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ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSI*net*

Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Nonlinear Adaptive Block Backstepping Control Using Command Filter and Neural Networks Approximation

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Abstract: A nonlinear adaptive block backstepping control approach is designed for a class of n-th order Multiple-input Multiple-output (MIMO) nonlinear systems with uncertainties and disturbances. The problem of “explosion of complexity” in traditional backstepping is avoided by using command filter to replace the differentiations of virtual control law. Radial Basis Function Neural Networks (RBF NNs) are employed to adaptively approximate the unknown nonlinear functions. The closed-loop system is guaranteed to be bounded and tracking errors are also proved to converge exponentially to a small residual set around the origin by Lyapunov approach. The nonlinear six Degrees-of-freedom (DOF) flight simulation on an Unmanned Aerial Vehicle (UAV) model is provided to demonstrate the effectiveness of the designed control scheme.

Key words: Adaptive control, block backstepping, command filter, RBF NNs, UAV

INTRODUCTION

In the past decades, backstepping (Kanellakopoulos *et al.*, 1991) has become one of the most popular adaptive and robust control design methods for nonlinear systems because it can guarantee global stabilities, tracking and transient performance for the broad class of strict-feedback system (Wang and Ge, 2001; Khalil, 2002). The essence of backstepping design is to select recursively some appropriate functions of state variables as pseudo control signals for lower dimension subsystems of the overall system. This control technique hinges on using a part of the system states as virtual controls to control the other states and offer a choice to accommodate the unmodeled nonlinear effects and parameter uncertainties.

Generally, the application of adaptive and robust techniques is limited by lack of accurate system dynamics. In particular, some adaptive learning parameter estimation or identification algorithms are utilized to eliminate uncertainties of dynamics (Van Oort *et al.*, 2010; Ren and Atkins, 2005; Li *et al.*, 2009; Leu and Chen, 2011; Peng, 2010, 2011; Karabacak and Eskikurt, 2011; Tong *et al.*, 2010a, b). Theoretically, a Radial Basis Function Neural Networks (RBF NNs) can approximate any continuous function to an arbitrary accuracy on any compact set as long as a sufficient number of neurons are employed

(Chen and Chen, 1995; Schilling *et al.*, 2001). As a result, backstepping control approaches had been presented that combine backstepping with Nns in the recent years (Shi, 2011; Mazenc and Bliman, 2006).

Varies applications of the adaptive backstepping control techniques demonstrate its superiority over classical controllers. However, an obvious drawback in the integrator backstepping design is the problem of “explosion of complexity” which is caused by the repeated differentiations of certain nonlinear functions such as virtual controls (Swaroop *et al.*, 2000). Dynamic Surface Control (DSC) technique is introduced to resolve this problem (Swaroop *et al.*, 2000; Zhang and Ge, 2008). In addition, the desired output and its first n derivatives must be available in tracking control for an n-th order systems. A command filtered approach for nonlinear systems which can resolve the two problems simultaneously, is proposed by Farrell *et al.* (2005, 2006, 2009).

In this study, a nonlinear adaptive block backstepping control approach is proposed.

SYSTEM FORMULATION AND PRELIMINARIES

Problem formulation: Let state variables $x^1 \in \mathbb{R}^{n_1}$, $x^2 \in \mathbb{R}^{n_2}, \dots, x^n \in \mathbb{R}^{n_m}$ and system input $u \in \mathbb{R}^m$, with $m \geq n_n$. Consider the following class of n-th order

Multiple-input Multiple-output (MIMO) nonlinear systems with uncertainties and disturbances:

$$\begin{aligned} \dot{x}_1 &= f_1(\bar{x}_1) + \Delta f_1(\bar{x}_1) + g_1(\bar{x}_1)x_2 + \Delta g_1(\bar{x}_1)x_2 + d_1 \\ &\vdots \\ \dot{x}_i &= f_i(\bar{x}_i) + \Delta f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + \Delta g_i(\bar{x}_i)x_{i+1} + d_i \\ &\vdots \\ \dot{x}_n &= f_n(\bar{x}_n) + \Delta f_n(\bar{x}_n) + g_n(\bar{x}_n)u + \Delta g_n(\bar{x}_n)u + d_n \quad y = x_1 \end{aligned} \quad (1)$$

where, $x_i = [x_{i1}, x_{i2}, \dots, x_{in}]^T$ are assumed to be available for measurement; y denotes the system output; The functions $f_i(\cdot)$ and $g_i(\cdot)$, $i = 1, 2, \dots, n$, are smooth nonlinear functions that are assumed to be known; $\Delta f_i(\cdot)$ and $\Delta g_i(\cdot)$ are smooth nonlinear functions caused by both parametric and nonparametric uncertainties; d_i denote the disturbances.

