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Observer-based Guaranteed Cost Fault-tolerant Controller Design for Networked Control Systems

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Abstract: The problem of integrity against sensor failures for networked control systems based on state observer is studied. Assuming that the time-delay is more than one sampling period, the system is modeled as a discrete time system with parametrical uncertainties. Based on the model, the state observe is designed and according to possible sensor failures, an augmented mathematic model for the networked control systems based on state observer is developed. Then in terms of the given quadratic performance index function, the integrity condition of the system is given and the designs for guaranteed cost fault-tolerant controller and observer are presented, by using the cooperative design approach of the controller and observer and the approach of bilinear matrix inequalities. An example is given to show the effectiveness of our method.

Key words: Sensor failures, stability, control law, bilinear matrix inequalities, cost function, time-delays

INTRODUCTION

Networked Control Systems are feedback control system wherein the control loops are closed through a real-time network (NCSs) (Zhang *et al*., 2001; Antsaklis and Baillieul, 2007; Hespanha *et al*., 2007). Components directly connected to network (including sensor and controller) are regarded as nodes of network. Compared with traditional point-to-point control, networked control systems have more advantages such as shared resources, long range manipulation, low cost and easy of system maintenance. Hence, it has good application prospect. Long range manipulation, long range teaching and experiment, wireless network robot and industrial Ethernet technology can all be ascribed to be control systems based on network. However, because of insertion of the communication network in the feedback control loop, problems of time-delay, data packet dropout and disordered time sequence arise, which can degrade performance of the systems and even breakdown the systems. So, it is significant to do research on the fault-tolerant control for the networked Control Systems.

When time-delay is longer than one sampling period, it is called long time-delay. At present, the research on long time-delays networked control systems is mainly focused on modeling, stability analysis and estimate and control the time-delay (Yang and Fang, 2009; Yang *et al*., 2006; Zhang and Yu, 2009), etc. However, little attention has been to the study on fault-tolerant control. A kind of networked control systems with random time-delays are modeled as a discrete-time jump linear system with Markov delay characteristics and the actuator failures of networked control systems are analyzed based on jump linear system theory and fault-tolerant control theory (Huo and Fang, 2006). The uncertainty of network-induced delays is converted to the uncertainty of the parameter matrix, the sufficient conditions for closed-loop networked control systems with uncertain disturbance possessing robust integrity against sensor or actuator failures are given and the robust fault-tolerant controller is designed by Li *et al*. (2007). For a class of networked control systems with time-varying delays, based on the integrity fault-tolerant control theory and the time-delay-dependent stability criteria, the sufficient conditions for systems with integrity against actuator failures are given and the robust fault-tolerant cont roller is designed by Guo *et al*. (2008). A procedure is proposed for controlling a system over a network using the concept of an NCS-Information-Packet which is an augmented vector comprising control moves and fault flags, then the problem of fault-tolerant control for networked control systems is studied (Klinkhieo *et al*., 2006). The conclusions of these references are based on state feedback. However, in actual systems, as the complexity of network, the all states of the system are not convenience enough to be obtained. Then the state observer is prone to be used. Therefore it is essential to do research on networked control systems with state

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observer. The problem of state feedback stabilization for a class of networked control systems based on observer state is investigated (Kong and Fang, 2006). Based on observer state, the fault detection of the system is studied (Bao *et al*., 2005; Zhu and Zhou, 2006; Lu *et al*., 2007). However, as so far, the research on fault-tolerant Control for networked control systems with long time-delays based on state observer is rare.

In this study, we aim at solving the problem of integrity against sensor failures for networked control systems with long time-delays. According to possible sensor failures, the state observer of the system is designed and an augmented mathematic model for the networked control systems based on state observer is developed. Then the cooperative design approach of the controller and the observer is given and the existence conditions of guaranteed cost fault-tolerant control law are testified by use of the Lyapunov stability theory combined with bilinear matrix inequalities. Furthermore, the designs for guaranteed cost fault-tolerant control law and the state observer are presented.

PROBLEM FORMULATION

Observer-based networked control systems studied in this study is shown in Fig. 1. Where, the sensor and controller are regarded as the same node. Network only exists between controller and actuator, then the system has control time-delay τ.

