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Application of the Residual Analysis Method for Data Fusion in Exterior Ballistics

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Abstract: In this study, in order to resolve the problem of data fusion in the trajectory calculation by many measurement elements, the method of calculating the parameters of exterior ballistics is proposed. The fusion is done by the nonlinear regression model of the reduced parameter model. The residual analysis method is used to diagnose and select the measurement elements which take part in solving fusion and the optimum trajectory is selected from the results trajectory. The result of experiments showed that the residual analysis method can diagnose the abnormal measurement elements in the fusion data processing effectively. The fusion result trajectory has good stability, the precision of the trajectory calculation is high and it has good effect of data fusion.

Key words: Eterior ballistics, data fusion, the reduced parameter modal, trajectory precision, residual analysis

INTRODUCTION

At present, the continuous wave radar is high precise equipment in the measurement system of exterior ballistics, the medium precise equipments include of the optical measuring and GPS (Global Positioning System), the low precise equipments include of monopulse radar, reflection radar and phased array radar, which provide the basis for the high precision parameter of exterior ballistics. The data processing mode of the original TTC (Tracking Telemetry and Command) system is that the exterior ballistic parameter is solved independently by the single equipment, which is hard to play the full capacity of the current TTC system, while the technology of data fusion can take full advantages of such a TTC system. In the measuring process, all the tracking equipments are used for the same trajectory. Therefore, all measured data contain the common real signal of the trajectory, which can lay the foundation for establishing the fusion model. As the number and the types of tracking equipment is more, we can establish a variety of fusion model by different combination forms of measurement elements, thus the realization condition is provided for combined solution trajectory and system error of the many combination fusion model.

In this study, the method of residual analysis is used to diagnose and select the measurement elements which take part in solving fusion. The analyzing problems include of how to fuse many types of measurement elements to diagnose and select the measurement elements which take part in solving fusion, how to obtain the optimum combination of the measurement elements, how to judge the fusion effect and how to provide the optimum high precision parameter of the exterior ballistics.

The reduced parameter model for parameter estimation of the trajectory: The data fusion of the trajectory parameter is mainly used the representation theory of the spline function (Wang, 1999). The essence of the spline function is the piecewise polynomial, in terms of the trajectory data, it is generally cubic function or quartic function. The spline polynomial is different from the normal algebraic polynomial, its function value is not only continuous in the split time points, but also its first derivative is continuous, which is accord with the physical background of the aircraft motion. Using spline function to represent the trajectory parameter can not only reflect the actual flight state of the vehicle, but also can reduce the expression error to a certain extent when the split time points is optimal selected, especially known the exact moment when the aircraft dynamic abruptly changed. Therefore, the natural problem in the data processing can be solved by reducing the truncation error to a certain extent.

Consider launching coordinate system, in x, y, z three directions, the trajectory parameter can be expressed as follows:

$$\begin{cases} x(t)=\sum_{j=1}^{N} c_j \psi_j(t), & \dot{x}(t)=\sum_{j=1}^{N_1} c_j \dot{\psi}_j(t) \\ y(t)=\sum_{j=1}^{N} c_{j+N} \psi_{j+N}(t), & \dot{y}(t)=\sum_{j=1}^{N_2} c_{j+N_1} \dot{\psi}_{j+N_1}(t), \qquad t_1 \le t \le t_m \\ z(t)=\sum_{j=1}^{N} c_{j+2N} \psi_{j+2N}(t), & \dot{z}(t)=\sum_{j=1}^{N_3} c_{j+N_1+N_2} \dot{\psi}_{j+N_1+N_2}(t) \end{cases} \quad (1)$$

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where, $\Psi_j(t)$, Ψ_{j+N_1}, $\Psi_{j+N_1+N_2}(t)$ is the basis function of the normalized B spline in three directions, respectively and decided by the knot sequence of every direction, which usually can be B spline of equidistance knots or unequal distance knots. c_j, c_{j+N_1}, $c_{j+N_1+N_2}$ is the spline coefficient of the trajectory parameter in three directions and [t1, tm] is the processing time split.