To ensure controllability, we will invoke the following assumption:

- There exists a constant g_{0i} , such that for $i = 1, 2, \dots, n$, each function $\|g_i(\cdot)\|$, where, $\|\cdot\|$ denotes the 2-norm of a vector or a matrix

The control objective is to design an adaptive control input u so that the output y follows a desired trajectory y_c with the constraint that all signals in the closed-loop system are semi-globally uniformly ultimately bounded. Generally, the assumption that y_c and its derivatives are all existent and bounded is required. If there is a n -th order system, y_c and its derivatives up to the $(n+1)^{th}$ order are all bounded is usually required (Shi, 2011). In the practices, this assumption is very stringent. In this study, Command Filtered Backstepping (CFBS) (Farrell *et al.*, 2009) is used and this approach requires the following less stringent assumption.

- The desired trajectory y_c and its derivative \dot{y}_c are continuous and bounded

In addition, to design the backstepping controller, another assumption is required.

- Farrell *et al.* (2009): for $i = 1, 2, \dots, n$, each function $f_i(\cdot)$ and $g_i(\cdot)$ and their first partial derivatives are continuous and bounded on any compact set $D_i \subset \mathbb{R}^i$

RBF Nns: In the system, there are some unknown uncertainties $\Delta f_i(\cdot)$ and $\Delta g_i(\cdot)$ and disturbances d_i . They

can be combined to form an unknown nonlinear function Δ_i as follows:

$$\begin{aligned} \Delta_i &= \Delta f_i + \Delta g_i x_{i+1} + d_i, \quad 1 \leq i \leq n-1 \\ \Delta_n &= \Delta f_n + \Delta g_n u + d_n \end{aligned} \quad (2)$$

To identify Δ_i , $i = 1, 2, \dots, n$, some identification models, e.g., fuzzy logical system, wavelet networks and neural networks can be applied. For Radial Basis Functions (RBF) NNs, the identification model $f(x)$ can be expressed as:

$$f(x) = W^{*T} \zeta(x) + \varepsilon(x) \quad (3)$$

where, $x \in \mathbb{R}^n$ is the input vector of RBF NNs; $W^* = [w_1, w_2, \dots, w_l]^T$ is the ideal weight vector, l denotes the node number; $\zeta(x) = [\zeta_1(x), \zeta_2(x), \dots, \zeta_l(x)]^T$ is the basis function vector; $\varepsilon(x)$ is the so-called NNs functional approximation error. $\zeta_i(x)$ is usually chosen as the Gaussian function:

$$\zeta_i(x) = \exp\left(-\frac{\|x - c_i\|^2}{b_i^2}\right), \quad i = 1, 2, \dots, l \quad (4)$$

where, $c_i = [c_{i1}, c_{i2}, \dots, c_{im}]^T$ is the center and b_i is the width of $\zeta_i(x)$.

Theoretically, the single-hidden-layer RBF NNs can approximate any continuous nonlinear function to any desired accuracy. This is known as the universal approximation capability (Funahashi, 1989). Even the approximation cannot always be perfect in the practices, there still exist integer N , the node number in the hidden layer, for arbitrary constant $\varepsilon_m > 0$, satisfying approximation error $\|\varepsilon(x)\| \leq \varepsilon_m$. To use RBF NNs in block backstepping, there is another assumption about weight vector W^* .

- Lee and Kim (2001): The weight vector W^* are bounded in the sense that

$$\|W^*\|_F \leq W_m \quad (5)$$

where, W_m is known positive constant and $\|\cdot\|_F$ denotes the Frobenious norm of a matrix.

THE DESIGN OF ADAPTIVE BLOCK BACKSTEPPING CONTROLLER

Step 1: Design a nominal control input α_1 . Recall first Eq. in 1 with 2, it can be written as:

$$\dot{x}_1 = f_1 + g_1 x_2 + \Delta_1 \quad (6)$$

The tracking error vector is defined as $\bar{x}_1 = x_1 - y_c$. By treating x_2 as a virtual control input and using the feedback linearization method, the nominal control input α_1 is designed as follows:

$$a_1 = -g_1^{-1}(k_1 \bar{x}_1 + f_1 + \hat{W}_1^T \zeta_1 - \dot{y}_c) \quad (7)$$

where, k_1 denotes the designed positive constant; \hat{W}_1 denotes the estimate of ideal weight matrix W_1^* and the estimate error is $\tilde{W}_1 = \hat{W}_1 - W_1^*$. Select adaptive update law of RBF NNs weight matrix as:

$$\dot{\hat{W}}_1 = \Xi_1(\zeta_1 \bar{x}_1^T - \sigma_1 \hat{W}_1) \quad (8)$$

where, $\Xi_1 \in R^{l \times l}$ denotes the invertible positive gain matrix and $\sigma_1 > 0$ is a small design parameter called σ -modification coefficient.

