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Stabilization for a Class of Time-delay Discrete Bilinear System

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Abstract: In this study, a novel controller is developed for controlling a class of time-delay discrete bilinear system. Utilizing the Schur complement and some variable transformation, the stabilization condition for the time-delay discrete bilinear system is formulated by Linear Matrix Inequalities (LMIs). Finally, the validity and applicability of the proposed scheme is demonstrated by a numerical simulation.

Key words: Lyapunov function, stabilization, time-delay discrete bilinear system, linear matrix inequality

INTRODUCTION

It is known that bilinear models can describe many physical systems and dynamical processes in engineering fields (Mohler, 1973, 1991; Elliott, 1999; Sun, 2007; Gibson *et al.*, 2005). The two main advantages of the bilinear system are that it provides a better approximation to a nonlinear system than a linear one. The other is that many real physical processes may be appropriately modeled as bilinear systems when the linear models are inadequate. Due to these two advantages, the stability analysis and control design for bilinear systems have been examined by many literature (Tang *et al.*, 2005; Tsai and Li, 2009; Chiou *et al.*, 2000; Zerrik *et al.*, 2004; Li *et al.*, 2008; Chen and Chen, 2008) over the past decades.

Besides bilinear system, time-delay phenomenon commonly exists in dynamic systems due to measurement, transmission, transport lags and computational delays. Thus, stabilization of time-delay systems is increasing being attentions in many studies (Yeh *et al.*, 2008; Tsai and Li, 2009; Li and Liu, 2009; Xiang-Shun and Hua-Jing, 2009).

The main contributions of this study are (1) designing a controller for the time-delay discrete bilinear system and (2) Describing the stabilization conditions for the time-delay discrete bilinear system via LMI (Zhang and Li, 2010).

SYSTEM DESCRIPTION AND CONTROLLER DESIGN

Here, we will introduce the time-delay discrete bilinear system and then develop its controller.

Firstly, we consider a class of time-delay discrete bilinear system can be described as:

$$x(k+1) = Ax(k) + A_d x(k-\tau) + Bu(k) + Nx(k)u(k) \qquad (1)$$

where, k is iteration instant, $x(k)\in R^{n\times n}$ is the state, $u(k)\in R$ is the control input; $A\in R^{n\times n}$, $A_d\in R^{n\times n}$, $B\in R^{n\times 1}$ and $B\in R^{n\times n}$.

The controller for the time-delay bilinear system Eq. 1 is formulated as follows:

$$\begin{aligned} u(k) &= \frac{\rho Dx(k)}{\sqrt{1+x^T(k)D^TDx(k)}} \\ &= \rho\sin\theta \\ &= \rho Dx\cos\theta \end{aligned} \qquad (2)$$

Where:

$$\sin\theta = \frac{Dx(k)}{\sqrt{1+x^T(k)D^TDx(k)}},\ \cos\theta = \frac{1}{\sqrt{1+x^T(t)D^TDx(t)}}$$

$$\theta\in[-\frac{\pi}{2},\frac{\pi}{2}],\ D\in R^{1\times n}$$

is a vector to be determined and $\rho>0$ is a scalar to be assigned.

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The control objective is to design a controller Eq. 2 to stabilize the time-delay discrete bilinear system Eq. 1. One substitutes Eq. 2 into Eq. 1 and then one can get the closed-loop system:

$$x(k+1) = (A + \rho N \sin\theta + \rho BD\cos\theta)x(k) + A_d x(k-\tau) \quad (3)$$

The main result on the asymptotic stability of the time-delay discrete bilinear system is presented next. Before discussing the proof, we first give the following results which will be used in the proof of our main results.

Lemma 1: Tanaka and Wang (2001) and Chiu and Chiang (2009) given any matrices A, B and ε with appropriate dimensions such that $\varepsilon>0$ we have:

$$A^T B + AB^T \le \varepsilon A^T A + \frac{1}{\varepsilon} B^T B \quad (4)$$

MAIN RESULT

Theorem 1: If there exist a symmetric and positive definite matrix P, a scalar ρ and a vector D, such that the following LMI (5) is satisfied, then the time-delay discrete bilinear system Eq. 1 is globally asymptotically stable via the feedback controller Eq. 2, the symmetric positions:

$$\begin{bmatrix} (\rho^2+1)A^TPA+S-P & * & * & * \\ A_d^TPA & (\rho^2\varepsilon_1^{-1}+1)A_d^TPA_d - S & * & * \\ PBD & 0 & \dfrac{-P}{(\rho^2+1+\varepsilon_1)} & * \\ PN & 0 & 0 & \dfrac{-P}{(\rho^2+1+\varepsilon_1)} \end{bmatrix} < 0 \quad (5)$$

where, * denotes the transposed elements in the symmetric positions.