We assume:

- The sensor is driven by time, which is sampled with known fixed T. The controller and the actuator are event-driven
- The time-delay τ is time varying and $\tau \in (0, \bar{d}T)$, where $\bar{d}$ τ Z^+
- A is nonsingular and (A,C) can be observed

From Huang *et al*. (2008), the time-delay τ can be represented by $\tau = dT+\Delta\tau_k \le \bar{d}T$, $0 \le \Delta\tau_k = h/2+\Delta\tau \le h$, where, h/2 is the mean and constant time delay; $-h/2 \le \Delta\tau \le h/2$ is the uncertain and time-vaying delay, including the offset of clock synchronization. $d \le \bar{d} = [\tau^{wc}/T]$, where, $[\Delta]$ is the least integer larger than Δ, τ^{wc} is the worst case time-delay and known.

Assume that the process to be monitored is an LTI system described by:

$$\begin{cases} \dot{x}(t) = A_f x(t) + B_f v(t) \\ y(t) = C_f x(t) \end{cases} \tag{1}$$

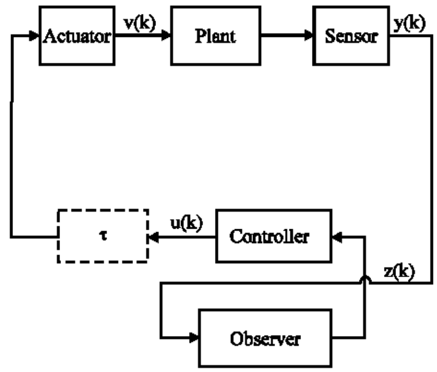

Fig. 1: Structure diagram of the observer-based networked control systems

where, $x(t) \in R^n$, $v(t) \in R^t$ and $y(t) \in R^P$ are state vector, input vector, measure output vector. A_f, B_f and C_f are known matrices of compatible dimensions.

Then the discrete-time NCS model with delays is obtained as follows:

$$\begin{cases} x(k+1) = Ax(k) + B_0(\Delta\tau_k)u(k-\bar{d}_{ca}) + B_1(\Delta\tau_k)u(k-\bar{d}_{ca}-1) \\ y(k) = Cx(k) \end{cases} \tag{2}$$

where, $u(k) \in R^l$ denote the control output vector.

According to Xie *et al*. (2009) , we obtain

$$B_0(\Delta\tau_k) = B - DF(\Delta\tau_k)E \tag{3}$$

$$B_1(\Delta\tau_k) = DF(\Delta\tau_k)E \tag{4}$$

and

$$F^T(\tau_k)F(\tau_k) \le I \tag{5}$$

where, B, D and E are constant matrices determined by eigenvalues and eigenvectors of matrix A_f and B_f, $F(\tau_k)$ is an uncertain matrix satisfying $F^T(\tau_k)F(\tau_k) \le I$ marked by F.

According to the system Eq. 2, the observer is designed as follow:

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k-\bar{d}_{ca}) + L(y(k) - \hat{y}(k)) \tag{6}$$

where, $L \in R^{n \times p}$ and $\hat{x}(k) \in R^n$ denote the state observer gain matrix and the state estimate, respectively.

Aiming at system Eq. 2, using state feedback control law based on observer as follows:

$$u(k) = K\hat{x}(k) \tag{7}$$

where, $K \in R^{l\times n}$.

In order to formulate the possible sensor failure faults, the fault model must be established first. Considering possible sensor failure faults, we can introduce a switched matrix H to the system (2) and lay the matrix M before the measure output vector y(k), where, $H = \text{diag}(h_1, h_2,...,h_m)$ and for I = 1,2,...,m

$$h_i = \begin{cases} 1 & \text{the ith sensor normal} \\ 0 & \text{the ith sensor failure} \end{cases}$$

Then the observer equation becomes:

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k-\bar{d}_{ca}) + L(Hy(k) - \hat{y}(k)) \tag{8}$$

Define the estimation error:

$$e(k) = x(k) - \hat{x}(k) \tag{9}$$

From Eq. 2, 8 and 9, we can obtain:

$$\begin{aligned} e(k+1) = (A-LC)e(k) + L(I-H)Cx(k) - \\ DFEu(k-\bar{d}_{ca}) + DFEu(k-\bar{d}_{ca}-1) \end{aligned} \tag{10}$$

Let an augmented vector:

$$w(k) = \begin{bmatrix} x^T(k) & e^T(k) & x^T(k-1) & e^T(k-1) \end{bmatrix}^T \tag{11}$$

From Eq. 2, 7 and 10, the augmented system based on observer can be represented by:

$$w(k+1) = \tilde{A}w(k) + \tilde{B}w(k-\bar{d}_{ca}) \tag{12}$$

Where:

$$\tilde{A} = \begin{bmatrix} A & 0 & 0 & 0 \\ L(I-H)C & (A-LC) & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} B & -B & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{D} = \begin{bmatrix} D \\ D \\ 0 \\ 0 \end{bmatrix}, \bar{E} = \begin{bmatrix} -E^T \\ E^T \\ E^T \\ -E^T \end{bmatrix}^T$$

Associating with the system (12), we define the following cost function:

$$J = \sum_{k=0}^{\infty} \left[x^T(k)Qx(k) + u^T(k)Ru(k) \right] \tag{13}$$

where, Q and R are given positive-definite symmetric matrices.