Step 1: Design a nominal control input α_i , $i = 2, 3, \dots, n-1$. Consider:

$$\dot{\bar{x}}_i = f_i + g_i x_{i+1} + \Delta_i \quad (9)$$

Define the tracking error vector \bar{x}_i and the compensated tracking error signals z_i as:

$$\bar{x}_i = x_i - x_{ie}, z_i = \bar{x}_i - \xi_i, i=1, 2, \dots, n \quad (10)$$

where, the variable ξ_i is the output of the following filter

$$\dot{\xi}_i = -k_i \xi_i + g_i(x_{i+1e} - a_i) + g_i^T \xi_{i+1} \quad (11)$$

with $\xi_i(0) = 0$. For $i = n$, define $\xi_n = 0$.

Then, the nominal control input α_i is designed as follows:

$$a_i = -g_i^{-1}(k_i \bar{x}_i + f_i + g_{i-1}^T z_{i-1} + \hat{W}_i^T \zeta_i - \dot{\bar{x}}_{ie}) \quad (12)$$

where, k_i denotes the designed positive constant; \hat{w}_i denotes the estimate of ideal weight matrix W_i^* and the estimate error is $\tilde{W}_i = \hat{W}_i - W_i^*$. Select adaptive update law of RBF NNs weight matrix as:

$$\dot{\hat{W}}_i = \Xi_i(\zeta_i \bar{x}_i^T - \sigma_i \hat{W}_i) \quad (13)$$

where, $\Xi_i \in R^{l \times l}$ denotes the invertible positive gain matrix and $\sigma_i > 0$ is σ -modification coefficient.

From Eq. 12, it's known that the derivative of virtual control input $\dot{\bar{x}}_{ie}$ is used. Unfortunately, computing $\dot{\bar{x}}_{ie}$ is a hard work. This situation will cause a problem called "explosion of complexity" and the assumption that y_c and its derivatives up to the $(n+1)^{th}$ order are all existent and bounded will be required.

To avoid this problem, the command filter, which is formal introduced in Ref. (Farrell *et al.*, 2009), is used. Pass α_{i-1} through a command filter (14) to produce the signals x_{ic} and \dot{x}_{ic} . Define the state space implementation of command filter as:

$$\begin{bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \end{bmatrix} = \begin{bmatrix} q_2 \\ -2\zeta_i \omega_i q_2 - \omega_i(q_1 - x_c) \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} x_{ic} \\ \dot{x}_{ic} \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

where, ω_i denotes the natural frequency of the command filter. The filter design parameters are $\omega_i > 0$ and $\zeta_i \in (0, 1)$. The designer would typically select $\omega_i > k_i$ for all i , so that x_{ic} and \dot{x}_{ic} will accurately track α_{i-1} and $\dot{\alpha}_{i-1}$, respectively.

Step n: Design control input u . Recall that:

$$\dot{\bar{x}}_n = f_n + g_n u + \Delta_n \quad (15)$$

Tracking error vector is defined as $\bar{x}_n = x_n - x_{ne}$. The control input u can be designed as follows:

$$u = -g_n^{-1}(k_n \bar{x}_n + f_n + g_{n-1}^T z_{n-1} + \hat{W}_n^T \zeta_n - \dot{\bar{x}}_{ne}) \quad (16)$$

where, k_n , \hat{W}_n , \tilde{W}_n are the same meaning as in step I. \dot{x}_{ne} can be produced with passing α_{n-1} through the command filter Eq. 14. Select adaptive update law of RBF NNs weight matrix like Eq. 13.

$$\dot{\hat{W}}_n = \Xi_n(\zeta_n \bar{x}_n^T - \sigma_n \hat{W}_n) \quad (17)$$

where, $\Xi_n \in R^{l \times l}$ denotes the invertible positive gain matrix and $\sigma_n > 0$.

STABILITY ANALYSIS

In Design Section, the adaptive block backstepping controller using CF and RBF NNs is designed. The stability of the close-loop system will be discussed here. The dynamics of the tracking error \bar{x}_i and the dynamics of the compensated tracking error z_i will be given firstly.