In general, the measurement elements of the equipment in the measuring network system include azimuth angle A_k, elevation angle E_k, distance R_k and range rate R_k, the four measurement elements can be expressed as:

$$A_k(t) = \mathrm{arctg}\frac{z(t)-z_k}{x(t)-x_k} + \alpha_k$$

$$E_k(t) = \mathrm{arctg}\frac{y(t)-y_k}{\sqrt{|x(t)-x_k|^2 + |z(t)-z_k|^2}} \tag{2}$$

$$R_k(t) = \sqrt{|x(t)-x_k|^2 + |y(t)-y_k|^2 + |z(t)-z_k|^2}$$

$$\dot{R}_k(t) = \frac{[x(t)-x_k]\dot{x}(t) + [y(t)-y_k]\dot{y}(t) + [z(t)-z_k]\dot{z}(t)}{R_k(t)}$$

where, α_k is the quadrant angle of every azimuth angle, $(x(t), y(t), z(t), \dot{x}(t), \dot{y}(t), \dot{z}(t))$ is the trajectory parameter of launching coordinate system at the time t, (x_k, y_k, z_k) is the coordinate of the measuring station k in the launching coordinate system.

From the Eq. 2, we can know that the measurement elements A_k, E_k, R_k and $\dot{R}_k$ is the nonlinear parameter of the trajectory parameter. From the Eq. (1), we can know that the trajectory parameter is the linear function of the spline coefficient β_P. So all the measurement elements is the nonlinear function of β_P. Define $a = (a_1, ..., a_K)^T$ as the system error parameter vector of all measurement elements to be estimated and here suppose that there is only constant error. So the measurement data of every time can be expressed as:

$$\begin{cases} \tilde{A}_j(t_i) = f_{ji}(\beta_p, a) + \varepsilon_j(t_i), & 1 \le j \le M_1 \\ \tilde{E}_j(t_i) = f_{ji}(\beta_p, a) + \varepsilon_j(t_i), & M_1 < j \le 2M_1 \\ \tilde{R}_j(t_i) = f_{ji}(\beta_p, a) + \varepsilon_j(t_i), & 2M_1 < j \le 2M_1 + M_2 \\ \tilde{\dot{R}}_j(t_i) = f_{ji}(\beta_p, a) + \varepsilon_j(t_i), & 2M_1 + M_2 < j \le 2M_1 + 2M_2 \end{cases} \tag{3}$$

Where:

$$F(t_i, \beta) = \begin{pmatrix} f_{1i}(\beta_p, a) \\ f_{2i}(\beta_p, a) \\ \cdots \\ f_{Mi}(\beta_p, a) \end{pmatrix}, \quad \varepsilon(i) = \begin{pmatrix} \varepsilon_1(t_i) \\ \varepsilon_2(t_i) \\ \cdots \\ \varepsilon_M(t_i) \end{pmatrix}$$

$\varepsilon(i)$ is the random error, denote

$$Y(i) = (\tilde{A}_1(t_i), \cdots, \tilde{A}_{M_1}(t_i), \cdots, \tilde{R}_{2M_1+M_2}(t_i), \cdots, \tilde{\dot{R}}_M(t_i))^T, \quad M = 2M_1 + 2M_2$$

and $\beta = \beta_P^T, a^T$, then the fusion model of exterior measuring data at the time t_i is:

$$Y(i) = F(t_i, \beta) + \varepsilon(i) \tag{4}$$

Combining the data at the $t_1, t_2, \cdots, t_m$ time, we can obtain:

$$Y = F(\beta) + \varepsilon \tag{5}$$

$$\text{where, } Y = \begin{pmatrix} Y(1) \\ Y(2) \\ \cdots \\ Y(m) \end{pmatrix}, \quad F(\beta) = \begin{pmatrix} F(t_1, \beta) \\ F(t_2, \beta) \\ \cdots \\ F(t_m, \beta) \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon(1) \\ \varepsilon(2) \\ \cdots \\ \varepsilon(m) \end{pmatrix} \text{ and } \varepsilon \sim N(0, \sigma^2 I)$$

The Eq. 5 is the integrative modeling in solving fusion for the trajectory parameter and it is a nonlinear modal. The parameters to be estimated are the spline coefficient and system error of measurement elements. Adopting the nonlinear regression method (Wei, 1989), we can obtain the trajectory parameter and system error of the measurement elements at the same time.

