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The Gaussian Sum Convolution PHD Filtering Algorithms for Nonlinear Models

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Abstract: A novel Probability Hypothesis Density (PHD) filter, called the Gaussian Sum Convolution PHD (GSCPHD) filter was proposed for nonlinear multi-target tracking problems. The PHD within the filter is approximated by a Gaussian sum, as in the Gaussian Mixture PHD (GMPHD) filter but the model may be nonlinear. This is implemented by a bank of convolution filters with Gaussian sum approximations to the predicted and posterior densities. The analysis results show the lower complexity, more amenable for parallel implementation of the GSCPHD filter than the convolution PHD (CPHD) filter and the ability to deal with complex observation model, small observation noise of the proposed filter over the existing Gaussian Mixture Particle PHD (GMPPHD) filter. Furthermore, the proposed GSCPHD filter was generalized to nonlinear non-Gaussian models, called as the generalized GSCPHD (GGSCPHD) filter. The multi-target tracking simulation results verify the effectiveness of the proposed methods.

Key words: Probability hypothesis density, nonlinear estimation, gaussian sum, monte carlo methods, tracking

INTRODUCTION

The main task of multi-target tracking involves joint estimation of the unknown and time-varying number of targets as well as their individual states (e.g., positions, velocities etc.) based on a sequence of noisy and cluttered observation sets (Bar-Shalom and Fortmann, 1988; Smith and Singh, 2006; Bar-Shalom and Li, 1995). Traditional multi-target tracking formulations involve the explicit associations between measurements and targets which makes multi-target tracking much harder task than single target tracking (Bar-Shalom and Fortmann, 1988; Smith and Singh, 2006; Bar-Shalom and Li, 1995; Bar-Shalom and Tse, 1975; Vihola, 2007).

The Random Finite Set (RFS) method to multi-target tracking is an emerging and promising alternative (Goodman *et al*., 1997; Mahler, 2000), where the collection of individual targets and observations are treated as a set-valued state and a set-valued observation, respectively (Mahler, 2003; Vo and Ma, 2006). Two new algorithms, called the Gaussian Mixture Particle PHD (GMPPHD) filter (Clark *et al*., 2007) and Convolution PHD (CPHD) filter (Panta and Vo, 2007), have been presented recently. The GMPPHD filter which is based on the particle filter scheme, requires the analytical availability of the likelihood function as well as the not too small observation noise, thus it is limited in practice (Rossi and Vila, 2004, 2006). And the convolution PHD filter which is based on convolution kernels, is complex and difficult to parallel implementation for the resampling scheme.

In this study, we firstly present the Gaussian sum convolution PHD (GSCPHD) filter which overcomes the drawbacks of the GMPPHD and convolution PHD filters. The algorithm approximates the intensity function by a weighted sum of Gaussians similar to the GMPHD and use Convolution Filter (CF) to compute the posterior means and covariances. Then the proposed GSCPHD filter is further generalized to nonlinear non-Gaussian models, called as the generalized GSCPHD (GGSCPHD) filter, by the Gaussian sum approximation of the state and measurement noise. The Simulation results demonstrate the good performance of the (G) GSCPHD filter when the observation noise is small, while the existing GMPPHD filter fails.

BACKGROUNDS

The PHD filter is an approximation developed to alleviate the computational intractability in the multi-target Bayes filter (Mahler, 2003; Vo and Ma, 2006). For a RFS X on χ with probability distribution P, its first order moment is a non-negative function c on χ, called the intensity or the PHD function, with the property that for any closed subset $S\subseteq\chi$:

$$\int |X \cap S| P(dX) = \int_S c(x)dx \quad (1)$$

where, |X| denotes the cardinality of X. Given the posterior intensity c_{k-1} at time k-1, the predicted intensity function

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$c_{k|k-1}$ (do not consider spawning) and the posterior intensity c_k can be given, respectively by Vo and Ma (2006):

$$c_{k|k-1}(x)=\int p_{S,k}(\zeta)f_{k|k-1}(x\mid\zeta)c_{k-1}(\zeta)d\zeta+b_k(x) \tag{2}$$

$$c_k(x)=\left[1-p_{D,k}(x)\right]c_{k|k-1}(x)+\sum_{z\in Z_k}\frac{p_{D,k}(x)h_k(z\mid x)c_{k|k-1}(x)}{\kappa_k(z)+\int p_{D,k}(\xi)h_k(z\mid\xi)c_{k|k-1}(\xi)d\xi} \tag{3}$$

where, $K_k(.)$ is the intensity of the (Poisson) clutter RFS; Z_k is the multi-target measurement available at time k; $b_k(.)$ denotes the intensity of spontaneous target birth; $p_{S,k}(\zeta)$ and $p_{D,k}(x)$ denote the survival probability and detection probability, respectively; $f_{k|k-1}(x|\zeta)$ and $h_k(z|x)$ are densities for state transition and measurement in single target model, respectively:

$$x_k = f_k(x_{k-1}) + w_k \tag{4}$$

$$Z_k = h_k(x_k) + v_k \tag{5}$$

where, w_k and v_k, respectively denote the Gaussian white process and measurement noises.