Error dynamics: The dynamics of tracking errors can be written as follows:

$$\begin{aligned} \dot{\tilde{x}}_1 &= \dot{x}_1 - \dot{x}_{1c} \\ &= f_1 + g_1 x_2 + \Delta_1 - \dot{x}_{1c} \\ &= f_1 + g_1 a_1 + W_1^T \zeta_1 + \varepsilon_1 - \dot{x}_{1c} + g_1 (x_2 - x_{2c}) + g_1 (x_{2c} - a_1) \\ &= -k_1 \tilde{x}_1 + W_1^T \zeta_1 - \hat{W}_1^T \zeta_1 + \varepsilon_1 + g_1 \tilde{x}_2 + g_1 (x_{2c} - a_1) \\ &= -k_1 \tilde{x}_1 + g_1 \tilde{x}_2 + g_1 (\tilde{x}_{2c} - x_{2c}) - \tilde{W}_1^T \zeta_1 + \varepsilon_1 \end{aligned} \quad (18)$$

$$\begin{aligned} \dot{\tilde{x}}_i &= \dot{x}_i - \dot{x}_{ic} \\ &= f_i + g_i a_i + \Delta_i - \dot{x}_{ic} + g_i (x_{i+1} - x_{i+1c}) + g_i (x_{i+1c} - a_i) \\ &= -k_i \tilde{x}_i - g_{i-1}^T z_{i-1} + W_i^T \zeta_i - \hat{W}_i^T \zeta_i + \varepsilon_i + g_i \tilde{x}_{i+1} + g_i (x_{i+1c} - a_i) \\ &= -k_i \tilde{x}_i - g_{i-1}^T z_{i-1} + g_i \tilde{x}_{i+1} + g_i (x_{i+1c} - a_i) - \tilde{W}_i^T \zeta_i + \varepsilon_i \end{aligned} \quad (19)$$

$$\begin{aligned} \dot{\tilde{x}}_n &= \dot{x}_n - \dot{x}_{nc} \\ &= f_n + g_n u + \Delta_n - \dot{x}_{nc} \\ &= -k_n \tilde{x}_n - g_{n-1}^T z_{n-1} + W_n^T \zeta_n - \hat{W}_n^T \zeta_n + \varepsilon_n \\ &= -k_n \tilde{x}_n - g_{n-1}^T z_{n-1} - \tilde{W}_n^T \zeta_n + \varepsilon_n \end{aligned} \quad (20)$$

Combine 10 and 11, the compensated tracking errors dynamics become:

$$\dot{z}_1 = \dot{\tilde{x}}_1 - \dot{\xi}_1 = -k_1 z_1 + g_1 z_2 - \tilde{W}_1^T \zeta_1 + \varepsilon_1 \quad (21)$$

$$\dot{z}_i = \dot{\tilde{x}}_i - \dot{\xi}_i = -k_i z_i - g_{i-1}^T z_{i-1} + g_i z_{i+1} - \tilde{W}_i^T \zeta_i + \varepsilon_i \quad (22)$$

$$\dot{z}_n = \dot{\tilde{x}}_n - \dot{\xi}_n = -k_n z_n - g_{n-1}^T z_{n-1} - \tilde{W}_n^T \zeta_n + \varepsilon_n \quad (23)$$

Stability properties: Now, consider the control Lyapunov function candidate:

$$V(t) = \frac{1}{2} \sum_{i=1}^n (z_i^T z_i + \text{tr}(\tilde{W}_i^T \Xi_i^{-1} \tilde{W}_i)) \quad (24)$$

The time derivative of the Lyapunov function is:

$$\begin{aligned} \dot{V} &= \sum_{i=1}^n (z_i^T \dot{z}_i + \text{tr}(\tilde{W}_i^T \Xi_i^{-1} \dot{\tilde{W}}_i)) \\ &= -\sum_{i=1}^n (k_i \|z_i\|^2 - z_i^T \tilde{W}_i^T \zeta_i + z_i^T \varepsilon_i + \text{tr}(\tilde{W}_i^T \Xi_i^{-1} \dot{\tilde{W}}_i)) \\ &= -\sum_{i=1}^n (k_i \|z_i\|^2 + \text{tr}(\tilde{W}_i^T \Xi_i^{-1} (\hat{W}_i^T - \Xi_i \zeta_i z_i^T)) + z_i^T \varepsilon_i) \end{aligned} \quad (25)$$

