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## Tracking Control for the Underactuated Overhead Crane System Based on Dynamic Equilibrium State Theory

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**Abstract:** With the Dynamic Equilibrium State (DES) theory, a robust adaptive control method is presented to realize the position tracking and anti-swing for the overhead crane system in presence of parameter variations and external disturbances. First, the basic idea of the DES theory is introduced. Then, the ideal DES reference trajectories are planned for the position of the trolley and the angle of the load. By using the cascade sliding mode and nonlinear-parameterized fuzzy logic systems, a robust adaptive fuzzy controller is designed to track the prescribed trajectories. The simulation results are included to indicate the effectiveness and robustness of the proposed controller.

**Key words:** Overhead crane, dynamic equilibrium state (DES), robust adaptive control, nonlinear-parameterized fuzzy logic systems

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### INTRODUCTION

Underactuated mechanical systems are characterized by the fact that they have fewer actuators than the degrees of freedom to be controlled. There are many advantages about the underactuated systems such as reducing weight, cost or energy consumption (White *et al.*, 2008). The overhead crane, a kind of typical underactuated mechanical system, is widely used in many factories, construction sites and harbors for lifting heavy cargos and transporting tools. Because of its property and external factors when the crane moves, the load will have unavoidable shimmy problem, causing unsafe factors of transportation. To ensure the production safety and enhance the work efficiency, higher requirements are added to tracking precision, anti-swing and robustness.

In recent years, there have been growing attention and increasing interest in underactuated overhead crane systems (Wang *et al.*, 2007; Xi and Hesketh, 2010; Yi *et al.*, 2005; Cho and Lee, 2002; Chang, 2007; Park *et al.*, 2008). Sliding Mode control has been widely applied to robust control of nonlinear systems. Wang *et al.* (2007) proposed a sliding mode anti-swing method for the linear model of the overhead crane, which divided the states of the system into two groups, constructed two layers sliding surface, derived the overall sliding mode control law and further designed the parameters of the controller. Xi and Hesketh (2010) addressed an integral sliding mode control method for discrete linear system of cranes in presence of uncertainties. Simulation and experiment

results showed the validity of the control strategy. The controller was easy to design by Wang *et al.* (2007) and Xi and Hesketh (2010) but the nonlinear characters of the model were ignored. Yi *et al.* (2005) presented a kind of cascade sliding mode control method for a class of underactuated system to simplify the controller design and stability analysis. The controller didn't consider the parameter variations and external disturbances, thus the use of the controller was limited.

For many years, the fuzzy control method has become a very popular control strategy for nonlinear systems. Some fuzzy-based methods were also addressed to control the overhead crane by Cho and Lee (2002), Chang (2007), Park *et al.* (2008). Unfortunately, Cho and Lee (2002) can not provide the ideal performance for the crane system, due to the internal uncertainties and external disturbances for the overhead crane. Chang (2007), Park *et al.* (2008) proposed an adaptive fuzzy controller based on linear parameter fuzzy systems for the cranes with nonlinear disturbances but the fuzzy set is unchangeable and parameter can not be used effectively.

Wang and Chen (1999), Wang and Wang (2006) and Jin and Wang (2007) addressed a kind of nonlinear tracking design method based on the DES theory. The traditional stability theory was confined the stability concept to the free system. The DES is a class of equilibrium state corresponding to the non-free system. It is not the origin or the constant but functions of input. The nonlinear tracking design method of the DES theory is established on the concept of DES asymptotic stability.

The most obvious feature is unified the stability and tracking problems. But the control scheme Wang and Chen (1999), Wang and Wang (2006) and Jin and Wang (2007) was only limited to a class of affine nonlinear systems, did not consider system uncertainties.

Taking the drawbacks above into consideration, this study combines nonlinear tracking design of DES theory and nonlinear-parameterized fuzzy logic systems to design a robust adaptive controller in view of many kinds of uncertainties like load changes and external disturbances of the overhead crane. The control approach realizes tracking to the desired DES trajectories of the overhead crane and enhances the suppression and adaptation to uncertainties of parameter variations and external disturbances of the control system. Simulation results show that the controller is validity.

