of the proposed method in estimating TOF of ultrasonic signal.

Key words: Hilbert transform, ultrasonic echo, wavelet denoising, TOF estimation, correction gauss newton method

INTRODUCTION

Many measurement systems based on ultrasound such as object location, ultrasonic thickness measurement and ultrasonic flowmeter depend on the reliable TOF of ultrasonic signal. Therefore, accurate estimation of TOF is essential in ultrasonic location system. Several conventional approaches are used to estimate TOF. Simple threshold detection is most frequently used to extract the TOF information in sonar ranging systems (Kleinschmidt and Magori, 1981). However, the estimation of TOF is rather crude and is usually larger than the actual TOF. An alternative to simple thresholding is parabolic fit method in which an iterative nonlinear least-squares procedure is employed to fit a parabola to the onset of the signal echo (Barshan and Kuc, 1992). This approach can reduce the bias considerably. However, the threshold level is difficult to determine in low SNR and that will disturb the precision and robustness. Another approach is the sliding window which is a suboptimal method (Barshan and Ayrulu, 1999). In addition, the cross-correlation method is frequently used to estimate TOF in ultrasonic applications (Barshan and Ayrulu, 1999). Model based estimation of TOF for its accurate and robust has been applied in ultrasonic non-destructive testing (NDT) (Dencks *et al*., 2008; Hagglund *et al*., 2009; Abdessalem *et al*., 2008). Based on the envelope of the ultrasonic signals to estimate TOA or TOF for its simple, robust and less computation has been applied in object location (Egana *et al*., 2008) and measurement (Angrisani and Moriello, 2006).

In this study, a new TOF estimation method through envelope is proposed. Wavelet denoising and HT are introduced in order to improve the estimation accuracy and reduce the computational burden.

PARAMETER ESTIMATION OF A SINGLE ECHO MODEL

If the transmitter is excited at t = 0 and signal duration is assumed to be T, the received continuous waveform can be modeled as Barshan and Kuc (1992):

$$x(t) = s(t;\theta)+w(t) \quad 0\le t\le T \tag{1}$$

where,

$$s(t;\theta) = a_0 (t-\tau)^2 e^{-a_1(t-\tau)} \cos(2\pi f_0 (t-\tau)),\ \theta = [a_0, a_1, \tau, f_0]$$

and w (t) is additive WGN having zero mean and variance σ_w^2.

The transmitted signal is assumed to be nonzero over the interval [0, T_s] and the maximum time delay is τ_{max}, then

Corresponding Author: Gang Wei, School of Electronic and Information Engineering, South China University of Technology, Guangzhou 510641, People's Republic of China

the observation interval is chosen to include the entire signal by letting $T = T_s+\tau_{max}$.Uniform sampling in time produces the sequence:

$$x(t_n)=\begin{cases} w(t_n) & 0\le t_n<\tau \\ s(t_n;\theta)+w(t_n) & \tau\le t_n\le T \end{cases} \quad (2)$$

where, $t_n = n\Delta$, n = 0, 1,..., N-1 and Δ is the sampling interval.

Our goal is to estimate TOF τ from the noisy echoes detected by the receiver. In order to simplify computation, the envelope of echo signal is extracted by rectification and lowpass filtering. After envelope detection Eq. 2 becomes:

$$u(t_n;\Theta)=\begin{cases} n(t_n) & 0\le t_n<\tau \\ a_0(t_n-\tau)^2 e^{-a_1(t_n-\tau)}+n(t_n) & \tau\le t_n\le T \end{cases} \quad (3)$$

where,

$$\Theta=[a_0,a_1,\tau]$$

n (t_n) is a WGN.

Therefore, we have to estimate the parameter vector Θ given the observations in u (t). In this study, a wavelet based method and the HT technique allow getting an improved echo signal's envelope which can be used to estimate the TOF of ultrasonic. The schema of the algorithm is shown in Fig. 1.

The algorithm used for enveope extraction of echo signal in WGN can be implemented in the following steps (Donoho, 1995):

Step 1: Apply the interval-adapted pyramidal filtering algorithm to the echo signal x (n) through Symmlet wavelet to get the approximated and detailed wavelet coefficients.