Note the following inequality:

$$\begin{aligned} \text{tr}(\tilde{W}_i^T \Xi_i^{-1} (\hat{W}_i^T - \Xi_i \zeta_i z_i^T)) &= \text{tr}(-\sigma_i \tilde{W}_i^T \hat{W}_i) \\ &= \text{tr}(-\sigma_i \tilde{W}_i^T (\tilde{W}_i + W_i^*)) \\ &\leq \text{tr}(-\sigma_i \tilde{W}_i^T \tilde{W}_i + \frac{\sigma_i}{2} \tilde{W}_i^T \tilde{W}_i) + \frac{\sigma_i}{2} \|W_i^*\|_F^2 \\ &\leq \text{tr}(-\frac{\sigma_i}{2} \tilde{W}_i^T \tilde{W}_i) + \frac{\sigma_i}{2} W_m^2 \end{aligned} \quad (26)$$

And from Young's inequality, we have the following inequality:

$$z_i^T \varepsilon_i \leq \|z_i\|^2 + \|\varepsilon_i\|^2 / 4 \quad (27)$$

Combine Eq. 26 and 27 with Eq. 25, the time derivative of the Lyapunov function could be written as:

$$\dot{V} \leq -\sum_{i=1}^n (k_i^* \|z_i\|^2 + \frac{\sigma_i}{2} \text{tr}(\tilde{W}_i^T \tilde{W}_i)) + \sum_{i=1}^n (\frac{\sigma_i}{2} W_m^2 + \|\varepsilon_i\|^2 / 4) \quad (28)$$

where, $k_i^* = (k_i - 1) > 0$.

Define:

$$k = \min_{1 \leq i \leq n} (2k_i^*, \sigma^i), \quad c = \sum_{i=1}^n (\frac{\sigma_i}{2} W_m^2 + \|\varepsilon_i\|^2 / 4)$$

Finally, the following equation is obtained for the derivative of the chosen Lyapunov candidate function Eq. 24:

$$\dot{V} \leq -kV + c \quad (29)$$

Equation 29 implies that $V < 0$, when $V \leq c/2k$. Multiplying Eq. 29 by e^{kt} yields:

$$\frac{d}{dt} (V(t)e^{kt}) \leq ce^{kt} \quad (30)$$

Integrating both sides of 30 over $[0, t]$, we obtain:

$$0 \leq V(t) \leq \frac{c}{k} + [V(0) - \frac{c}{k}] e^{-kt} \quad (31)$$

Therefore, all signals of the closed-loop system are uniformly ultimately bounded. Furthermore, it means that if the designed positive constants k_i are chosen suitably, tracking errors will converge exponentially to a small residual set around the origin.

APPLICATION TO UAV MODEL

Here, the approach proposed in design is used to design flight controller for a UAV model.

Let $x_1 = [\alpha, \beta, \mu]^T$, $x_2 = [p, q, r]^T$ and control input $u = [\delta_a, \delta_e, \delta_r]^T$. Where, α , β and μ , respectively denote angle of attack, sideslip angle and conical rotation angle; p , q and r , respectively denote roll, pitch and yaw rates about the body axes; δ_a , δ_e and δ_r denote deflections of aileron, elevator and rudder, respectively. Based on the assumption of the flat Earth and constant mass properties (Stevens and Lewis, 2003), the general nonlinear six

Degree of Freedom (DOF) dynamic of the UAV can be written as (Lijia *et al.*, 2010):

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 + h_1(x_1)u \\ \dot{x}_2 &= f_2(x_1, x_2) + g_2(x_1)u \\ y &= x_1 \end{aligned} \quad (32)$$

To design the adaptive block backstepping flight controller, some assumptions which have been presented in other papers (Li *et al.*, 2009; Lee and Kim, 2001) are given as follows:

- The control surface deflection has very small effects on the aerodynamic force component: $h_{1,u} \neq 0$
- There exist positive constants α_m and $\beta_m \in \mathbb{R}$ such that the magnitudes of g_1 and g_2 are bounded and invertible for all α and $\beta \in \mathbb{R}$, satisfying $|\alpha| \leq \alpha_m, |\beta| \leq \beta_m$

According to above assumptions, system Eq. 32 with uncertainties and disturbances can be transformed into a MIMO strict feedback system such as:

$$\begin{aligned} \dot{x}_1 &= f_1 + g_1 x_2 + \Delta_1 \\ \dot{x}_2 &= f_2 + g_2 u + \Delta_2 \\ y &= x_1 \end{aligned} \quad (33)$$

Where:

$$\Delta_1 = \Delta f_1 + \Delta g_1 x_2 + d_1 \text{ and } \Delta_2 = \Delta f_2 + \Delta g_2 u + d_2$$