THE (G) GSCPHD ALGORITHMS

Suppose that each target follows a non-linear Gaussian dynamic model and non-linear Gaussian measurement model with:

$$f_{k|k-1}(x_k|x_{k-1}) = N(x_k; f_k(x_{k-1}) + w_{km}, Q_k) \tag{6}$$

$$h_k(z_k|x_k) = N(z_k; h_k(x_k) + v_{km}, R_k) \tag{7}$$

where, $N(x; \mu, \Sigma)$ denotes the Gaussian distribution with the mean μ and covariance, Σ and Q_k are the mean and covariance of the process noise, respectively, v_{km} and R_k are the mean and covariance of the measurement noise, respectively. Also we suppose that the survival and detection probabilities are state independent and the intensity of the birth RFS is a Gaussian sum as in the GMPHD (Vo and Ma, 2006; Clark *et al*., 2007), i.e.:

$$b_k(x)=\sum_{n=1}^{N_{b,k}}\omega_{b,k}^{(n)}N\left(x;m_{b,k}^{(n)},P_{b,k}^{(n)}\right) \tag{8}$$

The GSCPHD filter

Proposition 1 (GSCPHD Predict): Suppose the posterior intensity at time k-1 is a Gaussian sum of the form as in GMPHD (Vo and Ma, 2006):

$$c_{k-1}(x)=\sum_{i=1}^{N_{k-1}}\omega_{k-1}^{(i)}N\left(x;m_{k-1}^{(i)},P_{k-1}^{(i)}\right) \tag{9}$$

then, after the prediction step of Eq. 2, the predictive intensity at time k is still a Gaussian sum given by:

$$c_{k|k-1}(x)=\sum_{i=1}^{N_{k|k-1}}\omega_{k|k-1}^{(i)}N\left(x;m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right) \tag{10}$$

Where:

$$N_{k|k-1}=N_{k-1}+N_{b,k},\ \omega_{k|k-1}^{(i)}=\begin{cases}\omega_{b,k}^{(i)} & 1\le i\le N_{b,k}\\ p_{S,k}\omega_{k-1}^{(i-N_{b,k})} & N_{b,k}+1\le i\le N_{k|k-1}\end{cases}$$
$$m_{k|k-1}^{(i)}=\begin{cases}m_{b,k}^{(i)} & 1\le i\le N_{b,k}\\ m_{S,k|k-1}^{(i-N_{b,k})} & N_{b,k}+1\le i\le N_{k|k-1}\end{cases},\ P_{k|k-1}^{(i)}=\begin{cases}P_{b,k}^{(i)} & 1\le i\le N_{b,k}\\ P_{S,k|k-1}^{(i-N_{b,k})} & N_{b,k}+1\le i\le N_{k|k-1}\end{cases} \tag{11}$$

and

$$m_{S,k|k-1}^{(i)}=\frac{1}{M}\sum_{j=1}^{M}x_{S,k|k-1}^{(i,j)},\ P_{S,k|k-1}^{(i)}=\frac{1}{M}\sum_{j=1}^{M}\left(x_{S,k|k-1}^{(i,j)}-m_{S,k|k-1}^{(i)}\right)\left(x_{S,k|k-1}^{(i,j)}-m_{S,k|k-1}^{(i)}\right)^T$$
$$x_{S,k|k-1}^{(i,j)}\sim N\left(x;f_k\left(x_{k-1}^{(i,j)}\right)+w_{km},Q_k\right),\ x_{k-1}^{(i,j)}\sim N\left(x_{k-1};m_{k-1}^{(i)},P_{k-1}^{(i)}\right),\ j=1,\cdots,M \tag{12}$$

Proof: Substitute Eq. 9 into Eq. 2, we have:

$$c_{k|k-1}(x)=p_{S,k}\sum_{i=1}^{N_{k-1}}\omega_{k-1}^{(i)}\int N\left(x;f_k(\zeta)+w_{km},Q_k\right)\cdot N\left(\zeta;m_{k-1}^{(i)},P_{k-1}^{(i)}\right)d\zeta+b_k(x) \tag{13}$$

We use Monte Carlo integration to compute the integrals in Eq. 13 since f_k is nonlinear and the integrals no longer have a closed form solution. For each component in Eq. 13, i = 1,⋯ N_{k-1}, we sample M particles from $N\left(\zeta;m_{k-1}^{(i)},P_{k-1}^{(i)}\right)$, i.e.:

$$x_{k-1}^{(i,j)}\sim N\left(x_{k-1};m_{k-1}^{(i)},P_{k-1}^{(i)}\right),\ j=1,\cdots,M \tag{14}$$

and from the strong law of large numbers (LLN) we obtain:

$$\frac{1}{M}\sum_{j=1}^{M}\left(N\left(x;f_k\left(x_{k-1}^{(i,j)}\right)+w_{km},Q_k\right)\right)\xrightarrow{a.s.}\int N\left(x;f_k(\zeta)+w_{km},Q_k\right)N\left(\zeta;m_{k-1}^{(i)},P_{k-1}^{(i)}\right)d\zeta \tag{15}$$

as $M\to\infty$, where $\to$ denotes almost sure (a.s.) convergence. Then the predicted intensity from the existing targets can be approximated with:

$$p_{S,k}\sum_{i=1}^{N_{k-1}}\omega_{k-1}^{(i)}N\left(x;m_{S,k|k-1}^{(i)},P_{S,k|k-1}^{(i)}\right) \tag{16}$$

where,

$$\begin{aligned} m_{S,k|k-1}^{(i)} &= \frac{1}{M}\sum_{j=1}^{M}x_{S,k|k-1}^{(i,j)},\; x_{S,k|k-1}^{(i,j)} \sim N\left(x;f_k\left(x_{k-1}^{(i,j)}\right)+w_{km},Q_k\right), \\ P_{S,k|k-1}^{(i)} &= \frac{1}{M}\sum_{j=1}^{M}\left(x_{S,k|k-1}^{(i,j)}-m_{S,k|k-1}^{(i)}\right)\left(x_{S,k|k-1}^{(i,j)}-m_{S,k|k-1}^{(i)}\right)^T \end{aligned} \tag{17}$$

where, A^T denotes the transpose of matrix A. Then we can rewrite Eq. 13 as the form:

$$c_{k|k-1}(x)=p_{S,k}\sum_{i=1}^{N_{k-1}}\omega_{k-1}^{(i)}N\left(x;m_{S,k|k-1}^{(i)},P_{S,k|k-1}^{(i)}\right)+b_k(x)=\sum_{i=1}^{N_{k|k-1}}\omega_{k|k-1}^{(i)}N\left(x;m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right) \tag{18}$$

where, $\omega_{k|k-1}^{(i)},m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}$ are shown in Eq. 11 and $\omega_{k-1}^{(i)}$, $m_{S,k|k-1}^{(i)}$, $P_{S,k|k-1}^{(i)}$ are corresponding to $\omega_{k-1}^{(i-N_{b,k})}$, $m_{S,k|k-1}^{(i-N_{b,k})}$, $P_{S,k|k-1}^{(i-N_{b,k})}$ in Eq. 11, respectively for consistency, b_k (x) is given in Eq. 8.

Proposition 2 (GSCPHD update): Suppose the predict intensity at time k is a Gaussian sum of the form:

$$c_{k|k-1}(x)=\sum_{i=1}^{N_{k|k-1}}\omega_{k|k-1}^{(i)}N\left(x;m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right) \tag{19}$$

Then, after the update step of (3), the posterior intensity at time k remains a Gaussian sum given by:

$$c_k(x)=\sum_{i=1}^{N_k}\omega_k^{(i)}N\left(x;m_k^{(i)},P_k^{(i)}\right) \tag{20}$$

where,

$$\begin{aligned} &N_k=\left(1+|Z_k|\right)N_{k|k-1},\; \omega_k^{(i)}=\begin{cases}\left[1-p_{D,k}\right]\omega_{k|k-1}^{(i)} & 1\le i\le N_{k|k-1}\\ W_k^{(i-N_{k|k-1})} & N_{k|k-1}+1\le i\le N_k\end{cases} \\ &m_k^{(i)}=\begin{cases}m_{k|k-1}^{(i)} & 1\le i\le N_{k|k-1}\\ m_{D,k}^{(i-N_{k|k-1})} & N_{k|k-1}+1\le i\le N_k\end{cases},\; P_k^{(i)}=\begin{cases}P_{k|k-1}^{(i)} & 1\le i\le N_{k|k-1}\\ P_{D,k}^{(i-N_{k|k-1})} & N_{k|k-1}+1\le i\le N_k\end{cases} \end{aligned} \tag{21}$$

where, $|Z_k|$ denotes the number of measurement at time k and

$$\begin{aligned} &W_k^{(i)}=p_{D,k}\omega_{k|k-1}^{(i)}\frac{1}{M'}\sum_{j'=1}^{M'}\omega_{D,k}^{(i,j')}\Big/\left(\kappa_k(z)+p_{D,k}\sum_{i=1}^{N_{k|k-1}}\omega_{k|k-1}^{(i)}\frac{1}{M'}\sum_{j'=1}^{M'}\omega_{D,k}^{(i,j')}\right),\; m_{D,k}^{(i)}=\sum_{j'=1}^{M'}\tilde{\omega}_{D,k}^{(i,j')}x_k^{(i,j')}, \\ &\tilde{\omega}_{D,k}^{(i,j')}=\omega_{D,k}^{(i,j')}\Big/\sum_{j'=1}^{M'}\omega_{D,k}^{(i,j')},\; P_{D,k}^{(i)}=\sum_{j'=1}^{M'}\tilde{\omega}_{D,k}^{(i,j')}\left(x_k^{(i,j')}-m_{D,k}^{(i)}\right)\left(x_k^{(i,j')}-m_{D,k}^{(i)}\right)^T,\; \omega_{D,k}^{(i,j')}=K_{h_n}^{z}\left(z-\hat{z}_k^{(i,j')}\right), \\ &\hat{z}_k^{(i,j')}\sim N\left(z;h_k\left(x_k^{(i,j')}\right)+v_{km},R_k\right),\; x_k^{(i,j')}\sim N\left(x;m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right),\; j'=1,\cdots,M' \end{aligned} \tag{22}$$

where, $K_{h_n}^{z}$ is Parzen-Rosenblatt kernel (Rossi and Vila, 2004).