### DYNAMIC EQUILIBRIUM STATE THEORY

As we all know, the stability is the main performance index of control systems. But traditional stability theory is only limited to the free system, isolated the equilibrium state from input. The proposal of the DES concept opens a new way to study the stability of non-free systems with input. The definition of a DES is as follows:

**Definition 1:** Consider a control system  $dx/dt = fx, u(t), t$ , the steady-forced part  $x_e(u(t))$  of the solution is called the system dynamic equilibrium state under the action of  $u(t)$  and is also called the system dynamic equilibrium  $x_e(t)$  for short Wang and Chen (1999).

The DES is the description of the non-free system's stability state. The introduction of the dynamical equilibrium state gives a new description to the dynamical relationships of the system. Classical control theory did not reveal the internal variables but study the relationship between input and output. Modern control theory studied the relationships between input, state and output. The state space method played a key role to reveal and understand many important characters of control systems. State controllability and observability are particularly important, which become two basic concepts of control theory. But conceptions of state controllability and observability in modern control theory did not combine with system stability. The DES theory studies the relationships between the four factors: input, DES, state and output. The relationships can be shown as Fig. 1.

It is pointed out that what the input of a control system affects directly is the DES of the system rather than the state. When the DES moves, the system state will track the movement of equilibrium. So if the movement of DES is controlled, the state and output can be controlled.

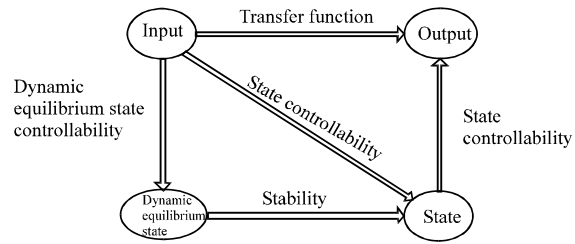


Fig. 1: The relationships between input, state, output and dynamic equilibrium state

In this sense, the control of input to the dynamic equilibrium is direct and the control to system state and output is indirect. That is the basic idea of equilibrium control (Wang and Wang, 2006; Jin and Wang, 2007).

The tracking design method of nonlinear system based on the DES theory is: Firstly, design a control law to make the DES move according to desired trajectory. Secondly, based on Lyapunov stability theory, make state asymptotically track DES in the optimal way and then achieve the control of state.

### PROBLEM STATEMENT

Figure 2 is a 2-D overhead crane system model. Assume that the trolley and load can be regarded as point masses, the trolley moves along the rail and the load moves in the x-y plane; at the same time, ignore the friction force in the trolley and the change of the length  $l$  of the steel wire rope. Ignore the mass of the steel wire rope.  $\theta$  is the sway angle of the load;  $x$  is the horizontal displacement of the trolley. The horizontal movement of the trolley is controlled by motor; The output control force is  $u$ .  $M$  and  $m$  are the masses of the trolley and the load, respectively.  $g$  is gravitational accelerating.

The plant model of the overhead crane by Yi *et al.* (2005) is:

$$\begin{cases} \dot{x}_1 = x_2 & \dot{x}_2 = f_1(x) + g_1(x)u + d_1(t) \\ \dot{x}_3 = x_4 & \dot{x}_4 = f_2(x) + g_2(x)u + d_2(t) \end{cases} \quad (1)$$

where,  $x_1 = x(t)$ ,  $x_2 = \dot{x}(t)$ ,  $x_3 = \theta(t)$ ,  $x_4 = \dot{\theta}(t)$ :

$$\begin{cases} f_1(x) = \frac{mL\dot{\theta}^2 \sin \theta + mg \sin \theta \cos \theta}{M + m \sin^2 \theta} \\ f_2(x) = \frac{-(m + M)g \sin \theta - mL\dot{\theta}^2 \sin \theta \cos \theta}{(M + m \sin^2 \theta)L} \\ g_1(x) = \frac{1}{M + m \sin^2 \theta} \\ g_2(x) = \frac{\cos \theta}{(M + m \sin^2 \theta)L} \end{cases}$$

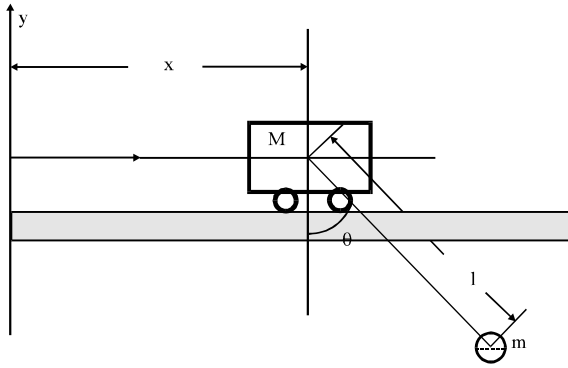


Fig. 2: Model of the overhead crane system

where,  $x = [x_1, x_2, x_3, x_4]$  are state variables;  $f_1(x), f_2(x)$  and  $g_1(x), g_2(x)$  are nominal bounded nonlinear functions;  $d_1(t), d_2(t)$  are external bounded disturbances and  $|d_1| \leq D_1, |d_2| \leq D_2$ .