Let the desired trajectory $y_c = x_{1c} = [\alpha_c \ \beta_c \ \mu_c]^T$. Now, the controller can be expressed as:

$$\begin{aligned} a_1 &= -g_1^{-1}(k_1 \tilde{x}_1 + f_1 + \hat{W}_1^T \zeta_1 - \dot{y}_c) \\ u &= -g_2^{-1}(k_2 \tilde{x}_2 + f_2 + g_1^T z_1 + \hat{W}_2^T \zeta_2 - \tilde{x}_{2c}) \\ \tilde{x}_1 &= x_1 - x_{1c}, \quad \tilde{x}_2 = x_2 - x_{2c}, \quad z_1 = \tilde{x}_1 - \zeta_1 \\ \dot{\zeta}_1 &= -k_1 \zeta_1 + g_1(x_{2c} - a_1) \end{aligned} \quad (34)$$

$$\begin{bmatrix} x_{2c} \\ \dot{x}_{2c} \end{bmatrix} = \frac{\begin{bmatrix} \omega^2 \\ s\omega^2 \end{bmatrix}}{s^2 + 2\zeta\omega s + \omega^2} a_1$$

where, s is the Laplacian.

The adaptive update law of RBF NNs weight matrix can be given as:

$$\dot{\hat{W}}_1 = \Xi_1(\zeta_1 \tilde{x}_1^T - \sigma_1 \hat{W}_1), \quad \dot{\hat{W}}_2 = \Xi_2(\zeta_2 \tilde{x}_2^T - \sigma_2 \hat{W}_2) \quad (35)$$

The schematic of the UAV flight control architecture can be given as Fig. 1. Note that there is a first order filter between command signals and desired trajectory y_c which is used to produce smooth desired trajectory and guarantee assumption 2.

SIMULATION RESULTS

This section presents the numerical flight simulation results from the application of the controller design to the UAV model of the previous section with aerodynamic coefficients uncertainties and continuous disturbances. The controller is evaluated on estimation accuracy and tracking performance. The control design has been implemented in the MATLAB/Simulink environment by means of Level-2 M-file S-function. The sampling time and delay time of states are both set to 5 m sec. In addition, all the control surface deflections are limited ± 25 deg.

The initial conditions of the engagement are given in Table 1.

The Lyapunov design only requires the controller gains to be negative definite but it is more natural to select the inner loop gains higher than the outer-loop gains to achieve good tracking performance (Van Oort *et al.*, 2010). Therefore, the gain matrix of update law and controller gains are selected as:

Table 1: Initial condition for simulations

State variable	Initial values	Control input	Initial values
V_0	238.9 m sec ⁻¹		
α_0	0.50 deg		
β_0	0.20 deg	δ_{a0}	0.35 deg
μ_0	0 deg	δ_{r0}	-0.06 deg
p_0	0.08 deg sec ⁻¹	$\delta_{\delta 0}$	0.58 deg
q_0	0 deg sec ⁻¹		
r_0	-0.07 deg sec ⁻¹		