Proof: Now, we choose a Lyapunov function candidate for this system as follows:

$$V(x(k)) = x(k)^T Px(k) + \sum_{\sigma=k-\tau}^{k-1} x^T(\sigma)Sx(\sigma) \quad (6)$$

where, P is a constant, symmetric and positive definite matrix.

Clearly the difference of Lyapunov function is:

$$\begin{aligned} \Delta V &= V(x(k+1)) - V(x(k)) \\ &= x^T(k+1)Px(k+1) + x^T(k)(S-P)x(k) - x^T(k-\tau(k))Sx(k-\tau(k)) \end{aligned} \quad (7)$$

By substituting Eq. 3 into Eq. 7, we can get Eq. 8:

$$\begin{aligned} \Delta V &= x^T(k+1)Px(k+1) - x^T(k)Px(k) \\ &= \big((A+\rho BD\cos\theta+\rho N\sin\theta)x(k) + A_d x(k-\tau(k))\big)^T P \\ &\quad \times\big((A+\rho BD\cos\theta+\rho N\sin\theta)x(k) + A_d x(k-\tau(k))\big) \\ &\quad + x^T(k)(S-P)x(k) - x^T(k-\tau(k))Sx(k-\tau(k)) \end{aligned} \quad (8)$$

Firstly, we assume Eq. 8 is negative and utilize Lemma 1, then we can obtain Eq. 9:

$$\begin{aligned} & x^T(k)\left[(\rho^2+1)A^TPA + (\rho^2+1+\varepsilon_1)(D^TB^TPBD + N^TPN) + S - P\right]x(k) \\ & +2x^T(t)A^TPA_d x(k-\tau(k)) + x^T(k-\tau(k))((\rho^2\varepsilon_1^{-1}+1)A_d^TPA_d - S)x(k-\tau(k)) < 0 \end{aligned} \quad (9)$$

Clearly, Eq. 9 can be expressed as Eq. 10:

$$\begin{bmatrix} x(k) \\ x(k-\tau) \end{bmatrix}^T \Delta \begin{bmatrix} x(k) \\ x(k-\tau) \end{bmatrix} < 0 \quad (10)$$

$$\text{where } \Delta = \begin{bmatrix} (\rho^2+1)A^TPA+S-P + (\rho^2+1+\varepsilon_1)(D^TB^TPBD+N^TPN) & * \\ A_d^TPA & (\rho^2\varepsilon_1^{-1}+1)A_d^TPA_d - S \end{bmatrix} \quad (11)$$

Utilizing the Schur Complement formula to Eq. 11, we can obtain:

$$\begin{bmatrix} (\rho^2+1)A^TPA+S-P & * & * & * \\ A_d^TPA & (\rho^2\varepsilon_1^{-1}+1)A_d^TPA_d - S & * & * \\ BD & 0 & \dfrac{-P^{-1}}{(\rho^2+1+\varepsilon_1)} & * \\ N & 0 & 0 & \dfrac{-P^{-1}}{(\rho^2+1+\varepsilon_1)} \end{bmatrix} < 0 \quad (12)$$

Evidently, the matrix inequality Eq. 12 is not an LMI but a quadratic-matrix inequality (QMI). In order to use the convex optimization technique, Eq. 12 must be converted to an LMI via some variable transformation. For this purpose, we adopt the following transformation matrix:

$$\mathrm{diag}[I \quad I \quad P \quad P] \quad (13)$$

and take a congruence transformation (Lo and Lin, 2006; Xu and Lam, 2005) on Eq. 11. This yields Eq. 14:

$$\begin{bmatrix} (\rho^2+1)A^TPA+S-P & * & * & * \\ A_d^TPA & (\rho^2\varepsilon_1^{-1}+1)A_d^TPA_d - S & * & * \\ PBD & 0 & \dfrac{-P}{(\rho^2+1+\varepsilon_1)} & * \\ PN & 0 & 0 & \dfrac{-P}{(\rho^2+1+\varepsilon_1)} \end{bmatrix} < 0 \quad (14)$$

Therefore, if Eq. 14 is satisfied, the closed loop time-delay discrete bilinear system is asymptotically stable. This completes the proof of the theorem.