Inspired by the reference model control method, we use the linear quadratic optimum control method to find the desired DES reference trajectories of the overhead crane.

The approximate linearization states equations of the overhead crane by Wang *et al.* (2007) are as follows:

$$\dot{x} = Ax + Bu = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{(m+M)g}{ML} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ -\frac{1}{ML} \end{bmatrix} u \quad (2)$$

Choose Q and R according to desired performance index, then the linear quadratic optimal feedback control law is given by:

$$u = -Kx \quad (3)$$

Substituting Eq. 3 to 2 and adding input r then the desired DES reference trajectories of the overhead crane system can be expressed by the linear reference model as follows:

$$\dot{x}_d = A_d x_d + B_d r = (A - BK) x_d + Bk_r \quad (4)$$

where,  $K_i$  is the first component of K.

$x_d$  is taken as the DES of the overhead crane and the trajectory tracking errors are defined as  $e_i = x_i - x_{di}, i = 1, 2, \dots, 4$ , such that:

$$\begin{cases} \dot{e}_1 = e_2 & \dot{e}_2 = f_1(x) + g_1(x)u + d_1(t) - \dot{x}_{d2} \\ \dot{e}_3 = e_4 & \dot{e}_4 = f_2(x) + g_2(x)u + d_2(t) - \dot{x}_{d4} \end{cases} \quad (5)$$

### CASCADE SLIDING MODE CONTROL

This section uses cascade sliding mode control by Yi *et al.* (2005) which can reduce the dimension of the sliding mode surface, simplify the system stability analysis and controller design. Define three layers sliding mode surface. Choose and to construct the first-layer surface:

$$s_1 = e_1 + c_1 e_2 \quad (6)$$

where,  $c_1 > 0$ , making polynomial  $s_1$  Hurwitz stable.

Then use  $s_1$  and to  $e_3$  construct the second-layer surface:

$$s_2 = c_2 e_3 + s_1 \quad (7)$$

Similarly, the third-layer surface can be written as:

$$s_3 = c_3 e_4 + s_2 \quad (8)$$

where,

$$c_i = C_i \operatorname{sgn}(e_{i+1} s_{i-1}) \quad (C_i > 0, i = 2, 3) \quad (9)$$

$$\dot{s}_3 = (c_1 g_1 + c_3 g_2)u + c_1 f_1 + c_3 f_2 + e_2 + c_2 e_4 - c_1 \dot{x}_{d2} - c_3 \dot{x}_{d4} + c_1 d_1 + c_3 d_2 \quad (10)$$

Choose  $\dot{s}_3 = 0$  the equivalent controller  $u_{eq}$  is:

$$u_{eq} = (c_1 \dot{x}_{d2} + c_3 \dot{x}_{d4} - c_1 f_1 - c_3 f_2 - e_2 - c_2 e_4 - c_1 d_1 - c_3 d_2) / (c_1 g_1 + c_3 g_2) \quad (11)$$

Using exponential reaching law, then the switching controller is given by:

$$u_{sw} = (-\eta \operatorname{sgn}(s_3) - ks_3) / (c_1 g_1 + c_3 g_2) \quad (12)$$

where,  $k > 0, \eta > c_1 D_1 + c_3 D_2$ .

Then, the total controller is designed as:

$$u = u_{eq} + u_{sw} \quad (13)$$

**Theorem 1:** Consider the overhead crane system represented by Eq. 1. Sliding surfaces are designed by Eq. 6, 7, 8; the desired DES reference trajectories are planned as Eq. 4. The control law is designed as Eq. 13, then the system is globally stable and the system states

asymptotically track the desired DES. The tracking errors are uniformly converging to zero.