Step 2: Apply the soft thresholding nonlinearity:

$$\eta_{soft}(x)=\begin{cases} \mathrm{sgn}(x)(|x|-\mathrm{Thresh}) & |x|\ge \mathrm{Thresh} \\ 0 & |x|<\mathrm{Thresh} \end{cases}$$

to the wavelet coefficients with the chosen threshold

$$\mathrm{Thresh}=\gamma_1\sigma\sqrt{2\log(n)/n},\ \gamma_1$$

is a constant.

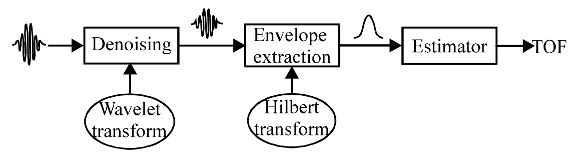

Fig. 1: Block diagram of the algorithm used in the TOF estimation

Step 3: Invert the pyramid filtering:

$$\hat{x}=W^{-1}(T(Wx))$$

where, W^{-1} is the inverse wavelet transform, T (Wx) is the vector of the thresholded coefficients of wavelet transform W of x and $\hat{x}$ is the denoised signal.

Step 4: The Fourier transform is applied to the signal x (n). The obtained frequency domain signal is denoted by X (k), k = 0, 1,..., N-1, where X (k) correspond to the negative frequencies for k = N/2,..., N-1 and the others correspond to the positive frequencies.

Step 5: Let the components corresponding to the negative frequencies be zeroed and the positive frequencies be doubled. The transfer formula can be then written as:

$$Z(k)=\begin{cases} X(k) & k=0 \\ 2X(k) & k=1,2,\cdots,N/2-1 \\ 0 & k=N/2,\cdots,N-1 \end{cases}$$

Step 6: Calculate the analytical signal z (n) of x (n) by applying inverse Fourier transform to Z (k).

Step 7: The envelope of echo signal u (t) is the modulus of z (n).

Account for the nonlinear function u (t), the Least Squares (LS) approach is applied. In LS approach we attempt to minimize the squared difference between the given u (t_n) and the assumed signal which is

$$s(t_n)=a_0(t_n-\tau)^2 e^{-a_1(t_n-\tau)}$$

Namely, the Least Squares Estimator (LSE) of Θ chooses the value that makes s (t_n) closest to the obsreved data u (t_n). Closeness is measured by the LS error criterion:

$$J(\Theta)=\sum_{n=0}^{N-1}\left(u(t_n)-s(t_n)\right)^2$$

where, the observation interval is assumed to be n = 0, 1,..., N-1 and the dependence of J on Θ is via:

$$s(t_n) = a_0 (t_n - \tau)^2 e^{-a_1(t_n-\tau)}$$

However, the error objective function is nonlinear in Θ, offering no explicit solution. We are forced to resort to iterative minimization procedures. Some typical ones are simplex search method (Nelder and Mead, 1965), Levenberg-Marquardt approach (Chong and Zak, 1996) and Gauss Newton (GN) algorithm (Kay, 1993). In general, these methods will produce the LSE if the initial guess for the parameter vector is sufficiently close to the true value. Otherwise, the optimal solution may not be attained and the algorithm may converge to one of the local minima. In addition, GN algorithm relies on the inversion of the gradient information matrix in the iteration procedures. If the determinant of this matrix is close to zero, the inverse will be ill conditioned and the algorithm may result in divergence.

In this study, a MGN method is proposed to estimate the value of parameter Θ. In order to overcome the weakness of the GN we have developed a scheme as follows:

Let:

$$v^k = \left(H^T(\Theta^k)H(\Theta^k)\right)^{-1} H^T(\Theta^k)\left(u - s(\Theta^k)\right)$$

be degressive direction of objective function and choose a very small positive number δ_k to make $J(\Theta^k + \delta^k v^k) < J(\Theta^k)$. Here, δ_k can be attained by a simple method and the calculation of this scheme is very small. Here,

$$H(\Theta) = \left[\frac{\partial s}{\partial a_0}, \frac{\partial s}{\partial a_1}, \frac{\partial s}{\partial \tau}\right]$$

denotes the gradient matrix. In this study, the second scheme is employed to estimate the value of Θ.