The application of residual analysis method for the fusion process control: In the fusion processing of the exterior ballistics by the reduced parameter model (Zhengming and Jubo, 1999; Weili *et al*., 2003), the key problem involved is the trade-offs of the measurement elements or the trade-offs of the time split and the whole fusion process which include modeling the system error, diagnosis and selection the measurement elements. Diagnosis and selection is based primarily on that most of the measurement elements is normal and the correct data fusion processing of the exterior ballistics must satisfy that the residuals of every measurement elements is close to the pure random error and apparently it is not include of the trend terms.

The basic method of analyzing and diagnosing the measurement elements is the residual analysis method. The method is mainly used to find and eliminate the measurement elements which include a batch of abnormal data and un-modeled system errors. The specific method as follows:

Consider the following two model, fusion model A with M-1 measurement elements and fusion model B with M measurement elements, according to equation:

$$Y^{(M-1)} = F(\beta^{(M-1)}) + \varepsilon^{(M-1)} \tag{6}$$

$$Y^{(M)} = F(\beta^{(M)}) + \varepsilon^{(M)} \quad (7)$$

Comparing Eq. 6 with Eq. 7, Eq. 6 is less of one measurement element $\tilde{Y}_M$, accordingly, it is less of m regression equations. The parameters to be estimated of Eq. 6 is $\beta^{(M-1)}$ and Eq. 7 is $\beta^{(M)}$, obviously, the parameters to be estimated of Eq. 7 is more.

Denote $\beta_P = \beta^{(M-1)}$, $\beta = \beta^{(M)}$, assume the component number is p and n, respectively, $p \le n$, the Eq. 6 and 7 can be simplified as:

$$Y_p = F_p(\beta_p) + \varepsilon_p \quad (8)$$

$$Y = F(\beta) + \varepsilon \quad (9)$$

Thus:

$$\left\|Y_P - F_P(\hat{\beta}_P)\right\|^2 = \min_{\alpha \in R^p} \left\|Y_P - F_P(\alpha)\right\|^2 \quad (10)$$

$$\left\|Y - F(\hat{\beta})\right\|^2 = \min_{\alpha \in R^n} \left\|Y - F(\alpha)\right\|^2 \quad (11)$$

Assume that there are observation data at time $(t_1, t_2, ..., t_m)$ of every measurement element, let:

$$\hat{\sigma}_P^2 = \frac{\left\|Y_P - F_P(\hat{\beta}_P)\right\|^2}{m(M-1) - p} \quad (12)$$

$$\hat{\sigma}^2 = \frac{\left\|Y - F(\hat{\beta})\right\|^2}{mM - n} \quad (13)$$

$$\tau = \left|\hat{\sigma}^2 - \hat{\sigma}_p^2\right| \quad (14)$$

For the given threshold τ_1:

- If $\tau \le \tau_1$, the measurement element is deemed to be usable
- If $\tau > \tau_1$, y_M, Y_M is deemed to be abnormal

The abnormal measurement elements are approximately divided into two kinds, one with low resolution, the other with system error. The random error of the former is high, the trade-offs can combine enough elements with medium and high precise at the solving time. The diagnosis of measurement elements with system error can be analyzed by combining the residual. When the measurement elements have no system error, the residual is close to random error and it is not include of apparent trend terms. Whereas, when the measurement elements have system error, the residual has strong trend terms after processed by fusion model with no system error. When the measurement elements have apparent system error and the error can be modeled and estimated, they are usable. Whereas, when the system error can't be modeled, the measurement elements are unusable. The diagnosis and selection of the rest elements can be processed by the same way.