**Proof:** Define the second Lyapunov function candidate as:

$$V_i = \frac{1}{2} s_i^2 \quad (i=1,2,3) \tag{14}$$

$$s_2^2 = c_2^2 e_3^2 + s_1^2 + 2c_2 e_3 s_1 \tag{15}$$

Substituting Eq. 9 into 15, one can obtain:

$$s_2^2 = c_2^2 e_3^2 + s_1^2 + 2C_2 |e_3 s_1| \geq s_1^2 \tag{16}$$

Similarly, there is:

$$s_3^2 \geq s_2^2 \geq s_1^2 \tag{17}$$

Then:

$$V_3 \geq V_2 \geq V_1 \geq 0 \tag{18}$$

$$\begin{aligned} \dot{V}_3 = s_3 [-ks_3 - \eta \operatorname{sgn}(s_3) + c_1 d_1 + c_3 d_2] = -ks_3^2 - |s_3| \\ (\eta - (c_1 D_1 + c_3 D_2)) \leq 0 \end{aligned} \tag{19}$$

Satisfying the sliding mode reaching conditions,  $V_3$  is bounded, i.e.,  $V_3 \in L_\infty$ . From Eq. 18 we can know that  $V_2 \in L_\infty$  and  $V_1 \in L_\infty$ , thus  $s_i \in L_\infty$  ( $i = 1, 2, 3$ ) and  $e_j \in L_\infty$  ( $j = 1, 2, 3$ ), i.e.,  $e_1 \in L_\infty, e_3 \in L_\infty$ .

It is assumed without loss of generality that the DES and their first and second time derivatives are uniformly bounded. We can obtain  $u \in L_\infty$ . Then, from Eq. 5, we know that  $\dot{e}_2 \in L_\infty$  and  $\dot{e}_4 \in L_\infty$ , so  $\dot{s}_i \in L_\infty$  ( $i = 1, 2, 3$ ). Integrating Eq. 19 with respect to time, yields:

$$k \lim_{T \rightarrow \infty} \int_0^T |s_3|^2 d\tau \leq V_3(0) - V_3(\infty) < \infty \tag{20}$$

Then  $s_3 \in L_2 \cap L_\infty$ . From Eq. 17 we can know,  $s_2 \in L_2 \cap L_\infty, s_1 \in L_2 \cap L_\infty$ . So by Barbalat's lemma, it can be shown that  $\lim_{t \rightarrow \infty} s_i = 0$  ( $i = 1, 2, 3$ ), so  $\lim_{t \rightarrow \infty} e_j = 0$  ( $j = 1, 2, 3$ ).

### ROBUST ADAPTIVE FUZZY CONTROL

**Nonlinear-parameterized fuzzy logic systems:** Considering an overhead crane system with the load variation and external disturbances, the control law (13) can not be obtained. A nonlinear-parameterized fuzzy logic system  $\hat{u}_{eq}(x, t)$  is designed to approximate  $u_{eq}(x)$  where  $\tau = (\lambda^T, \sigma^T, \omega^T)^T, \lambda^T, \sigma^T \in R^{M \times N}, \omega^T \in R^N$ .  $N$  represents

the number of fuzzy set inputs,  $M$  represents the rule number. In this study, the fuzzy system is implemented with singleton fuzzification and product inference and the defuzzifier is executed by the method of center of gravity. The input and output relation of the fuzzy system is obtained by Castro (1995) as:

$$u_{eq}(x, t) = \frac{\sum_{j=1}^M \omega_j \left( \prod_{i=1}^N \mu_{A_i^j}(x_i, \lambda_{ji}, \sigma_{ji}) \right)}{\sum_{j=1}^M \left( \prod_{i=1}^N \mu_{A_i^j}(x_i, \lambda_{ji}, \sigma_{ji}) \right)} \tag{21}$$

where,  $\omega_j$  is the parameter vector  $\mu_{A_i^j}(x_i, \lambda_{ji}, \sigma_{ji})$  and is called Gaussian membership function of fuzzy set  $A_i^j$ . One can defined as:

$$\mu_{A_i^j}(x_i) = \exp \left[ - \left( \frac{x_i - \lambda_{ji}}{\sigma_{ji}} \right)^2 \right] \tag{22}$$

$\lambda_{ji}, \sigma_{ji}$  and  $\omega_j$  are adjustable parameters of the Gaussian membership function.