Proof: By substituting Eq. 7 and 19 into Eq. 3 we get:

$$\begin{aligned} c_k(x)=&\left[1-p_{D,k}\right]\sum_{i=1}^{N_{k|k-1}}\omega_{k|k-1}^{(i)}N\left(x;m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right) \\ &+\sum_{z\in Z_k}\sum_{i=1}^{N_{k|k-1}}\frac{p_{D,k}\omega_{k|k-1}^{(i)}N\left(z;h_k(\xi)+v_{km},R_k\right)N\left(x;m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right)}{\kappa_k(z)+p_{D,k}\sum_{i=1}^{N_{k|k-1}}\omega_{k|k-1}^{(i)}\int N\left(z;h_k(\xi)+v_{km},R_k\right)N\left(\xi;m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right)d\xi} \end{aligned} \tag{23}$$

Again importance sampling is performed to compute the integrals. For i = 1,··· $N_{k|k-1}$ and $z\in Z_k$, draw M' particles from an importance density. We use the components of the predicted intensity as importance density, i.e.,

$$x_k^{(i,j')}\sim N\left(x;m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right),\; j'=1,\cdots,M' \tag{24}$$

and then calculate the particle weights through the convolution kernels according to:

$$\omega_{D,k}^{(i,j')}=K_{h_n}^{z}\left(z-\hat{z}_k^{(i,j')}\right) \tag{25}$$

where,

$$\hat{z}_k^{(i,j')}\sim N\left(z;h_k\left(x_k^{(i,j')}\right)+v_{km},R_k\right) \tag{26}$$

and $K_{h_n}^{z}$ is Parzen-Rosenblatt kernel, h_n is called the kernel bandwidth (Rossi and Vila, 2004). From the results in corollary 1, the second part on the right side of Eq. 23 is approximated with:

$$c_{D,k}(x)=\sum_{z\in Z_k}\sum_{i=1}^{N_{k|k-1}}\left(p_{D,k}\omega_{k|k-1}^{(i)}\frac{1}{M'}\sum_{j'=1}^{M'}\omega_{D,k}^{(i,j')}N\left(x;m_{D,k}^{(i)},P_{D,k}^{(i)}\right)\Big/\left(\kappa_k(z)+p_{D,k}\sum_{i=1}^{N_{k|k-1}}\omega_{k|k-1}^{(i)}\frac{1}{M'}\sum_{j'=1}^{M'}\omega_{D,k}^{(i,j')}\right)\right) \tag{27}$$

where,

$$m_{D,k}^{(i)}=\sum_{j'=1}^{M'}\tilde{\omega}_{D,k}^{(i,j')}x_k^{(i,j')},\; \tilde{\omega}_{D,k}^{(i,j')}=\omega_{D,k}^{(i,j')}\Big/\sum_{j'=1}^{M'}\omega_{D,k}^{(i,j')},\; P_{D,k}^{(i)}=\sum_{j'=1}^{M'}\tilde{\omega}_{D,k}^{(i,j')}\left(x_k^{(i,j')}-m_{D,k}^{(i)}\right)\left(x_k^{(i,j')}-m_{D,k}^{(i)}\right)^T \tag{28}$$

By substituting Eq. 27 into Eq. 23 we can approximate the PHD update with:

$$c_k(x)=\left[1-p_{D,k}\right]\sum_{i=1}^{N_{k|k-1}}\omega_{k|k-1}^{(i)}N\left(x;m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right)+\sum_{z\in Z_k}\sum_{i=1}^{N_{k|k-1}}W_k^{(i)}N\left(x;m_{D,k}^{(i)},P_{D,k}^{(i)}\right)=\sum_{i=1}^{N_k}\omega_k^{(i)}N\left(x;m_k^{(i)},P_k^{(i)}\right) \tag{29}$$

where, $\omega^{(i)}_k$, $m^{(i)}_k$, $P^{(i)}_k$ shown in Eq. 21, note that $W^{(i)}_k$, $m^{(i)}_{D,k}$, $P^{(i)}_{D,k}$ in Eq. 29 are corresponding to $W_k^{(i-N_{k|k-1})}$, $m_{D,k}^{(i-N_{k|k-1})}$, $P_{D,k}^{(i-N_{k|k-1})}$ in Eq. 21, respectively for consistency.

Corollary 1: The mean of the weights given by Eq. 25 converges to the integral given in Eq. 23.

Proof: From the kernel theory by Rossi and Vila (2004), we have:

$$K_{hn}^{z}\left(z-\hat{z}_k^{(i,j')}\right)\xrightarrow{a.s.}N\left(z;h_k\left(x_k^{(i,j')}\right)+v_{km},R_k\right) \quad (30)$$

as $h\to 0$, $Mh_n^d/\log M\to\infty$ and

$$E\left[K_{hn}^{z}\left(z-\hat{z}_k^{(i,j')}\right)-N\left(z;h_k\left(x_k^{(i,j')}\right)+v_{km},R_k\right)\right]^2\to 0 \quad (31)$$

as $h\to 0$, $Mh_n^d\to\infty$. Then:

$$\frac{1}{M'}\sum_{j'=1}^{M'}\omega_{D,k}^{(i,j')}\to\frac{1}{M'}\sum_{j'=1}^{M'}N\left(z;h_k\left(x_k^{(i,j')}\right)+v_{km},R_k\right) \quad (32)$$

and from the LLN we get:

$$\frac{1}{M'}\sum_{j'=1}^{M'}N\left(z;h_k\left(x_k^{(i,j')}\right)+v_{km},R_k\right)\to\int N\left(z;h_k\left(\xi\right)+v_{km},R_k\right)N\left(\xi;m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right)d\xi \quad (33)$$

Corollary 2: Equation 27 converges to the sum part in Eq. 23.