In summary, the MGN algorithm used for parameter estimation of narrowband echo in WGN can be implemented in the following steps:

Step 1: Make an initial guess for the parameter vector $\Theta^{(0)}$ and a required accuracy ε>0. Set k = 0

Step 2: Compute the value of the model $s(\Theta^{(k)})$ and the gradient $H(\Theta^{(k)})$

Step 3: Compute the correction factor $\Delta\Theta = \left(H^T(\Theta^{(k)})H(\Theta^{(k)})\right)^{-1} H^T(\Theta^{(k)})\left(u - s(\Theta^{(k)})\right)$

Step 4: Set $\alpha = 1$, $\beta = 10^{-5}$

Step 5: If $J(\Theta^{(k)} + \alpha\Delta\Theta) < J(\Theta^{(k)}) + 2\alpha\beta(\Delta\Theta)^T H^T(\Theta^{(k)})\left(u - s(\Theta^{(k)})\right)$, then let $\Theta^{(k+1)} = \Theta^{(k)} + \alpha\Delta\Theta$ and go to step 6, else let $\alpha = \alpha/2$ and go to step 5

Step 6: Check convergence criterion: if $\left\|\Theta^{(k+1)} - \Theta^{(k)}\right\| < \varepsilon$, then stop

Step 7: Set k→k+1 and go to step 2

Parameter estimation results and analysis: The performance of the proposed MGN algorithm for parameter estimation is tested by a simulated narrowband echo with the parameter vector θ = (1, 17, 0.1, 25) and $\Theta = [1,17,0.1]$. Therefore, the amplitude α_0 = 1, the time scaling factor α_1 = 17 kHz, the arrival time τ = 0.1 ms and the center frequency f_0 = 25 kHz. Gaussian White noise with SNR of 20, 10 and 5 dB is added to this signal, respectively. Then, the MGN algorithm is applied to estimate the parameters of the echo signal envelope. The initial guess $\Theta^{(0)} = [0.6,10,0]$ and $\varepsilon = 10^{-6}$ are provided for the parameter vector and the accuracy in the algorithms. To assess the statistical performance of the algorithm, 100 Monte Carlo simulations are carried out. The simulated and estimated results with SNR of 20, 10 and 5 dB are plotted in Fig. 2. For comparison, the original echo signal envelope used in simulation (solid line) is also plotted in the Figure along with the estimated echoe envelope (discontinued blue line). The estimation results are listed in Table 1 in terms of estimated parameters and the number of iterations.

From the Table 1, it can be observed that the estimated parameters are achieved with very high accuracy. Especially, the estimation of a noise-free echo is achieved with 100% accuracy. Although a good fit is achieved in high SNR, bad estimation is obtained in low SNR as expected. Figure 2 confirms the effectiveness, robustness and high accuracy of the MGN parameter estimation method.

Table 1: Parameter estimation results using the MGN method

SNR		Amplitude	Time scaling factor (kHz)	Arrival time (ms)	No. of iterations
	Real value	1	17	0.1	
	Initial guess		0.6	10	0
Noise free	Mean	1	17	0.1	6
	Variance	0	0	0	
20dB	Mean	1.0019	17.1440	0.1004	6
	Variance	3.3572E-4	0.0174	6.8594E-7	
10dB	Mean	0.9842	17.2698	0.1006	7
	Variance	0.0042	0.2199	8.9203E-6	
5dB	Mean	0.9428	17.2882	0.1008	8
	Variance	0.0094	0.6081	2.4835E-5	