Because a fuzzy system has the capability of universal approximation for any nonlinear function to the desired accuracy by Castro (1995). The optimal fuzzy approximation  $u_{eq}^*(x, t^*)$  is further designed to approximate the  $u_{eq}(x)$ , such that:

$$u_{eq}(x) = u_{eq}^*(x, t^*) + \epsilon \tag{23}$$

Let us define the optimal approximation parameters in the fuzzy system as follows:

$$t^* = \operatorname{argmin}_{t \in \Omega_t} \left( \sup_{x \in \Omega_x} |u_{eq}(x) - \hat{u}_{eq}(x, t)| \right) \quad U \subset R^N \tag{24}$$

where,  $\Omega_t$  and  $\Omega_x$  are bounded compact sets of adjustable parameters  $\tau$  and fuzzy input vector  $x$ .  $\epsilon$  is a optimal approximation error of the fuzzy logic system. Generally speaking, assume that the optimal approximation error is bounded, satisfying  $|\epsilon| \leq \bar{\epsilon}$ .

Suppose that  $\hat{\lambda}, \hat{\sigma}, \hat{\omega}$  are the estimated vectors of  $\lambda^*, \sigma^*, \omega^*$ . The optimal fuzzy approximation  $u_{eq}^*(x, t^*)$  is expanded based on Taylor series about an estimation point of  $\hat{u}_{eq}(x, \hat{t})$  (Wang and Wang, 2010; Lee and Zak, 2004). Then:

$$\begin{aligned} \hat{u}_{eq}(x, \hat{t}) = \sum_{j=1}^M \left\{ \sum_{i=1}^N \left[ O(\hat{\lambda}_{ji}^2) + O(\hat{\sigma}_{ji}^2) \right] + O(\hat{\omega}_j^2) \right\} + \\ \sum_{j=1}^M \left\{ \sum_{i=1}^N \left[ \frac{\partial \hat{u}_{eq}}{\partial \lambda_{ji}^*} \hat{\lambda}_{ji} + \frac{\partial \hat{u}_{eq}}{\partial \sigma_{ji}^*} \hat{\sigma}_{ji} \right] + \frac{\partial \hat{u}_{eq}}{\partial \omega_j^*} \hat{\omega}_j \right\} \end{aligned} \tag{25}$$

where, a control approximation error  $\hat{u}_{eq}(x, t)$  is defined as  $\hat{u}_{eq}(x, t) = u_{eq}^*(x, t) - \hat{u}_{eq}(x, t)$ ; the corresponding differences between the optimal and estimated parameters are defined as  $\hat{\lambda}_{ji} = \lambda_{ji}^* - \hat{\lambda}_{ji}$ ,  $\hat{\sigma}_{ji} = \sigma_{ji}^* - \hat{\sigma}_{ji}$  and  $\hat{\omega}_j = \omega_j^* - \hat{\omega}_j$ ,  $O(\hat{\lambda}_{ji}^2)$ ,  $O(\hat{\sigma}_{ji}^2)$  and  $O(\hat{\omega}_j^2)$  are high-order terms, respectively and:

$$\frac{\partial \hat{u}_{eq}}{\partial \lambda_{ji}^*} = \frac{2(x_i - \hat{\lambda}_{ji})(\hat{\omega}_j - \hat{u}_{eq})}{\hat{\sigma}_{ji}^2} \frac{\partial \hat{u}_{eq}}{\partial \omega_j^*},$$

$$\frac{\partial \hat{u}_{eq}}{\partial \sigma_{ji}^*} = \frac{2(x_i - \hat{\lambda}_{ji})(\hat{\omega}_j - \hat{u}_{eq})}{\hat{\sigma}_{ji}^3} \frac{\partial \hat{u}_{eq}}{\partial \omega_j^*},$$

$$\frac{\partial \hat{u}_{eq}}{\partial \omega_j^*} = \frac{\prod_{i=1}^N \exp \left[ - \left( \frac{x_i - \hat{\lambda}_{ji}}{\hat{\sigma}_{ji}} \right)^2 \right]}{\sum_{j=1}^M \left[ \prod_{i=1}^N \exp \left[ - \left( \frac{x_i - \hat{\lambda}_{ji}}{\hat{\sigma}_{ji}} \right)^2 \right] \right]}$$