Proof: The denominator is apparent because of the LLN and Eq. 33. The numerator comes from the following approximation (Clark *et al*., 2007):

$$N\left(z;h_k\left(x\right)+v_{km},R_k\right)N\left(x;m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right)\approx\left(\int N\left(z;h_k\left(\xi\right)+v_{km},R_k\right)N\left(\xi\, m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right)d\xi\right)N\left(x;m_{D,k}^{(i)},P_{D,k}^{(i)}\right) \quad (34)$$

Then by using Eq. 33 we have:

$$N\left(z;h_k\left(x\right)+v_k^{(j)},R_k^{(j)}\right)N\left(x;m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right)\to\left(\frac{1}{M'}\sum_{j'=1}^{M'}\omega_{D,k}^{(i,j')}\right)N\left(x;m_{D,k}^{(i)},P_{D,k}^{(i)}\right) \quad (35)$$

The GSCPHD in non-Gaussian models: We know that any density can be approximated as close as required by a linear combination of Gaussian densities (Alspach and Sorenson, 1972; Anderson and Moore, 1979). Suppose w_k and v_k in Eq. 4 and 5 are not Gaussian, we may write them as the following Gaussian sums (Alspach and Sorenson, 1972; Anderson and Moore, 1979):

$$p\left(w_k\right)=\sum_{l=1}^{N_{w,k}}\omega_{w,k}^{(l)}N\left(w_k;w_k^{(l)},Q_k^{(l)}\right) \quad (36)$$

$$p\left(v_k\right)=\sum_{j=1}^{N_{v,k}}\omega_{v,k}^{(j)}N\left(v_k;v_k^{(j)},R_k^{(j)}\right) \quad (37)$$

where,

$$\sum_{l=1}^{N_{w,k}}\omega_{w,k}^{(l)}=\sum_{j=1}^{N_{v,k}}\omega_{v,k}^{(j)}=1 \quad (38)$$

Thus Eq. 6 and 7 may be rewritten as:

$$f_{k|k-1}\left(x_k\mid x_{k-1}\right)=\sum_{l=1}^{N_{w,k}}\omega_{w,k}^{(l)}N\left(x_k;f_k\left(x_{k-1}\right)+w_k^{(l)},Q_k^{(l)}\right) \quad (39)$$

$$h_k\left(z_k\mid x_k\right)=\sum_{j=1}^{N_{v,k}}\omega_{v,k}^{(j)}N\left(z_k;h_k\left(x_k\right)+v_k^{(j)},R_k^{(j)}\right) \quad (40)$$

Then by applying the similar way used in the GSCPHD filter and the properties of Gaussian, we may directly have the following propositions of the Generalized GSCPHD (GGSCPHD) filter.

Proposition 3 (GGSCPHD predict): Suppose the posterior intensity at time k-1 is a Gaussian sum of the form:

$$c_{k-1}\left(x\right)=\sum_{i=1}^{N_{k-1}}\omega_{k-1}^{(i)}N\left(x;m_{k-1}^{(i)},P_{k-1}^{(i)}\right) \quad (41)$$

then, after the prediction step of Eq. 2, the predictive intensity at time k is still a Gaussian sum given by:

$$\mathbf{c}_{k|k-1}\left(\mathbf{x}\right)=\sum_{i=1}^{N_{k|k-1}}\omega_{k|k-1}^{(i)}N\left(\mathbf{x};\mathbf{m}_{k|k-1}^{(i)},\mathbf{P}_{k|k-1}^{(i)}\right)=\mathbf{p}_{S,k}\sum_{i=1}^{N_{k-1}}\omega_{k-1}^{(i)}\sum_{l=1}^{N_{w,k}}\omega_{w,k}^{(l)}N\left(\mathbf{x};\mathbf{m}_{S,k|k-1}^{(i,l)},\mathbf{P}_{S,k|k-1}^{(i,l)}\right)+\mathbf{b}_k\left(\mathbf{x}\right) \quad (42)$$

where,

$$\mathbf{N}_{k|k-1}=\mathbf{N}_{k-1}\mathbf{N}_{w,k}+\mathbf{N}_{b,k},\ \omega_{k|k-1}^{(i)}=\begin{cases}\omega_{b,k}^{(i)} & 1\le i\le N_{b,k}\\ \mathbf{p}_{S,k}\omega_{k-1}^{(i-N_{b,k})}\omega_{w,k}^{(l)} & N_{b,k}+1\le i\le N_{k|k-1}\end{cases}$$