It is worth noting that in the processing of selection, the measurement elements set which take part in fusion should be with high precise, stable and without large error terms. In the fist fusion, the measurement elements should be as few as possible and as excellent as possible, ensuring that most of measurement elements are with normal precise, so the diagnosis and selection is correct for the very few abnormal measurement elements.

Application of the residual analysis method for fusion effect evaluation: In order to test the fusion effects, the data fusion results will be evaluated. Because the real exterior ballistics value can not be obtained, the precise of the trajectory parameter is not given exactly by comparing with the real data. At present, mutual evaluation is mainly adopted. The main basis is that no matter how many measurement elements participate in fusion, ultimately the trajectory parameter is excusive. The difference of all evaluation value of the trajectory obtained by different fusion model should be small and the evaluation results are from all kinds of model which are constructed by different combination of the measurement elements. The estimation results will be stable at a certain change range of weight.

The main approach of mutual evaluation is the residual analysis method. Firstly, analyze the residual sequence of the data fusion trajectory by inverse calculation measurement elements and the real credibility of the practical trajectory can be confirmed. Then, the stability of the final trajectory can be confirmed by comparing the result trajectory which obtained by fusion model constructed by different combination of measurement elements. If the model is correct, the combined solution value of the trajectory by all kinds model is accord with that of the system error. Therefore, whether the combined solution results can keep stable and consistence or not is the important criteria to diagnose error and evaluate data quality. Specific approaches are as follows:

After using residual analysis method, the measurement elements which affect the precise of the trajectory parameters are eliminated. For the rest of the measurement elements, select different combination and different optimum knots in the fusion processing, so the many fusion schemes are constructed for fusion solution.

Here, it is assumed that selecting measurement elements is no longer involved in, but the optimum of the result trajectory obtained by the different fusion schemes should be done. Specific approaches is simplified as follows.

Let the trajectory parameter be:

$$X(t_i)=(x(t_i),y(t_i),z(t_i),\dot{x}(t_i),\dot{y}(t_i),\dot{z}(t_i))^T$$
$$=(x_1(t_i),x_2(t_i),x_3(t_i),x_4(t_i),x_5(t_i),x_6(t_i))$$

Suppose that there areusion model A and B, the trajectory parameter of fusion solution is $\hat{x}_A$ and $\hat{x}_B$ respectively, ψ is the spline basis function, theestimation value of the spline coefficients is $\hat{\beta}_A,\hat{\beta}_B$. From Eq. 1, we can obtain:

$$X_K(t_i)=\psi(t_i)\hat{\beta}_K=(\hat{x}_{1,K}(t_i),\cdots,\hat{x}_{6,K}(t_i)) \quad \text{(K=A or B)}$$

Let $X_K=(X_K(t_1)^T,\cdots,X_K(t_m)^T)^T$ then:

$$\hat{X}_K=\psi\hat{\beta}_K=(X_K(t_1)^T,\cdots,X_K(t_m)^T)^T$$

where, $\hat{\beta}_A,\hat{\beta}_B$ is estimation of the spline coefficients of the trajectory parameter obtained from Eq. 10 and 11. For the given precision index , $\tau=(\tau_1,\tau_2,\tau_3,\tau_4,\tau_5,\tau_6)^T$ the mean of residual error is:

$$S_j=\frac{1}{m}\sum_{i=1}^{m}\Delta x_j(t_i)$$

The residual variance is

$$\sigma_j=\left(\frac{1}{m-1}\sum_{i=1}^{m}(\Delta x_j(t_i)-\Delta\bar{x}_j)^2\right)^{\frac{1}{2}}\le\tau_j, j=1,\cdots,6$$

where, $\Delta x_j(t_i)=\hat{x}_{j,A}(t_i)-\hat{x}_{j,B}(t_i)$.

If S_j is equal to zero or close to zero and $\sigma_j\le\tau_j, j=1,\cdots,6$, it is considered that the fusiontrajectory is consistency and stability, which indicate that the constructed model and the estimation value is correct. Otherwise, the fusion trajectory is not consistency.