**Robust adaptive fuzzy controller:** Based on the above fuzzy logic systems, the robust adaptive fuzzy controller of the overhead crane is defined as:

$$u = \hat{u}_{eq}(x, t) - ks_3 - \eta \text{sgn}(s_3) \tag{26}$$

where,

$$|s_3| \leq \frac{2(c_1 G_1 + c_3 G_2)^2}{c_1 G_{11} + c_3 G_{22}} (\eta - \bar{\epsilon}) \tag{27}$$

The parameter adaptation laws for the fuzzy system are chosen as:

$$\dot{\hat{\lambda}}_{ji} = -\Upsilon_{\lambda_{ji}} \frac{\partial \hat{u}_{eq}}{\partial \lambda_{ji}^*} s_3 \tag{28}$$

$$\dot{\hat{\sigma}}_{ji} = -\Upsilon_{\sigma_{ji}} \frac{\partial \hat{u}_{eq}}{\partial \sigma_{ji}^*} s_3 \tag{29}$$

$$\dot{\hat{\omega}}_j = -\Upsilon_{\omega_j} \frac{\partial \hat{u}_{eq}}{\partial \omega_j^*} s_3 \tag{30}$$

where,  $j = 1, 2, \dots, M$ ;  $i = 1, 2, \dots, N$ ;  $k, \eta > 0$ ;

$$\Upsilon_{\lambda_{ji}}, \Upsilon_{\sigma_{ji}}, \Upsilon_{\omega_j} > 0; |\epsilon^0 + \epsilon| \leq \bar{\epsilon}; \epsilon^0 = \sum_{j=1}^M \left\{ \sum_{i=1}^N [O(\hat{\lambda}_{ji}^2) + O(\hat{\sigma}_{ji}^2)] + O(\hat{\omega}_j^2) \right\}$$

**Theorem 2:** Consider the actual dynamic models of the overhead crane system in presence of parameter variations and external disturbances represented by Eq. 1. The ideal dynamic equilibrium states reference trajectories are designed as Eq. 4. IF the adaptive fuzzy control law is designed as Eq. 26 with the adaptation laws showed in 28, 29 and 30 then the stability of the entire adaptive fuzzy controller system can be guaranteed. The

parameter of the fuzzy logic system will remain bounded and the tracking errors asymptotically converge to a neighborhood of zero.

**Proof:** Define the second function candidate as:

$$V_3 = \frac{1}{2(c_1 g_1 + c_3 g_2)} s_3^2 + \frac{1}{2} \sum_{j=1}^M \left[ \sum_{i=1}^N \left[ \frac{\hat{\lambda}_{ji}^2}{\Upsilon_{\lambda_{ji}}} + \frac{\hat{\sigma}_{ji}^2}{\Upsilon_{\sigma_{ji}}} \right] + \frac{\hat{\omega}_j^2}{\Upsilon_{\omega_j}} \right] \tag{31}$$

The derivative of Eq. 31 with respect to time can be represented as:

$$\dot{V}_3 = \frac{1}{(c_1 g_1 + c_3 g_2)} s_3 \dot{s}_3 - \frac{1}{2} s_3^2 \frac{c_1 \dot{g}_1 + c_3 \dot{g}_2}{(c_1 g_1 + c_3 g_2)^2} + \sum_{j=1}^M \left[ \sum_{i=1}^N \left[ \frac{\hat{\lambda}_{ji}^2}{\Upsilon_{\lambda_{ji}}} \dot{\hat{\lambda}}_{ji} + \frac{\hat{\sigma}_{ji}^2}{\Upsilon_{\sigma_{ji}}} \dot{\hat{\sigma}}_{ji} \right] + \frac{\hat{\omega}_j^2}{\Upsilon_{\omega_j}} \dot{\hat{\omega}}_j \right] \tag{32}$$

Substituting Eq. 10 into 32 and using 13 and 23, then one can obtain:

$$\begin{aligned} \dot{V}_3 &= s_3(u - u_{eq}^*(x, t) - \epsilon) - \frac{1}{2} s_3^2 \frac{c_1 \dot{g}_1 + c_3 \dot{g}_2}{(c_1 g_1 + c_3 g_2)^2} \\ &= s_3(-ks_3 - \eta \text{sgn}(s_3) - \hat{u}_{eq}(x, t) - \epsilon) - \frac{1}{2} s_3^2 \frac{c_1 \dot{g}_1 + c_3 \dot{g}_2}{(c_1 g_1 + c_3 g_2)^2} \\ &\quad - \sum_{j=1}^M \left[ \sum_{i=1}^N \left[ \frac{\hat{\lambda}_{ji}^2}{\Upsilon_{\lambda_{ji}}} \dot{\hat{\lambda}}_{ji} + \frac{\hat{\sigma}_{ji}^2}{\Upsilon_{\sigma_{ji}}} \dot{\hat{\sigma}}_{ji} \right] + \frac{\hat{\omega}_j^2}{\Upsilon_{\omega_j}} \dot{\hat{\omega}}_j \right] \end{aligned} \tag{33}$$