EXAMPLE

Here, the proposed method is used to design a controller for a class of time-delay discrete bilinear system. The time-delay discrete bilinear system is described as follows:

$$x(k+1) = Ax(k) + A_d x(k-\tau) + Bu(k) + Nx(k)u(k) \quad (15)$$

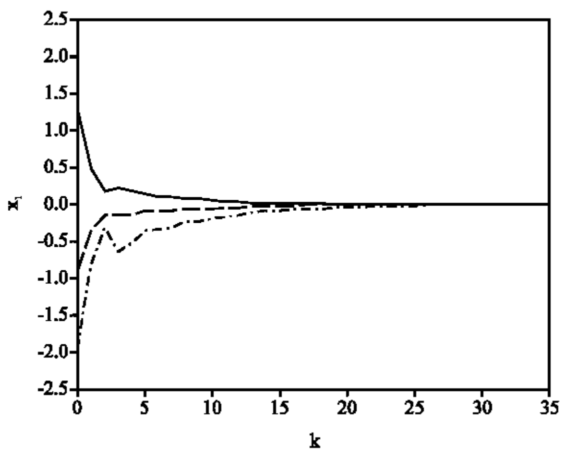

Fig. 1: State responses of x_1 under three different initial conditions ($x(0) = [1.3\ -1.5]^T$: solid line, $x(0) = [-0.9\ 1.2]^T$: dash line, $x(0) = [-2\ -0.5]^T$: dash-dot line)

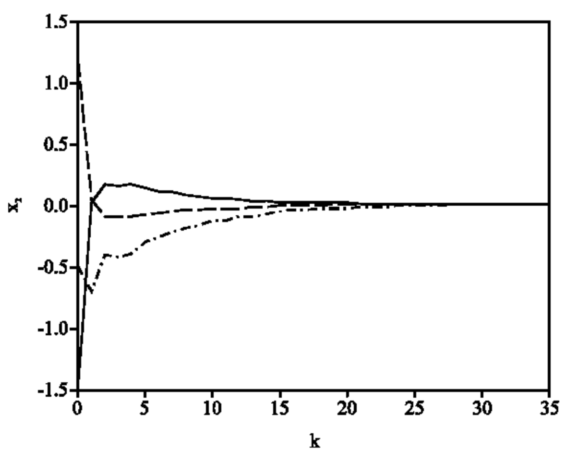

Fig. 2: State responses of x_2 under three different initial conditions ($x(0) = [1.3\ -1.5]^T$: solid line, $x(0) = [-0.9\ 1.2]^T$: dash line, $x(0) = [-2\ -0.5]^T$: dash-dot line)

Where:

$$A = \begin{bmatrix} 0.4021 & 0.0613 \\ 0.3 & 0.25 \end{bmatrix}, A = \begin{bmatrix} 0.2139 & 0.0938 \\ 0.1 & 0.2 \end{bmatrix}$$
$$B = \begin{bmatrix} 0.3 \\ -0.01 \end{bmatrix}, N = \begin{bmatrix} 0.02 & 0 \\ 0 & -0.03 \end{bmatrix}$$

and $\tau = 1$.

Let $\rho = 0.3$ and choose the controller gain matrices as:

$$D = [-0.1 \quad -0.4] \quad (16)$$

Applying ρ-value and all these given matrices to inequalities Eq. 5 mentioned in Theorem 1 and utilizing the LMI tool box (Tanaka and Wang, 2001), one can figure out the common positive-definite matrices:

$$P = \begin{bmatrix} 3.1581 & 0.1779 \\ 0.1779 & 2.8253 \end{bmatrix} \text{and } S = \begin{bmatrix} 1.2502 & -0.0313 \\ -0.0313 & 1.3813 \end{bmatrix}$$

Figure 1 and 2 show the simulation results of applying the control law (2) to the time-delay discrete bilinear system (15) under three different initial conditions $x(0) = [1.3\ -1.5]^T$, $x(0) = [-0.9\ 1.2]^T$ and $x(0) = [-2\ -0.5]^T$. Figure 1 shows state responses of x_1 and Fig. 2 shows state responses of x_2. From these two figures, one can find that the states x_1 and x_2 converge to the equilibrium state. Simulation results show that the proposed control scheme for time-delay discrete bilinear system is effective and feasible.

CONCLUSIONS

This study has presented a control scheme for the time-delay discrete bilinear system. For stabilizing the time-delay discrete bilinear system, some sufficient conditions have been derived to guarantee the stability of the overall time-delay discrete bilinear system via LMIs. Finally, a numerical simulation has been adopted to demonstrate the feasibility and effectiveness of the proposed schemes.

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