If S_j is greater or, it is $\sigma_j>\tau_j, j=1,\cdots,6$ considered that there are measurement elements affecting the precise in the model A or B, then the analysis should be combined with residual plot or other trajectory, such as the rendezvous trajectory obtained by least square method point by point or by other fusion scheme and abandon the measurement elements which have influence on the trajectory precise. In order to realize it, there should be a wealth of data processing experience and fully understanding of error characteristic of the measurement elements. Here there is no longer detailed analysis.

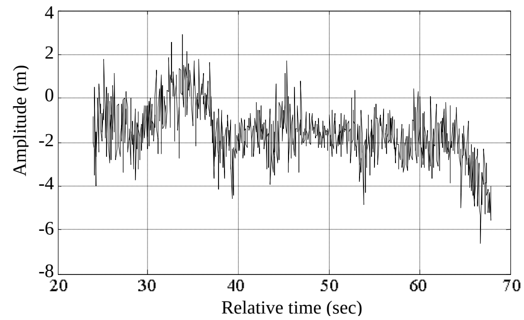

Fig. 1: Residual plot of No. 1measurement elements

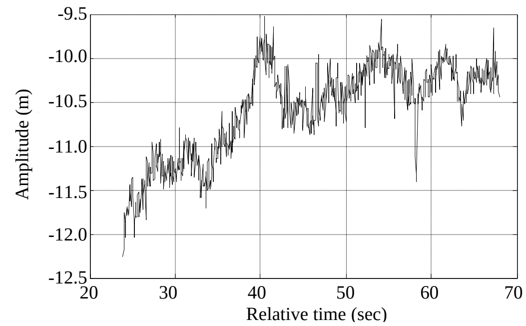

Fig. 2: Residual plot of No. 2 measurement elements

Application and analysis: Applying the above residual analysis method, the data fusion of the exterior ballistics is calculated as follows. The collecting time is 60 sec, the data set is 6 of A measurement element, 6 of E measurement element, 8 measurement elements for ranging and 8 for velocity measuring. Tracking the data is not at the whole time. In the fusion calculation, it is found that the residual of two measurement elements is abnormal, assuming that they are No. 1 measurement element and No. 2 measurement element. Perhaps there may be system errors to be estimated, as shown in Fig. 1 and 2.

So the following fusion processing strategy is adopted:

Options 1: The constant value system errors of No. 1 and No. 2 will not be estimated

Options 2: The constant value system errors of No. 1 will be estimated and the constant value system errors of No. 2 will be estimated

Options 3: The constant value system errors of No. 2 will be estimated and the constant value system errors of No. 1 will be estimated

Table 1: The constant system error estimation of different fusion options (unit:metre)

		Option 1	Option 2	Option 3	Option 4
Constant system error	No. 1		-2.781804		-1.355489
	No. 2			-16.190518	-16.174390
Residual standard deviation		0.948818	0.809811	0.659864	0.64148

Table 2: The statistic residual mean of different fusion options

	Residual mean of distance (unit: m)			Residual mean of velocity (unit: m sec^{-1})		
	X	Y	Z	VX	VY	VZ
trajectory 1 - trajectory 2	0.098212	-0.069521	0.140421	-0.000007	-0.002526	0.000447
trajectory 1 - trajectory 3	0.094478	0.846401	-5.966185	0.026633	0.026421	-0.081890
trajectory 1 - trajectory 4	0.117413	0.829258	-5.927334	0.026606	0.025803	-0.081704
trajectory 2 - trajectory 3	-0.003734	0.915922	-6.106607	0.026641	0.028947	-0.082337
trajectory 2 - trajectory 4	0.019201	0.898779	-6.067755	0.026613	0.028329	-0.082151
trajectory 3 - trajectory 4	0.022935	-0.017142	0.038852	-0.000028	-0.000619	0.000186