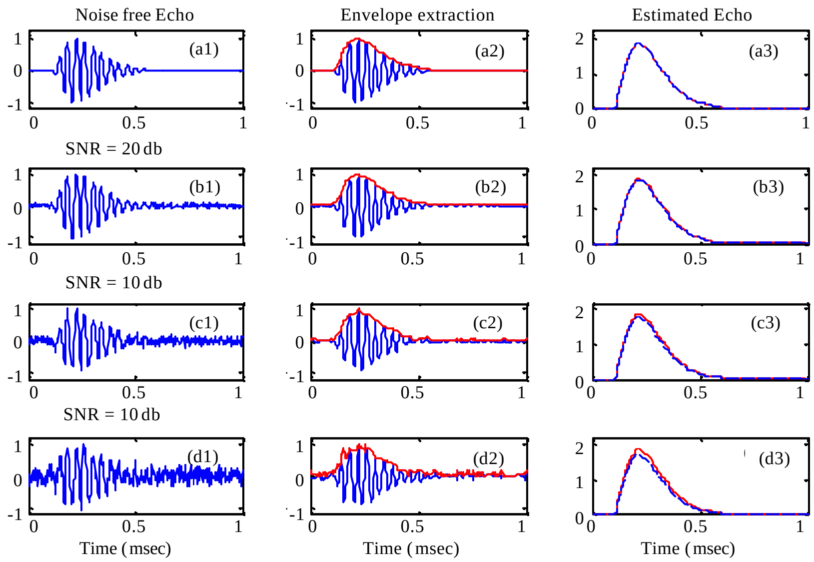

Fig. 2: (a) The echo with noise-free, 20 dB, 10 dB, 5 dB plotted in blue line (see subfigure (a1) (b1) (c1) and (d1)). (b) The envelope extraction of echo signal with noise-free, 20 dB, 10 dB, 5 dB plotted in solid red line and the denoised echo plotted in blue line (see subfigure (a2) (b2) (c2) and (d2)). (c) The estimated echo envelope of the signal with noise-free, 20 dB, 10 dB, 5 dB plotted in discontinued blue line and the original echo envelope plotted in solid red line (see subfigure (a3) (b3) (c3) and (d3))

PERFORMANCE ANALYSIS THROUGH CRLB BOUNDS

In the above section, it can be seen that the estimated parameters by the MGN method can achieve high accuracy. According to estimation theory (Kay, 1998), the variance of any unbiased estimator is greater than or equal to the Cramer-Rao Low Bound (CRLB). Hence, the CRLB for a vector parameter Θ can be defined as:

$$\mathrm{var}\left(\hat{\Theta}_i\right) \geq \left[I^{-1}(\Theta)\right]_{ii}$$

where, $I(\Theta)$ is the Fisher information matrix. For the observed signal envelope model (3) $u(t;\Theta)$ is normally distributed as $N\left(u(\Theta),\sigma^2 I\right)$, the Fisher information matrix can be written as (Feder and Weinstein, 1988):

$$I(\Theta) = \frac{1}{\sigma^2} H^T(\Theta) H(\Theta)$$

where, $H(\Theta)$ represents the gradients of the echo model. The detailed analytical derivation of the gradients, Fisher information matrix and the CRLB are given in the Appendix A which gives the CRLB bounds on the variances of the estimated parameters under noise:

$$\mathrm{var}\left(\hat{a}_0\right) \geq \frac{27a_0^2}{4f_s\varsigma} \tag{4a}$$

$$\mathrm{var}\left(\hat{a}_1\right) \geq \frac{a_1^2}{f_s\varsigma} \tag{4b}$$

$$\mathrm{var}\left(\hat{\tau}\right) \geq \frac{15}{4a_1^2 f_s\varsigma} \tag{4c}$$

where, f_s is the sampling frequency and ς denotes the SNR.

The performance of estimation is assessed by a Monte-Carlo method. A Gaussian echo with the parameter vector θ = (1, 17, 0.1, 25) and $\Theta = [1,17,0.1]$ is simulated. The sampling frequency is 500 kHz. Guassien white noise with different SNR is added to the signal. The parameters of the echo signal envelope are estimated by the MGN algorithm and the initial guess is $\Theta^{(0)} = [0.6,10,0]$. This procedure is followed 100 times. Using the formulas (4a) to (4c), the CRLBs for estimation of paramerters are computed for echoes with SNR of 30, 25, 20, 17, 15, 13, 10, 7, 5 and 3 dB. Figure 3 shows the results on the estimation variances. It can be observed that the variances of all the estimators attain the CRLB if the SNR is sufficiently high.