Definition 1: Consider the system (12), If there exist a control law based on the observer and a scalar J' such that the overall networked control systems is asymptotically stable with $H \in \Omega$ and the closed-loop value of the cost function (13) satisfies $J \le J'$, then J' is said to be a guaranteed cost and u(k) is said to be a guaranteed cost fault-tolerant controller based on the observer for the system (12).

For the convenience of notations, (*) is denoted as an ellipsis for terms that are induced by symmetry in the rest of this study.

MAIN RESULTS

Lemma 1 Zhang *et al*. (2001) If A, P and Q, are finite-dimension constant matrices, then $Q = Q^T$, $P = P^T > 0$ we have:

$$A^TPA + Q < 0 \Leftrightarrow \begin{bmatrix} Q & A^T \\ A & -P^{-1} \end{bmatrix} < 0 \text{or} \begin{bmatrix} -P^{-1} & A \\ A^T & Q \end{bmatrix} < 0$$

Lemma 2 Zhang *et al*. (2001) Given matrices W, M, N of appropriate dimensions and with W symmetric, then:

$$W + N^TF^T(k)M^T + MF(k)N < 0$$

For all F(k) satisfying $F^T(k)F(k) \le I$, if and only if there exists a scalar $\varepsilon > 0$ such that:

$$W + \varepsilon MM^T + \varepsilon^{-1}N^TN < 0$$

Lemma 3 Lu *et al*. (2007) Given vectors a, b and matrices $\bar{N}$, X, Y, Z of appropriate dimensions and with X, Z symmetric, then:

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0 \tag{14}$$

we have

$$-2a^T\bar{N}b \le \inf_{X,Y,Z} \left\{ \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y-\bar{N} \\ Y^T-\bar{N}^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right\}$$

Theorem 1: Consider the system (12), if there exist symmetry positive-definite matrices P_1, S_1, S_2, symmetry matrix X, matrices K, L, Y, P_2, P_3, such that:

$$\begin{bmatrix} \varphi_0 & -Y+P^T\begin{bmatrix}0\\ \tilde{B}\end{bmatrix} \\ * & -S_1 \end{bmatrix} < 0 \quad (15)$$

$$\begin{bmatrix} X & Y \\ Y^T & S_2 \end{bmatrix} \geq 0 \quad (16)$$

then the system (12) is asymptotically stable and the associated cost function satisfies $J \leq J'$, where:

$$J' = w^T(0)P_1w(0) + \sum_{l=-\bar{d}_{ca}}^{-1} w^T(l)S_1w(l) + \sum_{i=-\bar{d}_{ca}}^{-1}\sum_{l=i}^{-1}\delta^T(l)S_2\delta(l) \quad (17)$$

$\tilde{A}, \tilde{B}, \bar{K}, \bar{E}, \tilde{K}, \bar{I}, \bar{Q}$ have been defined above, $\delta(l) = w(l+1)-w(l)$, $\tilde{K} = [K \ \ -K \ \ 0 \ \ 0]$,

$$\varphi_0 = \bar{d}_{ca}X + \begin{bmatrix} S_1+\bar{I}\bar{Q}\bar{I}+\tilde{K}^TR\tilde{K} & 0 \\ 0 & \bar{d}_{ca}S_2+P_1 \end{bmatrix} + P^T\begin{bmatrix} 0 & I \\ \tilde{A}-I & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ \tilde{A}-I & -I \end{bmatrix}^T P + \begin{bmatrix} Y & 0 \end{bmatrix} + \begin{bmatrix} Y & 0 \end{bmatrix}^T$$

$$\bar{Q} = \text{diag}(Q,Q,Q,Q),\ \bar{I} = \text{diag}(I,0,0,0),\ P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$$

Proof: Defining the Lyapunov function:

$$V(k) = V_1(k)+V_2(k)+V_3(k)$$

where:

$$V_1(k) = w^T(k)P_1w(k)$$

$$V_2(k) = \sum_{l=-\bar{d}_{ca}}^{-1} w^T(k+l)S_1w(k+l)$$

$$V_3(k) = \sum_{i=-\bar{d}_{ca}}^{-1}\sum_{l=i}^{-1}\delta^T(k+l)S_2\delta(k+l)$$