$$\mathbf{m}_{k|k-1}^{(i)}=\begin{cases}\mathbf{m}_{b,k}^{(i)} & 1\le i\le N_{b,k}\\ \mathbf{m}_{S,k|k-1}^{(i-N_{b,k},l)} & N_{b,k}+1\le i\le N_{k|k-1}\end{cases},\ \mathbf{P}_{k|k-1}^{(i)}=\begin{cases}\mathbf{P}_{b,k}^{(i)} & 1\le i\le N_{b,k}\\ \mathbf{P}_{S,k|k-1}^{(i-N_{b,k},l)} & N_{b,k}+1\le i\le N_{k|k-1}\end{cases} \quad (43)$$

and

$$m_{S,k|k-1}^{(i,l)}=\frac{1}{M}\sum_{j=1}^{M}x_{k-1}^{(i,j,l)},\ x_{k-1}^{(i,j,l)}\sim\mathbf{N}\left(x;f_k\left(x_{k-1}^{(i,j)}\right)+w_k^{(l)},Q_k^{(l)}\right),\ P_{S,k|k-1}^{(i,l)}=\frac{1}{M}\sum_{j=1}^{M}\left(x_{k-1}^{(i,j,l)}-m_{S,k|k-1}^{(i,l)}\right)\left(x_{k-1}^{(i,j,l)}-m_{S,k|k-1}^{(i,l)}\right)^T \quad (44)$$

Proposition 4 (GGSCPHD update): Suppose the predict intensity at time k is a Gaussian sum of the form:

$$c_{k|k-1}\left(x\right)=\sum_{i=1}^{N_{k|k-1}}\omega_{k|k-1}^{(i)}N\left(x;m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right) \quad (45)$$

then, after the update step of Eq. 3, the posterior intensity at time k remains a Gaussian sum given by:

$$c_k(x)=\sum_{i=1}^{N_k}\omega_k^{(i)}N\left(x;m_k^{(i)},P_k^{(i)}\right)=\left[1-p_{D,k}\right]\sum_{i=1}^{N_{k|k-1}}\omega_{k|k-1}^{(i)}N\left(x;m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right)+\sum_{z\in Z_k}\sum_{i=1}^{N_{k|k-1}}\sum_{j=1}^{N_{v,k}}W_k^{(i,j)}N\left(x;m_{D,k}^{(i,j)},P_{D,k}^{(i,j)}\right) \tag{46}$$

where,

$$N_k=\left(1+|Z_k|N_{v,k}\right)N_{k|k-1},\ \omega_{k|}^{(i)}=\begin{cases}\left[1-p_{D,k}\right]\omega_{k|k-1}^{(i)} & 1\le i\le N_{k|k-1}\\ W_k^{(i-N_{k|k-1},j)} & N_{k|k-1}+1\le i\le N_k\end{cases}$$
$$m_k^{(i)}=\begin{cases}m_{k|k-1}^{(i)} & 1\le i\le N_{k|k-1}\\ m_{D,k}^{(i-N_{k|k-1},j)} & N_{k|k-1}+1\le i\le N_k\end{cases},\ P_k^{(i)}=\begin{cases}P_{k|k-1}^{(i)} & 1\le i\le N_{k|k-1}\\ P_{D,k}^{(i-N_{k|k-1},j)} & N_{k|k-1}+1\le i\le N_k\end{cases} \tag{47}$$

and

$$W_k^{(i,j)}=p_{D,k}\omega_{k|k-1}^{(i)}\omega_{v,k}^{(j)}\frac{1}{M'}\sum_{j'=1}^{M'}\omega_{D,k}^{(i,j,j')}\Bigg/\left(\kappa_k(z)+p_{D,k}\sum_{i=1}^{N_{k|k-1}}\sum_{j=1}^{N_{v,k}}\omega_{k|k-1}^{(i)}\omega_{v,k}^{(j)}\frac{1}{M'}\sum_{j'=1}^{M'}\omega_{D,k}^{(i,j,j')}\right)$$
$$m_{D,k}^{(i,j)}=\sum_{j'=1}^{M'}\tilde{\omega}_{D,k}^{(i,j,j')}x_k^{(i,j')},\ \tilde{\omega}_{D,k}^{(i,j,j')}=\omega_{D,k}^{(i,j,j')}\Bigg/\sum_{j=1}^{M}\omega_{D,k}^{(i,j,j')},\ P_{D,k}^{(i,j)}=\sum_{j'=1}^{M'}\tilde{\omega}_{D,k}^{(i,j,j')}\left(x_k^{(i,j')}-m_{D,k}^{(i,j)}\right)\left(x_k^{(i,j')}-m_{D,k}^{(i,j)}\right)^T$$
$$\omega_{D,k}^{(i,j,j')}=K_{h_k}^{z}\left(z-\hat{z}_k^{(i,j,j')}\right),\ \hat{z}_k^{(i,j,j')}\sim N\left(z;h_k\left(x_k^{(i,j')}\right)+v_k^{(j)},R_k^{(j)}\right) \tag{48}$$

Also we may have the corollaries similar to corollary 1 and 2.