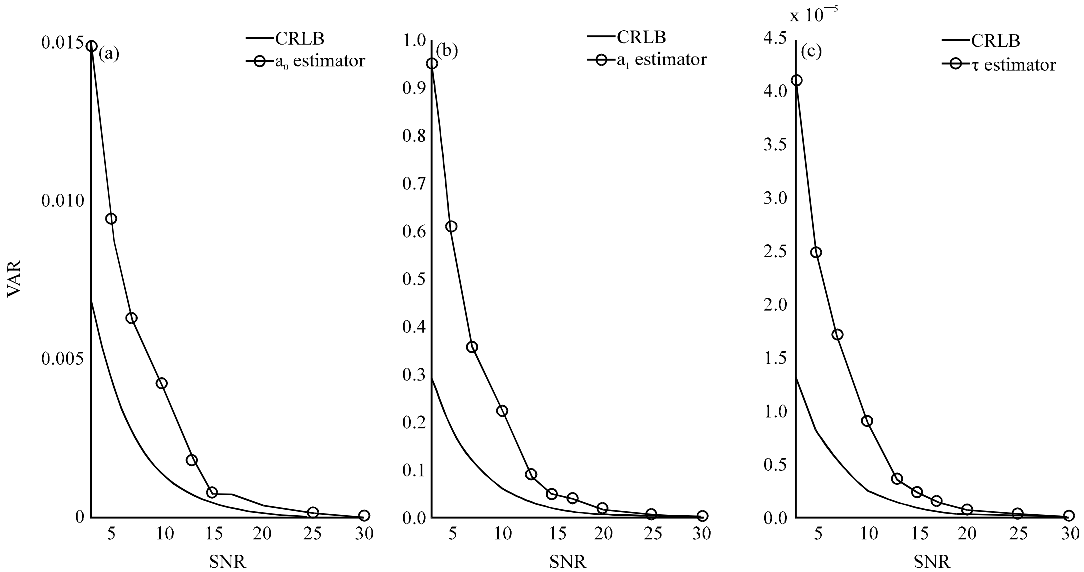

Fig. 3: Comparison of variance of the a_0, a_1 and τ estimators to the CRLB with different SNR

CONCLUSION

In this study, we have developed a method for estimating TOF of the ultrasonic echoes by the signal envelope. It is based on the envelope of the received signal whose parameters only contain TOF, time scaling factor and amplitude. Therefore, it can reduce the computational burden. In addition, a wavelet denoising method is developed to enhance the extracted envelope. At the same time, in order to overcome the weakness of GN, a MGN method is developed to estimate the parameters of an echo in WGN. Through computer simulations and analytical CRLB derivations, it has been shown that the estimated parameters are optimal and unbiased. According to numerical simulations, we can note that the proposed method can improve accuracy in the estimation of TOF of the ultrasonic echo and can be effectively applied in object location.

ACKNOWLEDGMENT

This study is supported by the National Natural Science Foundation of China (No. U1035003).

APPENDIX A

Derivation of echo parameter crlb bounds: The signal echo is defined through the following model:

$$s(t;\theta)=\begin{cases}0 \quad 0\le t<\tau & 0\le t<\tau\\ a_0(t-\tau)^2 e^{-a_1(t-\tau)}\cos(2\pi f_0(t-\tau)) \quad \tau\le t<\infty & \tau\le t<\infty\end{cases} \tag{A1}$$

where, $\theta = (a_0, a_1, \tau, f_0)$ denotes the parameter vector.

The envelope of the echo is written by:

$$u(t;\Theta)=\begin{cases}0 & 0\le t<\tau\\ a_0(t-\tau)^2 e^{-a_1(t-\tau)} & \tau\le t<\infty\end{cases} \tag{A2}$$

where, $\Theta=[a_0,a_1,\tau]$ is a vector parameter.