From

$$w(k-\bar{d}_{ca}) = w(k) - \sum_{l=-\bar{d}_{ca}}^{-1}\delta(k+l)$$

we can obtain

$$(\tilde{A}+\tilde{B}-I)w(k) - \delta(k) - \tilde{B}\sum_{l=-\bar{d}_{ca}}^{-1}\delta(k+l) = 0 \quad (18)$$

The differentiate of $V_1(k)$ satisfies the relation:

$$\Delta V_1(k) = w^T(k+1)P_1w(k+1) - w^T(k)P_1w(k) = 2w^T(k)P_1\delta(k) + \delta^T(k)P_1\delta(k)$$

Let $\xi^T(k) = [w^T(k)\ \delta^T(k)]$ and from Eq. 18, we have:

$$2w^T(k)P_1\delta(k) = 2\xi^T(k)P^T\begin{bmatrix} \delta(k) \\ (\tilde{A}+\tilde{B}-I)w(k) - \delta(k) - \tilde{B}\sum_{l=-\bar{d}_{ca}}^{-1}\delta(k+l) \end{bmatrix}$$

Where:

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$$

Then, for any matrices X, Y and S_2 satisfying Eq. 14, based on Lemma 3, we can obtain:

$$-2\sum_{l=-\bar{d}_{ca}}^{-1}\xi^T(k)P^T\begin{bmatrix}0\\ \tilde{B}\end{bmatrix}\delta(k+l) \leq \bar{d}_{ca}\xi^T(k)X\xi(k) + \sum_{l=-\bar{d}_{ca}}^{-1}\delta^T(k+l)S_2\delta(k+l) \rightarrow \leftarrow +2\xi^T(k)\left\{Y - P^T\begin{bmatrix}0\\ \tilde{B}\end{bmatrix}\right\}(w(k)-w(k-\bar{d}_{ca}))$$

The differentiate of $V_2(k)$ and $V_2(k)$ satisfy the relation as follows, respectively:

$$\Delta V_2(k) = w^T(k)S_1w(k) - w^T(k-\bar{d}_{ca})S_1w(k-\bar{d}_{ca})$$

$$\Delta V_3(k) = \bar{d}_{ca}\delta^T(k)S_2\delta(k) - \sum_{l=-\bar{d}_{ca}}^{-1}\delta^T(k+l)S_2\delta(k+l)$$

Therefore:

$$\begin{aligned}\Delta V(k) &= V(k+1) - V(k) = \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) \\ &\leq \delta^T(k)P_1\delta(k) + 2\xi^T(k)P^T\begin{bmatrix} \delta(k) \\ (\tilde{A}+\tilde{B}-I)w(k) - \delta(k) \end{bmatrix} + \\ &\bar{d}_{ca}\xi^T(k)X\xi(k) + \bar{d}_{ca}\delta^T(k)S_2\delta(k) + \\ &2\xi^T(k)\left\{Y - P^T\begin{bmatrix}0\\ \tilde{B}\end{bmatrix}\right\}(w(k)-w(k-\bar{d}_{ca})) + \\ &w^T(k)S_1w(k) - w^T(k-\bar{d}_{ca})S_1w(k-\bar{d}_{ca})\end{aligned}$$

Let $\theta^T(k) = [\xi^T(k)\ \ w^T(k-\bar{d}_{ca})]$, we have:

$$\Delta V(k)=\theta^T(k)\begin{bmatrix}\hat{\varphi}_0 & -Y+P^T\begin{bmatrix}0\\ \tilde{B}\end{bmatrix}\\ 0 & -S_1\end{bmatrix}\theta(k)$$

Where

$$\hat{\varphi}_0=\varphi_0-\begin{bmatrix}\bar{I}Q\bar{I}+\tilde{K}^TR\tilde{K} & 0\\ 0 & 0\end{bmatrix}$$

From Eq. 15, we can obtain:

$$\Delta V(k)<-\theta^T(k)\begin{bmatrix}\begin{bmatrix}\bar{I}Q\bar{I}+\tilde{K}^TR\tilde{K} & 0\\ 0 & 0\end{bmatrix} & 0\\ 0 & 0\end{bmatrix} \tag{19}$$

$$\theta(k)=-[x^T(k)Qx(k)+u^T(k)Ru(k)]$$

Obviously, $\Delta V(k)<0$. That means when inequalities Eq. 15 and 16 is satisfied, the system (12) remains asymptotically stable.