Substituting Taylor series expansion Eq. 25 and the adaptation laws 28, 29 and 30 into 33, one can conclude that:

$$\begin{aligned} \dot{V}_3 &= -ks_3^2 - \eta |s_3| - s_3(\epsilon^0 + \epsilon) - \frac{1}{2} s_3^2 \frac{c_1 \dot{g}_1 + c_3 \dot{g}_2}{(c_1 g_1 + c_3 g_2)^2} \\ &\quad - \sum_{j=1}^M \left[ \sum_{i=1}^N \left[ \left( \frac{\hat{\lambda}_{ji}}{\Upsilon_{\lambda_{ji}}} + \frac{\partial \hat{u}_{eq}}{\partial \lambda_{ji}^*} s_3 \right) \hat{\lambda}_{ji} + \left( \frac{\hat{\sigma}_{ji}}{\Upsilon_{\sigma_{ji}}} + \frac{\partial \hat{u}_{eq}}{\partial \sigma_{ji}^*} s_3 \right) \hat{\sigma}_{ji} \right] + \left( \frac{\hat{\omega}_j}{\Upsilon_{\omega_j}} + \frac{\partial \hat{u}_{eq}}{\partial \omega_j^*} s_3 \right) \hat{\omega}_j \right] \end{aligned} \tag{34}$$

Then:

$$\begin{aligned} \dot{V}_3 &\leq -ks_3^2 - |s_3| \left[ \eta - \bar{\epsilon} - |s_3| \frac{c_1 G_{11} + c_3 G_{22}}{2(c_1 G_1 + c_3 G_2)^2} \right] \\ &\leq -ks_3^2 - \beta |s_3| \leq -ks_3^2 \leq 0 \end{aligned} \tag{35}$$

where,

$$\beta = \eta - \bar{\epsilon} - |s_3| \frac{c_1 G_{11} + c_3 G_{22}}{2(c_1 G_1 + c_3 G_2)^2} > 0 \tag{36}$$

From Theorem 1, we can know that  $s_i \in L_2 \cap L_\infty$  ( $i = 1, 2, 3$ ) and  $\dot{s}_i \in L_\infty$  ( $i = 1, 2, 3$ ). Thus, by Barbalat's lemma, it can be showed that when  $t \rightarrow \infty$ ,  $s_i(t) \rightarrow 0$ . Namely all the signals of the closed-loop system are bounded when  $t \rightarrow \infty$ ,  $e_j(t) \rightarrow 0$  ( $j = 1, 2, 3, 4$ ).

**SIMULATION RESULTS**

The constants of the crane model are chosen as:  $M = 2$  kg,  $m = 1$  kg. The parameter variation is  $\Delta m = 0.2$  sin ( $2\pi t$ ). The initial condition of the crane is  $x_0 = [0.3 \ 0 \ 0.2 \ 0]$ . The structure parameters of fuzzy logic systems are  $N = 4, M = 7$ . The initial values of the Gaussian fuzzy

membership function are averagely chosen between  $[-100, 100]$ . Adaptive parameters are  $\bar{\epsilon} = 0.1, \gamma = 0.5$ ; The parameters of the controller are:  $c_1 = 5, c_2 = 10, c_3 = 0.05, k = 20, \eta = 5$ .