Discussion: Similar to the GMPHD filter (Vo and Ma, 2006), the proposed (G) GSCPHD filters also suffer from computational troubles caused by the increasing number of Gaussian components. Therefore, the method from (Vo and Ma, 2006) is used to reduce the number of Gaussian components by truncating components having weak weights and merging the closest components into one.

We know that the GMPPHD filter is based on the particle filter scheme, so it requires the analytical availability of the likelihood function as well as the not too small observation noise, thus it is limited in practice (Rossi and Vila, 2004). The convolution PHD filter (Panta and Vo, 2007), though it is based on convolution kernels and can deal with the situations that the observation noise is small or the likelihood is not analytically available, it is complex and difficult to parallel implementation because of the resampling scheme. The methods proposed in this study apply the convolution kernels without the resampling step during recursion, thus they have the ability to deal with complex observation model, small observation noise of the GSCPHD over the GMPPHD filter and the lower complexity, more amenable for parallel implementation than the convolution PHD filter.

SIMULATION RESULTS

Consider a two dimensional scenario with an unknown and time varying number of targets observed in clutter over the surveillance region ($-\pi/2$, $-\pi/2$) rad $\times$(0, 2000) m. The state dynamics are given by Vo and Ma (2006) and Clark *et al*. (2007):

$$\bar{x}_k=f(\varpi_{k-1})\bar{x}_{k-1}+Gw_{k-1},\ \varpi_k=\varpi_{k-1}+\Delta u_{k-1}$$

where, $\bar{x}_k=\left[x_{k,1},x_{k,2},x_{k,3},x_{k,4}\right]^T$ and $[x_{k,1}, x_{k,2}]^T$ is the position, $[x_{k,3}, x_{k,4}]^T$ is the velocity, ϖ_k is the turn rate, the target state variable $x_k=[\bar{x}_k,\varpi_k]^T$ and:

$$f(\varpi)=\begin{bmatrix}1 & 0 & \sin\varpi\Delta/\varpi & -(1-\cos\varpi\Delta)/\varpi\\ 0 & 1 & (1-\cos\varpi\Delta)/\varpi & \sin\varpi\Delta/\varpi\\ 0 & 0 & \cos\varpi\Delta & -\sin\varpi\Delta\\ 0 & 0 & \sin\varpi\Delta & \cos\varpi\Delta\end{bmatrix},\ G=\left[\Delta^2/2,0;0,\Delta^2/2;\Delta,0;0,\Delta\right],\ w_{k-1}\sim N\left(\cdot;0,\sigma_w^2 I_2\right)$$

$u_{k-1}\sim N$ (.;0, σ^2_n) with Δ = 1s, σ_w = 20 m/s^2 and σ_u = (π/180)rad/s. A consists of noisy bearing and range observations is given by:

$$z_k=\left(\arctan\left(x_{k,2}/x_{k,1}\right),\sqrt{\left(x_{k,1}\right)^2+\left(x_{k,2}\right)^2}\right)^T+v_k$$

We assume the spontaneous birth RFS is Poisson with intensity $b_k(x)$ = 0.1N (x; $m^{(1)}_{b,k}$, $P_{b,k}$) + 0.1N (x; $m^{(2)}_{b,k}$, $P_{b,k}$), where $m^{(1)}_{b,k}$ = $[0,1000,0,0,0]^T$, $m^{(2)}_{b,k}$ = $[-500,1000,0,0,0]^T$, $P_{b,k}$ = diag ($[2500,2500,2500,2500, (6\pi/180)^2]^T$). The probability of target survival and detection are $P_{S,k}$ = 0.99 and $P_{D,k}$ = 0.98, respectively, clutter is uniformly distributed over the surveillance region with an average 20 points per scan. We use 100 particles both in the proposed (G)GSCPHD filters and the GMPPHD filter. The number of Gaussian components at each time step is capped to a maximum of 100 components, the pruning is performed with a weight threshold of 10^{-5} and merging is performed with a threshold of 4.

We consider two scenarios, i.e., Gaussian and Gaussian sum noises, where in Gaussian noise scenario, $v_k\sim n$ (.;0, R_k), with R_k = diag ($[\sigma^2_\theta, \sigma^2_r]^T$}, σ_θ = (2π/180)rad/s, σ_r = 20m in Gaussian sum scenario, $v_k\sim 0.8N$ (.;$v^{(1)}_k$, $R^{(1)}_k$)+0.2N ($v^{(2)}_k$, $R^{(2)}_k$), with, $v^{(1)}_k$ = $[0,0]^T$, $v^{(1)}_k$ = [(2π/180)rad/s, 3m]^T, $R^{(1)}_k$ = diag ($[(\sigma^{(1)}_\theta)^2, \sigma^{(1)}_r)^2]^T$, $\sigma^{(1)}_\theta$ = (2π/180)rad/s, $\sigma^{(1)}_r$ = 20 m, $R^{(2)}_k$ = diag ($[(\sigma^{(2)}_\theta)^2, \sigma^{(2)}_r)^2]^T$, $\sigma^{(2)}_\theta$ = (3π/180)rad/s, $\sigma^{(2)}_r$ = 5m.