Now we shall prove that cost function is less than or is equal to J', where J' is defined in Theorem 1. From Eq. 19, we have:

$$\Delta V(k)<-(x^T(k)Qx(k)+u^T(k)Ru(k)) \tag{20}$$

That means:

$$x^T(k)Qx(k)+u^T(k)Ru(k)<-\Delta V(k) \tag{21}$$

By summing the inequality Eq. 16 from 0 to ∞, then:

$$\sum_{k=0}^{\infty}(x^T(k)Qx(k)+u^T(k)Ru(k))<V(0)-\lim_{k\to\infty}V(k) \tag{22}$$

Because the system is asymptotic stability, then

$$\lim_{k\to\infty}V(k)=0$$

So, from Eq. 22, we can obtain:

$$J=\sum_{k=0}^{\infty}(\theta^T(k)Q\theta(k)+u^T(k)Ru(k))\le V(0)=w^T(0)P_1w(0)+\sum_{l=-d_{ca}}^{-1}w^T(l)S_1w(l)+\sum_{i=-d_{ca}}^{-1}\sum_{l=i}^{-1}\delta^T(l)S_2\delta(l)$$

This completes the proof of theorem 1.

It is noted that the inequalities of theorem 1 are not linear matrix inequality. In order to obtain the controller gain L and the observer gain L, the inequalities of theorem 1 can be transformed to bilinear matrix inequality, which can be solved with MATLAB and PENBMI (Kocvara and Stingl, 2004).

Theorem 2: Consider the system (12), if there exist a constant scalar $\mu>0$, a scalar $\varepsilon>0$, symmetry positive-definite matrices W_1, $\bar{S}_1$, $\bar{S}_2$, matrices K, L, W_2, W_3, W_3, X_1, X_2, X_3, such that:

$$\begin{bmatrix}\psi_{11} & \psi_{12} & 0 & 0 & \bar{d}_{ca}W_2^T & W_2^T & W_1\bar{I} & W_1\tilde{K}^T\\ * & \psi_{22} & (1-\mu)\bar{B}\bar{K}W_1 & 0 & \bar{d}_{ca}W_3^T & W_3^T & 0 & 0\\ * & * & -\bar{S}_1 & (\bar{B}\bar{K}W_1)^T & 0 & 0 & 0 & 0\\ * & * & * & -\varepsilon I & 0 & 0 & 0 & 0\\ * & * & * & * & -\bar{d}_{ca}\bar{S}_2 & 0 & 0 & 0\\ * & * & * & * & * & -W_1 & 0 & 0\\ * & * & * & * & * & * & -\bar{Q}^{-1} & 0\\ * & * & * & * & * & * & * & -R^{-1}\end{bmatrix}<0 \tag{23}$$

$$\begin{bmatrix}-X_1 & -X_2 & 0\\ * & -X_3 & -\mu\bar{B}\bar{K}W_1\\ * & * & -W_1\bar{S}_2^{-1}W_1\end{bmatrix}\le 0 \tag{24}$$

then the system (12) is asymptotically stable and the associated guaranteed cost J' satisfies (17), where $\tilde{A}$, $\tilde{B}$, $\bar{K}$, $\bar{E}$, $\tilde{K}$, $\bar{I}$, $\bar{Q}$ have been defined above,

$$\psi_{11}=\bar{d}_{ca}X_1+W_2+W_2^T+\bar{S}_1,$$
$$\psi_{12}=\bar{d}_{ca}X_2+W_1(\tilde{A}-I)^T-W_2^T+W_3+\mu(\bar{B}\bar{K}W_1)^T$$
$$\psi_{22}=\bar{d}_{ca}X_3-W_3-W_3^T+\varepsilon\bar{D}\bar{D}^T$$

Proof: From (15), we can obtain:

$$\begin{bmatrix}\varphi_0 & -Y+P^T\begin{bmatrix}0\\ \bar{B}\bar{K}\end{bmatrix}\\ * & -S_1\end{bmatrix}+\begin{bmatrix}P^T\begin{bmatrix}0\\ \bar{D}\end{bmatrix}\\ 0\end{bmatrix}F\begin{bmatrix}0 & \bar{E}\bar{K}\end{bmatrix}+\left\{\begin{bmatrix}P^T\begin{bmatrix}0\\ \bar{D}\end{bmatrix}\\ 0\end{bmatrix}F\begin{bmatrix}0 & \bar{E}\bar{K}\end{bmatrix}\right\}^T<0$$