In this appendix the CRLB for the vector parameter $\Theta=[a_0,a_1,\tau]$ is derived. The partial derivatives of the echo envelope can be written as:

$$\frac{\partial u(t;\Theta)}{\partial a_0}=(t-\tau)^2 e^{-a_1(t-\tau)} \tag{A3}$$

$$\frac{\partial u(t;\Theta)}{\partial a_1}=-a_0(t-\tau)^3 e^{-a_1(t-\tau)} \tag{A4}$$

$$\frac{\partial u(t;\Theta)}{\partial \tau}=-2a_0(t-\tau)e^{-a_1(t-\tau)}+a_0a_1(t-\tau)^2 e^{-a_1(t-\tau)} \tag{A5}$$

The Fisher information matrix is given as:

$$I(\Theta)=\frac{1}{\sigma^2}H^T(\Theta)H(\Theta) \tag{A6}$$

where, σ^2 denotes the noise variance and $H(\Theta)$ is the gradient matrix (i.e., Jacobian). The gradient matrix can be given in terms of partial derivatives:

$$H(\Theta)=\left[\frac{\partial u}{\partial a_0},\frac{\partial u}{\partial a_1},\frac{\partial u}{\partial \tau}\right] \tag{A7}$$

The Jacobian of the discrete signal envelope obtained by sampling $u(t;\Theta)$ with a sampling frequency f_s can be rewritten as:

$$H(\Theta)=\left[\frac{\partial \bar{u}}{\partial a_0},\frac{\partial \bar{u}}{\partial a_1},\frac{\partial \bar{u}}{\partial \tau}\right]$$

where, $\bar{u}$ denotes the discrete signal envelope. Each element of the matrix $H^T(\Theta)H(\Theta)$ can be computed explicitly using the following approximation:

$$\left[H^T(\Theta)H(\Theta)\right]_{ij}=\left[\frac{\partial \bar{u}}{\partial \Theta_i}\right]^T\left[\frac{\partial \bar{u}}{\partial \Theta_j}\right]\cong f_s\int_\tau^\infty \frac{\partial u(t;\Theta)}{\partial \Theta_i}\frac{\partial u(t;\Theta)}{\partial \Theta_j}dt$$

where, Θ_i denotes the i-th parameter in the parameter vector Θ and I = 1, 2, 3, j = 1, 2, 3.

To simplify mathematical expressions, we define the following variables:

$$A_{ij}=\int_\tau^\infty \frac{\partial u(t;\Theta)}{\partial \Theta_i}\frac{\partial u(t;\Theta)}{\partial \Theta_j}dt$$

For the bandpass condition, the energy of the echo can be written as:

$$E=\frac{3\pi a_0^2}{8a_1^5}$$

Therefore, A_{ij} can be caculated as the following expressions:

$$A_{11}=\int_\tau^\infty (t-\tau)^4 e^{-2a_1(t-\tau)}dt=\frac{3}{4a_1^5}=\frac{2}{\pi a_0^2}E$$

$$A_{12}=\int_\tau^\infty -a_0(t-\tau)^5 e^{-2a_1(t-\tau)}dt=-\frac{15a_0}{8a_1^6}=-\frac{5}{\pi a_0 a_1}E$$

$$A_{13}=\int_\tau^\infty [-2a_0(t-\tau)^3 e^{-2a_1(t-\tau)}+a_0a_1(t-\tau)^4 e^{-2a_1(t-\tau)}]dt=0$$

$$A_{22}=\int_\tau^\infty a_0^2(t-\tau)^6 e^{-2a_1(t-\tau)}dt=\frac{45a_0^2}{8a_1^7}=\frac{15}{\pi a_1^2}E$$

$$A_{23}=\int_\tau^\infty [2a_0^2(t-\tau)^4 e^{-2a_1(t-\tau)}-a_0^2a_1(t-\tau)^5 e^{-2a_1(t-\tau)}]dt=-\frac{1}{\pi}E$$

$$A_{33}=\int_\tau^\infty [4-4a_1(t-\tau)+a_1^2(t-\tau)^2]a_0^2(t-\tau)^2 e^{-2a_1(t-\tau)}dt=\frac{a_0^2}{4a_1^3}=\frac{2a_1^2}{3\pi}E$$