In the case that the parameters are given above, all the (G)GSCPHD and GMPPHD filters work well. However, when the covariance of the observation noise is reduced, e.g., σ_r = 1 m in Gaussian scenario or $\sigma^{(1)}_r$ = 1 m Gaussian sum scenario, the GMPPHD filter fails while the proposed (G) GSCPHD filters still works well (Fig. 1, 2), where the true states are denoted by solid lines and estimated states denoted by circles. Note that Fig. 1a and b correspond to Gaussian noise scenario, Fig. 2a and b correspond to Gaussian sum noise scenario, Fig. 1a and 2a correspond to the GMPPHD filter, Fig. 1b and 2b correspond to the GSCPHD and GGSCPHD filters,

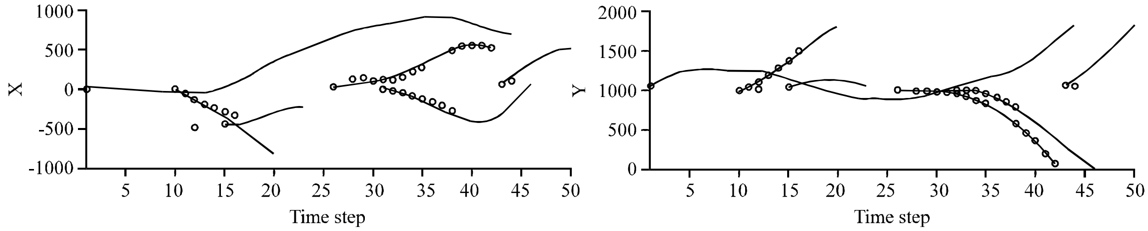

Fig. 1a: Results of the GMPPHD filter in Gaussian noise

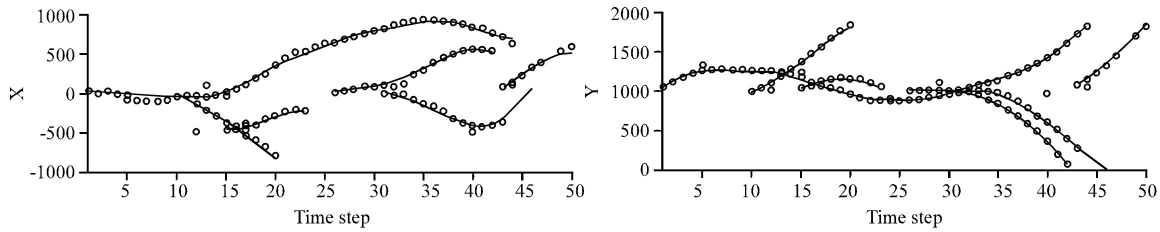

Fig. 1b: Results of the GSCPHD filter in Gaussian noise

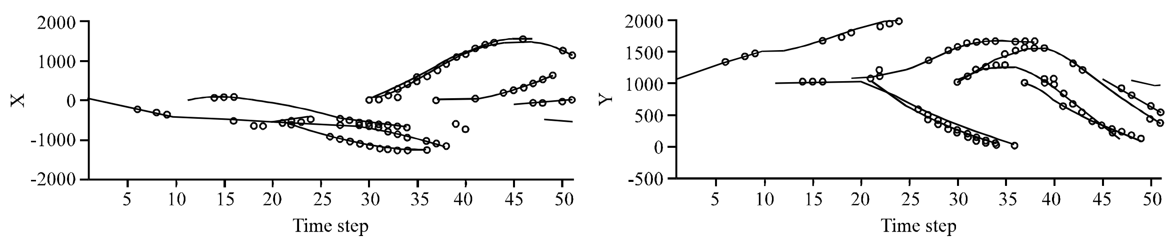

Fig. 2a: Results of the GMPPHD filter in Gaussian sum noise

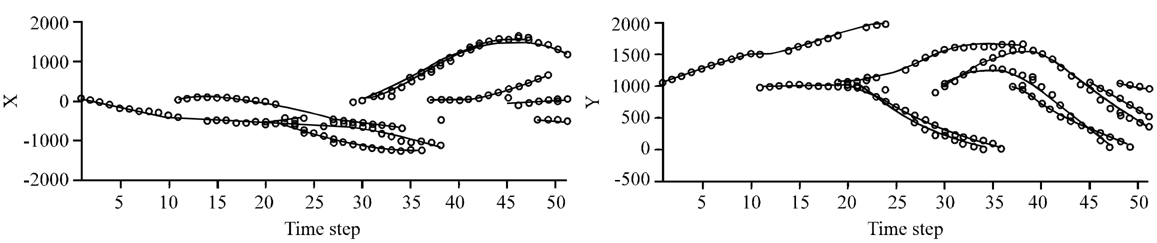

Fig. 2b: Results of the GGSCPHD filter in Gaussian sum noise

respectively. The top plot of each figure corresponds to x position, the bottom y position.

CONCLUSIONS

The (G)GSCPHD filters overcome the inabilities of the existing GMPPHD filter which is limited in the applications to scenarios that have low-noisy observations or lack the knowledge of the likelihood function. Simulation results are also presented to demonstrate the good performance of the (G)GSCPHD filters when observation noise is small while the existing GMPPHD filter fails. Moreover, the (G)GSCPHD filters are more convenient for VLSI implementation and feasible for practical real-time applications than the convolution PHD filter since there is no resampling step.

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