In light of Lemma 2 and using the *Lemma* 1, the inequality Eq. 13 can be equivalently rewritten as:

$$\begin{bmatrix}\varphi_0+\varepsilon P^T\begin{bmatrix}0 & 0\\ 0 & \bar{D}\bar{D}^T\end{bmatrix}P & -Y+P^T\begin{bmatrix}0\\ \bar{B}\bar{K}\end{bmatrix} & 0\\ * & -S_1 & (\bar{E}\bar{K})^T\\ * & * & -\varepsilon I\end{bmatrix}<0 \tag{25}$$

Pre-and post-multiplying both sides of Eq. 25 with

$$\mathrm{diag}\{(P^{-1})^T, P_1^{-1}, I, I\}$$

and its transpose, we have:

$$\begin{bmatrix} (P^{-1})^T\varphi_0 P^{-1} + \varepsilon\begin{bmatrix} 0 & 0 \\ 0 & \bar{D}\bar{D}^T \end{bmatrix} & -(P^{-1})^T Y P_1^{-1} + \begin{bmatrix} 0 \\ \bar{B}\bar{K} \end{bmatrix} P_1^{-1} & 0 \\ * & -P_1^{-1}S_1P_1^{-1} & P_1^{-1}(\bar{E}\bar{K})^T \\ * & * & -\varepsilon I \end{bmatrix} < 0 \qquad (26)$$

From Chen *et al.* (2003), let

$$Y = \mu P^T \begin{bmatrix} 0 \\ \bar{B}\bar{K} \end{bmatrix}$$

where, μ is a constant scalar. For

$$P^{-1} = \begin{bmatrix} P_1^{-1} & 0 \\ -P_3^{-1}P_2P_1^{-1} & P_3^{-1} \end{bmatrix}$$

we have:

$$(P^{-1})^T\{[Y \quad 0] + [Y \quad 0]^T\}P^{-1} = \mu\begin{bmatrix} 0 \\ \bar{B}\bar{K} \end{bmatrix}[P_1^{-1} \quad 0] + \mu\left\{\begin{bmatrix} 0 \\ \bar{B}\bar{K} \end{bmatrix}[P_1^{-1} \quad 0]\right\}^T \qquad (27)$$

Let

$$W_1 = P_1^{-1},\ W_3 = P_3^{-1},\ W_2 = -P_3^{-1}P_2P_1^{-1},\ (P^{-1})^T X P^{-1} = \begin{bmatrix} X_1 & X_2 \\ X_2^T & X_3 \end{bmatrix},$$
$$\bar{S}_2 = S_2^{-1},\ \bar{S}_1 = P_1^{-1}S_1P_1^{-1}$$

and using the Lemma 1 repetitiously, from Eq. 26 and 27, we can obtain (23).

Similar to the proof process of Eq. 23, Pre-and post-multiplying both sides of Eq. 25 with

$$\mathrm{diag}\{(P^{-1})^T, P_1^{-1}\}$$

we can obtain system (24).

This completes the proof of theorem 2.

The upper bound system (17) of guaranteed cost depends on the initial values of the system (12), which is inconvenient for the parameterized design of controller and observer. In order to delete the dependence, the initial values of the system (12) are assumed arbitrarily but satisfy the following condition:

$$S = \{w(l) \in R^{4n} : w(l) = Uv_i, \quad v_i^T v_i \le 1, l = -\bar{d}_{ca}, -\bar{d}_{ca}+1, \cdots, 0\} \qquad (28)$$

where, U is given matrix.

Then finding the minimum of this upper bound can be formulated into an generalized eigenvalue minimization problem subject to linear matrix equality constraints. So, we can obtain:

$$J \le \lambda_{max}(U^T W_1^{-1} U) + \bar{d}_{ca}\lambda_{max}(U^T W_1^{-1}\bar{S}_1 W_1^{-1} U) + \frac{\bar{d}_{ca}(\bar{d}_{ca}+1)}{2}\lambda_{max}(U^T \bar{S}_2^{-1} U)$$

That means

$$\begin{bmatrix} -\alpha I & U^T \\ * & -W_1 \end{bmatrix} < 0, \begin{bmatrix} -\beta I & U^T \\ * & -W_1\bar{S}_1^{-1}W_1 \end{bmatrix} < 0, \begin{bmatrix} -\gamma I & U^T \\ * & -\bar{S}_2 \end{bmatrix} < 0$$

where, α, β and γ are scalars.