The dynamic equilibrium state reference trajectories are chosen as the following two cases. (1) Choose the weighting matrix of LQR for (4) as  $R = 1, Q = \text{diag} [100 \ 100 \ 100 \ 100]$  then  $K = [10 \ 14.3393 \ -22.3522 \ -3.8103]$ . External disturbances are chosen random signals and signals intensity is five times. (2) Choose  $Q = \text{diag} [200 \ 50 \ 200 \ 50]$  then  $K = [14.1421 \ 13.6537 \ -17.8679 \ -1.4473]$ . External disturbances are set to be sine waves, i.e.,  $d_1(t) = d_2(t) = 0.3 \sin(\pi t)$ . Figure 3 and 4 are simulating curves

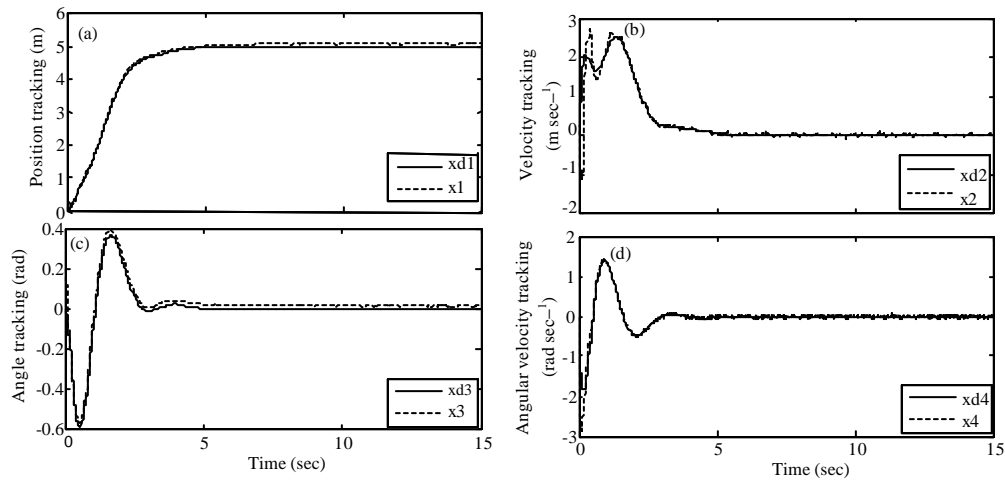


Fig. 3 (a-d): Simulation results of DES trajectories tracking in case 1; (a) position, (b) velocity, (c) swing angle and (d) angular velocity

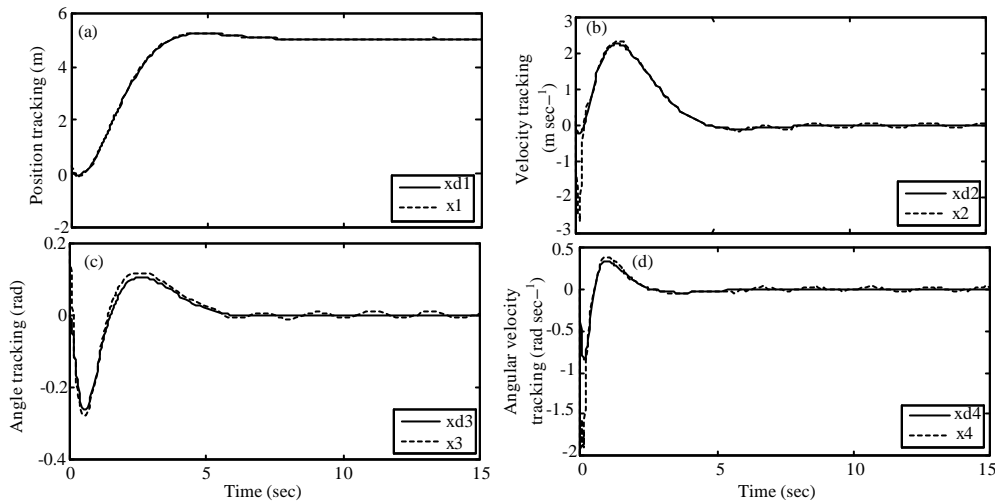


Fig. 4 (a-d): Simulation results of DES trajectories tracking in case 2, (a) position, (b) velocity, (c) swing angle and (d) angular velocity

of system states tracking desired DES. From simulation results we can see that, in presence of parameters variation (mass of the load changed) and external disturbances, the overhead crane realized global asymptotic tracking.

### CONCLUSION

In present study, the desired DSE reference trajectories are scheduled for the underactuated overhead crane system and the robust adaptive fuzzy controller is designed to realize the asymptotical tracking. Because of the use of the nonlinear-parameterized fuzzy logic system, fuzzy set can be changed during the adaptive process and adjustable parameters can be utilized effectively. The controller has stronger robustness and adaptive ability to parameters variation (mass of the load changed) and external disturbances, achieving anti-swing control. The proposed adaptive fuzzy tracking control method can be also used for many other applications.

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