Using all of the above result, the Fisher information matrix can be given as:

$$I(\Theta)=\frac{1}{\sigma^2}H^T(\Theta)H(\Theta)=\frac{f_sE}{\pi\sigma^2}\begin{bmatrix}\frac{2}{a_0^2} & -\frac{5}{a_0a_1} & 0\\ -\frac{5}{a_0a_1} & \frac{15}{a_1^2} & -1\\ 0 & -1 & \frac{2a_1^2}{3}\end{bmatrix} \quad \text{(A8)}$$

Here, we define:

$$\varsigma=\frac{E}{\pi\sigma^2}$$

as the SNR and the above matrix $I(\Theta)$ can be inverted analytically to obtain the inverse Fisher information matrix:

$$I^{-1}(\Theta)=\frac{1}{f_s\varsigma}\begin{bmatrix}\frac{27}{4}a_0^2 & \frac{5}{2}a_0a_1 & \frac{15a_0}{4a_1}\\ \frac{5}{2}a_0a_1 & a_1^2 & \frac{3}{2}\\ \frac{15a_0}{4a_1} & \frac{3}{2} & \frac{15}{4a_1^2}\end{bmatrix} \quad \text{(A9)}$$

Hence, the analytical CRLB on the variances of parameters is given by:

$$\mathrm{var}(\hat{a}_0)\geq\frac{27a_0^2}{4f_s\varsigma} \quad \text{(A10)}$$

$$\mathrm{var}(\hat{a}_1)\geq\frac{a_1^2}{f_s\varsigma} \quad \text{(A11)}$$

$$\mathrm{var}(\hat{\tau})\geq\frac{15}{4a_1^2f_s\varsigma} \quad \text{(A12)}$$

REFERENCES

Abdessalem, B., D. Redouane and G. Abderrezak, 2008. Detection of delamination defects in CFRP materials using ultrasonic signal processing. Ultrasonics, 48: 731-738.

Angrisani, L. and R.S.L. Moriello, 2006. Estimating ultrasonic time-of-flight through quadrature demodulation. IEEE Trans. Instrum. Meas., 55: 54-62.

Barshan, B. and B. Ayrulu, 1999. Performance comparison of four time of flight estimation methods for sonar signals. IEEE Electron. Lett., 34: 1616-1617.

Barshan, B. and R. Kuc, 1992. A Bat-like sonar system for obstacle localization. IEEE Trans. Syst. Man Cybern., 22: 636-646.

Chong, E.K.P. and S.L. Zak, 1996. An Introduction to Optimization. John Wiley and Sons, Inc., New York.

Dencks, S., R. Barkmann, F. Padilla, P. Laugier and C. Gluer, 2008. Model based estimation of quantitative ultrasound variables at the proximal femur. IEEE Trans. Ultrason. Ferroelectr. Freq. Control, 55: 1304-1315.

Donoho, D.L., 1995. Denoising by soft-thresholding. IEEE Trans. Inform. Theor., 41: 613-627.

Egana, A., F. Seco and R. Ceres, 2008. Processing of ultrasonic echo envelopes for object location with nearby receivers. IEEE Trans. Instrum. Meas., 57: 2751-2755.

Feder, M. and E. Weinstein, 1988. Parameter estimation of superimposed signals using the EM algorithm. IEEE Trans. Acoust. Speech Signal Process., 36: 477-489.

Hagglund, F., J. Martinsson and J.E. Carlson, 2009. Model-based estimation of thin multi-layered media using ultrasonic measurements. IEEE Trans. Ultrason. Ferroelectr. Freq. Control, 56: 1689-1702.

Kay, S.M., 1993. Fundamentals of Statistical Signal Processing. Prentice Hall, New Jersey, USA.

Kay, S.M., 1998. Fundamentals of Statistical Signal Processing: Estimation Theory. Prentice-Hall, Englewood Cliffs, New Jersy.

Kleinschmidt, P. and V. Magori, 1981. Ultrasonic remote sensors for noncontact object detection. Siemens Forschungs-und Entwicklungsberichte, 10: 110-118.

Nelder, J.A. and R. Mead, 1965. A simplex method for function minimization. Comput. J., 7: 308-313.