For $W_1 > 0$, we have:

$$-W_1\bar{S}_2^{-1}W_1 \le -2W_1 + \bar{S}_2,\ -W_1\bar{S}_1^{-1}W_1 \le -2W_1 + \bar{S}_1$$

Then the guaranteed cost fault-tolerant controller and observer gain and the minimization guaranteed cost can be obtained by solving the following minimization problem with MATLAB and PENBMI:

$$\min_{T,Y,\bar{S}_{d_{sc}},\bar{L}_{d_{sc}}} \alpha + \bar{d}_{ca}\beta + \frac{\bar{d}_{ca}(\bar{d}_{ca}+1)}{2}\gamma$$

$$\text{subject to} \begin{cases} \text{inequality}(23) \\ \begin{bmatrix} -X_1 & -X_2 & 0 \\ * & -X_3 & -\mu\bar{B}\bar{K}W_1 \\ * & * & -2W_1 + \bar{S}_2 \end{bmatrix} \le 0 \\ \begin{bmatrix} -\alpha I & U^T \\ * & -W_1 \end{bmatrix} < 0, \begin{bmatrix} -\beta I & U^T \\ * & -2W_1 + \bar{S}_1 \end{bmatrix} < 0, \begin{bmatrix} -\gamma I & U^T \\ * & -\bar{S}_2 \end{bmatrix} < 0 \end{cases}$$

SIMULATION EXAMPLE

Consider a system described by (1), where:

$$A_f = \begin{bmatrix} -2 & 0.1 \\ -0.8 & -1.2 \end{bmatrix}, B_f = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix}, C_f = \begin{bmatrix} 0.3 & 0.1 \\ 0.4 & 0.8 \end{bmatrix}$$

assume that the initial conditions are T = 0.1s, d = 3, $\Delta\tau_k \in [1,1.1]$, $x(0) = [0.8\ \ -0.5]^T$, Q = I, R = 0.1I, U = I, μ = 1.

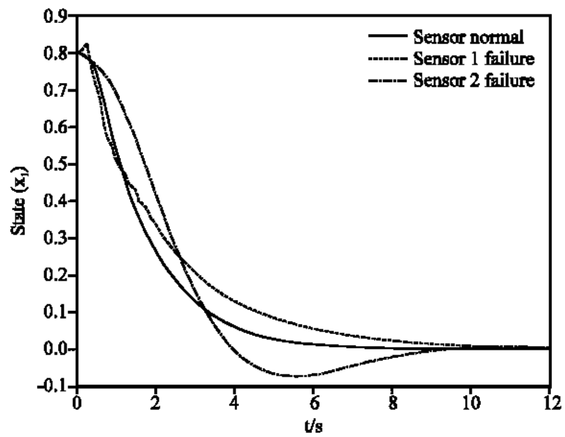

Fig. 2: Zero-input response of state x_1

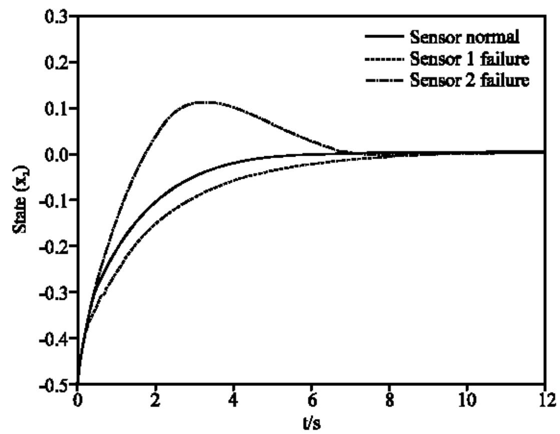

Fig. 3: Zero-input response of state x_2

From (3) and (4), based on Zhang *et al.* (2001), we have:

$$A=\begin{bmatrix}0.8184 & 0.0085\\ -0.0682 & 0.8866\end{bmatrix},\ B=\begin{bmatrix}0.0906 & 0.0095\\ -0.0036 & 0.0939\end{bmatrix}$$

$$D=\begin{bmatrix}0.3448 & 0.1100\\ 0.4040 & 0.7512\end{bmatrix},\ E=\begin{bmatrix}-1.8593 & 0.0864\\ 1.4293 & -1.0771\end{bmatrix}$$

By solving the minimization problem with MATLAB and PENBMI, we can obtain:

$$\alpha+\bar{d}_{ca}\beta+\frac{\bar{d}_{ca}(\bar{d}_{ca}+1)}{2}\gamma=1.2576\times10^3$$

$$L=\begin{bmatrix}-0.1465 & 0.1050\\ 0.2164 & 0.4372\end{bmatrix},\ K=\begin{bmatrix}-0.7427 & -1.1331\\ -0.2792 & -1.0606\end{bmatrix}$$

In cases of sensor normal and possible failures, the switched matrices L_0 = diag(1,1), L1 = diag(0, 1) and L_2 = diag(1, 0) indicate actuator normal and sensor 1, 2 failure, respectively. In the case of L_0, L_1, L_2, zero-input response of state x_1, x_2 are shown in Fig. 2 and 3. The curves of zero-input response state x_1, x_2 in Fig. 2 and 3 illustrate that the networked control systems against possible sensor failure faults is asymptotically stable. It reveals that the presented method makes the networked control systems possess integrity againstsensor failures and the minimization guaranteed cost is $J' = 1.2576\times10^3$.