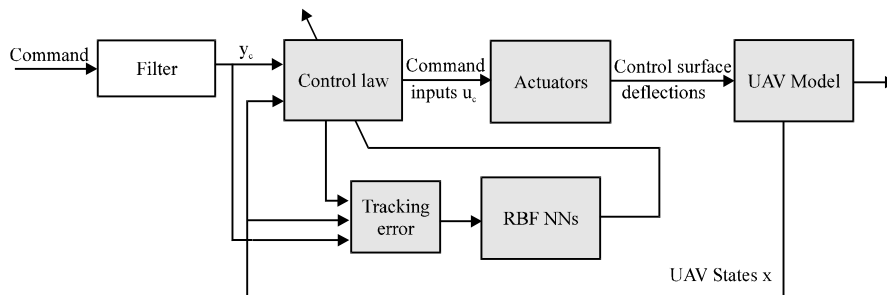


Fig. 1: Schematic overview of the control architecture

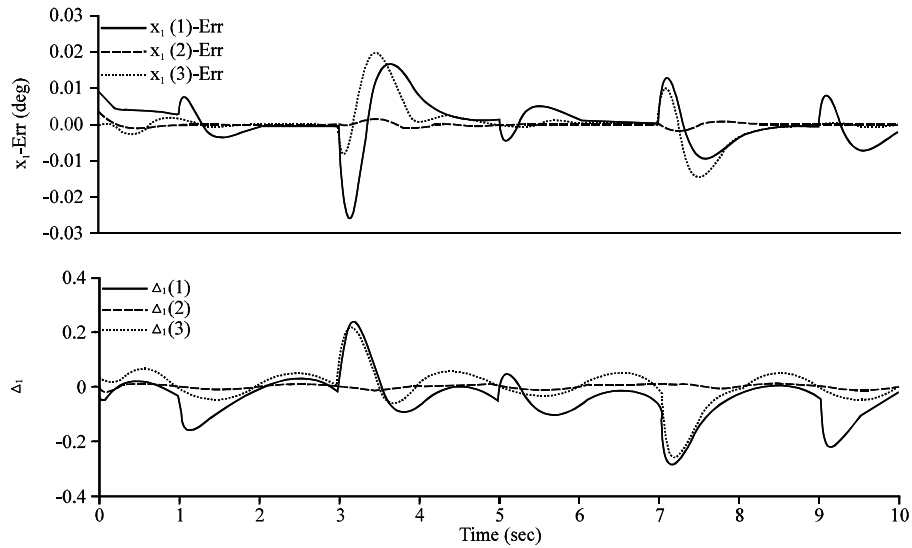


Fig. 2: Time histories of tracking error \tilde{x}_1 and unknown nonlinear function Δ_1

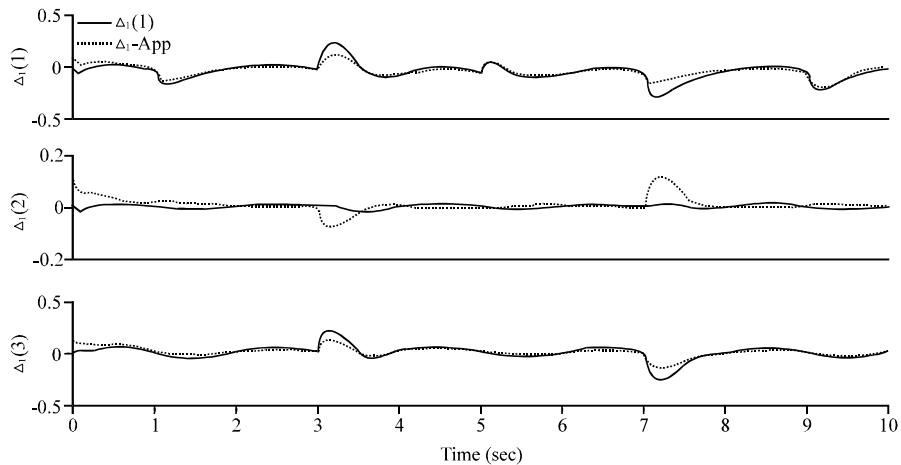


Fig. 3: Approximation performance of the RBF NNs for Δ_1

$$\begin{aligned} \Xi_1 &= \text{diag}([0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]), \\ \Xi_2 &= \text{diag}([10^{-2}, 10^{-2}, 10^{-2}, 10^{-2}, 10^{-1}, 10^{-2}, 10^{-1}, 10^{-1}]), \\ k_1 &= 8, k_2 = 6 \end{aligned}$$

In approximating Δ_1 and Δ_2 , the RBF NNs contain 8 nodes with centers of receptive field c_i evenly spaces in $(-0.5+0.5)$; The widths are all initially selected as $b_i = 1$. The initial aerodynamic coefficients are all chosen as two times of accurate values and disturbances are both selected as $\sin(\pi t) [0.05, 0.01, 0.05]^T$. Figure 2 shows time histories of the tracking error x_1 and unknown nonlinear function Δ_1 , which are used, respectively as inputs and ideal approximation targets of RBF NNs. The approximation performance for Δ_1 is shown in Fig. 3. What

is worth noting is that as there is similarly approximation performance for Δ_2 , it's not given in this study.

Consider quickly approximation is needed, the size of RBF NNs are small. As a result, the unknown nonlinear function estimation error cannot be very well. However, Fig. 3 still shows that RBF NNs has enough approximation accuracy for the unknown nonlinear function.

The effectiveness of the designed control scheme is demonstrated on the nonlinear six Degrees-Of-Freedom (DOF) flight simulation on the UAV model, of which all the aerodynamic coefficients are shifted of 100% and the disturbances are also added in.