Table 3: The statistic residual standard deviation of different fusion options

	Residual standard deviation of distance (unit: m)			Residual standard deviation of velocity (unit: m sec^{-1})		
	X	Y	Z	X	VY	VZ
trajectory 1 - trajectory 2	0.012152	0.061059	0.012940	0.003674	0.001581	0.004274
trajectory 1 - trajectory 3	0.592409	0.632478	1.518622	0.307035	0.018034	0.376056
trajectory 1 - trajectory 4	0.591991	0.617857	1.514690	0.307330	0.017800	0.374726
trajectory 2 - trajectory 3	0.591822	0.692298	1.528975	0.304493	0.019006	0.380140
trajectory 2 - trajectory 4	0.591348	0.677623	1.525033	0.304783	0.018753	0.378809
trajectory 3 - trajectory 4	0.002876	0.014932	0.004337	0.000686	0.000382	0.001363

Options 4: The constant value system errors of No. 1 and No. 2 will be estimated

The statistical constant value system errors of the above fusion options are shown in Table 1 and the residual standard deviation are obtained by the Eq. 12 and 13. The comparisons by the fusion calculation of the 4 options are shown in Table 2 and 3.

Comparing the statistical results of Table 1, 2 and 3, we can conclude:

- The measurement element residual, the trajectory parameter residual and system error of different combinations is approximately same and they are within the accuracy requirements. It is shown that every option is feasible to the trajectory parameters to be estimated
- The constant system error of No. 1 measurement element is approximately same for different combinations, but the value is small. From comparisons of the fusion result trajectory, it can be seen that the difference between trajectory 1 and trajectory 2 is small and difference between trajectory 3 and trajectory 4 is small. It is indicated that whether such the constant value system error is to be estimated or not, does not affect the precise of the trajectory
- The residual mean value and residual mean square deviation value of trajectory 1 and trajectory 3 in the Z direction is large and the value of trajectory 2 and trajectory 4 is large too, which indicates that estimating the constant value system error of trajectory 2 or not influence the result trajectory
- The estimation value of constant value system error of No. 2 measurement element is very close for different combinations and the value is larger, about 14 m. It is indicated that there is really constant value system error on No. 2. So the trajectory 1 and trajectory 2 can not be adopted from the point of view of optimized selection
- The difference of trajectory 3 and trajectory 4 is very small and the residual mean and the residual variance is small too, which can be negligible. So the trajectory 3 and trajectory 4 can be considered as the optimum result trajectory
- Finally, from the respects of the residual of the measurement elements, the trajectory parameter residual, system error estimation, the number of parameters to be estimated, fitting with the engineering background, etc. the trajectory 3 is selected as the result trajectory

CONCLUSION

The method of fusion to calculate the exterior ballistic parameter is proposed in this study, which apply the reduced parameter nonlinear regression model. The key problems are analyzed and discussed by the residual analysis method, including selection and diagnosis of the measurement elements, optimum the result trajectory in the fusion process. Simulation by calculation show that the residual analysis can effectively diagnose the abnormal measurement elements in the fusion data processing and can detect and identify the measurement elements with low precise and system error. The residual analysis of the result trajectory by fusion can achieve the aim of optimum the trajectory, the result trajectory has good stability and have good effects in data fusion. In the solving practical data fusion process, the residual analysis method is applied repeatedly, ensuring the precise of the exterior ballistic parameters can meet the requirement. The final calculating scheme can be determined by fusion calculation, measurement element diagnosis and trajectory analysis repeatedly.

REFERENCES

Wang, Z.M., 1999. Calibration and Evaluation of Trajectory Tracking Data. National Defence and Technology University Press, Changsha, China.

Wei, B.C., 1989. Modern Nonlinear Regression Analysis. Southeast University Press, NanJing, China.

Weili, C., S. Changlin and Y. Zhengmao, 2003. A new algorithm for data fusion of the trajectory computing only with the velocity and its application. J. Spacecraft TT C Technol., 22: 32-36.

Zhengming, W. and Z. Jubo, 1999. The reduced parameter modelof trajectory tracking data and its application. Sci. China Series E, 29: 146-154.