CONCLUSIONS

Focusing on networked control systems with long time-delays, a control method based on state observer against sensor failures is investigated in this study. According to possible sensor failures, an augmented mathematic model for the networked control systems based on state observer is developed. Then the cooperative design approach of the controller and the observer is given and the designs for guaranteed cost fault-tolerant control law and the state observer are presented in terms of the Lyapunov stability theory combined with bilinear matrix inequalities. Compare with fault-tolerant controller design based on state feedback, the advantage of the presented fault-tolerant control method considers that the all states of the system are not convenience enough to be obtained, so it has practical significance to the application of the networked control systems.

REFERENCES

Antsaklis, P. and J. Baillieul, 2007. Special issue on technology of networked control systems. Proc. IEEE., 95: 5-8.

Bao, Y., D. Qiu-Qiu, Y.L. Cui and H.Z. Wu, 2005. Fault detection based on robust states observer on networked control systems. Proceedings of the 5th International Conference on Control and Automation, Jun. 26-29, Piscataway, New Jersey, USA., pp: 1237-1241.

Chen, W.H., Z.H. Guan and X. Lu, 2003. Delay-dependent guaranteed cost control for uncertain discrete-time systems with delay. IEE Proc. Control Theory Applied, 150: 412-416.

Guo, Y.N., Q.Y. Zhang and D.W. Gong, 2008. Robust fault-tolerant control of networked control systems with Time-varying delays. Control Decision, 23: 689-692.

Hespanha, J.P., P. Naghshtabrizi and Y. Xu, 2007. A survey of recent results in networked control systems. Proc. IEEE., 95: 138-162.

Huang, H., X.D. Han and Z.Q. Wang, 2008. Fault detection of networked control systems with time-delay and data packet dropout. J. Nanjing Univ. Sci. Technol., 32: 387-392.

Huo, Z.H. and H.J. Fang, 2006. Fault-tolerant Control of networked control system s with random time-delays. Inform. Control, 35: 584-588.

Klinkhieo, S., C. Kambhampati and R.J. Patton, 2006. Information packets and MPC enable fault-tolerance in network control. Proceedings of the IEEE Conference on Emerging Technologies and Factory Automation, Sept. 20-22, Prague, pp: 689-694.

Kocvara, M. and M. Stingl, 2004. PENBMI, version 2.0. http://www.penopt.com/.

Kong, D.M. and H.J. Fang, 2006. A class of networked control systems based on observer state feedback stabilization. Inform. Control, 35: 684-689.

Li, W., Y.J. Li and W.R. Liu, 2007. Robust fault tolerant control for networked control systems with uncertain disturbance. Proceedings of the 2007 IEEE International Conference on Mechatronics and Automation, Aug. 5-8, Harbin, China, pp: 2813-2819.

Lu, M., X.B. Wu, Z.l. Xu and W.l. Hu, 2007. Fault detection for networked control systems with long time delays of asynchronous clock. Inform. Control, 36: 386-396.

Xie, C.Y., W.H. Fan and W.L. Hu, 2009. Modeling and control method of a class of networked control systems with short time-delay. J. Nanjing Univ. Sci. Technol., 33: 156-160.

Yang, F. and H.J. Fang, 2009. Control structure design of networked control systems based on maximum allowable delay bounds. J. Franklin Inst., 346: 626-635.

Yang, F., W. Wang, Y.S. Hung and M. Gani, 2006. H8 control for networked systems with random communication delays. IEEE Trans. Automatic Control, 51: 511-518.

Zhang, W.A. and L. Yu, 2009. A robust control approach to stabilization of networked control systems with time-varying delays. Automatica, 45: 2440-2445.

Zhang, W., M.S. Branicky and S.M. Phillips, 2001. Stability of networked control systems. IEEE Control Syst. Mag., 21: 85-99.

Zhu, Z. and X. Zhou, 2006. Robust fault detection on networked control systems with uncertain long time-delay. Proc. 6th World Cong. Intel. Control Automation, 2: 5670-5674.