Figure 4 shows that the angle of attack, sideslip angle and conical rotation angle commands tracking is quite

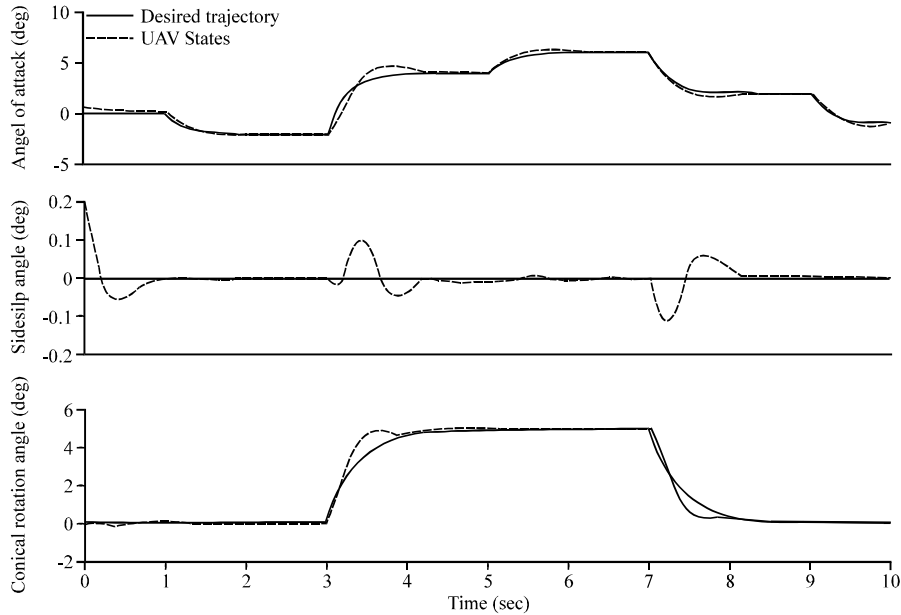


Fig. 4: Tracking performance of the designed controller

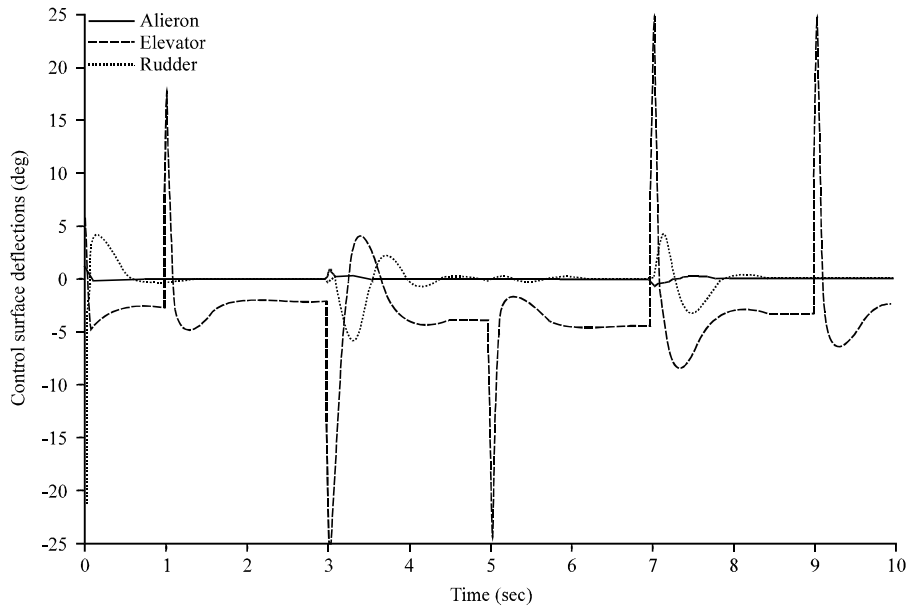


Fig. 5: Time histories of control surface deflections

good despite the unknown nonlinear function. Figure 5 shows the efficiency of control is comparatively high, since there are little saturation of control surface deflections.

CONCLUSION

This study has been concerned with designing a nonlinear adaptive block backstepping control system

capable of tracking desired trajectory while uncertainties and disturbances existing in system model. The command filtering approach is extended on MIMO systems to avoid the problem of “explosion of complexity” in traditional backstepping. RBF NNs are employed to adaptively approximate the unknown nonlinear functions composed of unknown uncertainties and disturbances. According to stability analysis using Lyapunov function, the closed-loop system is guaranteed to be bounded and tracking

errors are also proved to converge exponentially to a small residual set around the origin. The effectiveness of the proposed control approach is demonstrated in the tracking problem of UAV nonlinear model with aerodynamic coefficients uncertainties and disturbances.

ACKNOWLEDGMENTS

This research was supported by National Natural Science Foundation of China (No. 60874093). And authors also greatly appreciate all the anonymous reviewers for their comments.

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