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The Extended State Particle Filter for Unknown Process Models

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Abstract: In this study, a new dynamic state space model was established by using the polynomial predictive idea and state dimension extension. We call the new model extended model, which was established without the exact knowledge of the original state dynamics, i.e., we may use the proposed extended model to describe the state dynamics no matter we know original state propagation well or not. A correspondent Extended State Particle Filter (ESPF) was then presented based on the proposed extended model. In the ESPF, the sum of the extended particle weights was applied to test whether the filter is convergent or not. Simulation results demonstrate that the proposed method still works well while the existed Particle Filter (PF) diverges in the situations that the state dynamics are not known well.

Key words: Particle filtering, Kalman filtering, state space model, simulation, tracking

INTRODUCTION

The problem of extracting data from a measured signal is important in a great number of applications, for instance, tracking and navigation. There exist many methods, among which the Kalman filter algorithm is probably the most popular one (Hernane *et al.*, 2010; Tao *et al.*, 2009; Grewal, 2001). However, in order to get the optimal solutions, the Kalman filter assumes that both the process and measurement models are linear and Gaussian, which is not always the case in the real world. In more common situations, numerical approximation methods are used, for example, the Extended Kalman Filter (EKF) (Hernane *et al.*, 2010; Grewal, 2001). The EKF firstly linearizes the dynamic state space model using a first-order truncated Taylor series expansion around the current estimates and then uses the KF. However, the linearization process of the EKF is liable to large errors threatening the convergence of the algorithm, particularly for models with high nonlinearity.

The shortcomings of the EKF have led to the independent development of a number of closely related Gaussian approximate derivativeless filters, all based on novel deterministic sampling methods for the propagation of Gaussian random variables through nonlinear systems. One of them is the Unscented Kalman Filter (UKF) (Julier and Uhlmann, 2004), which is founded on the intuition that it is easier to approximate a probability distribution than it is to approximate an arbitrary nonlinear function or transformation. The UKF consistently outperforms the EKF in terms of state estimation accuracy

and estimate consistency. Another famous filter is called the Central Difference Filter (CDF) (Ito and Xiong, 2000), which is based on the Stirling's interpolations formula.

A recently-popularized technique for numerical approximation, termed as the Particle Filter (PF) (Arulampalam *et al.*, 2002; Doucet *et al.*, 2001; Gustafsson, 2010; Ahmed *et al.*, 2010), it offers a general tool for the state estimation of nonlinear non-Gaussian systems. The core idea behind the PF is to use samples (particles) to approximate the concerned distributions. The PF has better estimation performance than the Kalman related filter in nonlinear non-Gaussian models, however, it needs much more computation.

Note that all the filters above work well only if the state transition density or dynamics is explicitly known. When the state dynamics is not known exactly, the filters may not provide correct results, sometimes even be divergent. Polynomial predictive filter is an effective way to predict future signal values (Valiiviita *et al.*, 1999; Tanskanen, 2000). Future estimates of the signal are produced by extrapolating and it turns out that the coefficients of the filter are not dependent on the signal, but only on the values assigned to the degree of polynomial model, the number of latest samples and the forward prediction step. In this study, we firstly use polynomial predictive filter to construct the process model by extending the dimension of the state, i.e., the extended state at the current time step consists the original state at the current time step and those of several backward time steps. In this way, we may obtain the extended state dynamics whether the original state transition density is

known exactly or not. Similarly we extend the measurement model. And a new dynamic state space model-the extended state space models-is established by the combination of the extended process and measurement model. Then a new particle filter, termed as the Extended State Particle Filter (ESPF), is presented based on the proposed extended model. Simulation results show that in the situations that the state dynamics are not known well, the proposed method still works well while the existed Particle Filter (PF) diverges.

BACKGROUND

Here, we recall the polynomial predictive filter (Tanskanen, 2000). Suppose that the signal x_k can be modeled as a polynomial of the degree L , then we have:

$$x_k = \sum_{l=0}^L p_l k^l \tag{1}$$

where, p_l is the polynomial coefficients. We may have:

$$x_k = \sum_{l=0}^L p_l k^l + w_k \tag{2}$$

when signal x_k does not fit the polynomial model exactly, where w_k is an error term. It turns out that predictions using polynomial models can be done by taking a weighted average of the past few values of the signal and that the weighted coefficients are not dependent on the signal, but only on the values assigned to polynomial degree, number of prediction steps and the length of the filter (Valiiviita *et al.*, 1999; Tanskanen, 2000), i.e., for exactly polynomial signals, a future value x_{k+N} can be obtained as:

$$x_{k+N} = \sum_{m=0}^{M-1} h_m x_{k-m} \tag{3}$$

where, N and M denote the number of prediction steps and the length of the filter respectively, h_m are the filter coefficients. Substituting Eq. 1 into 2 yields:

$$\sum_{l=0}^L p_l [k+N]^l = \sum_{m=0}^{M-1} h_m \sum_{l=0}^L p_l [k-m]^l \tag{4}$$

Then, we have:

$$p_l [k+N]^l = \sum_{m=0}^{M-1} h_m p_l [k-m]^l \tag{5}$$

which yields,

$$[-N]^l = \sum_{m=0}^{M-1} h_m m^l \tag{6}$$

for $l = 1, \dots, L$. We know that in the case of FIR filter, the noise gain can be expressed as (Tanskanen, 2000):

$$NG = \sum_{m=0}^{M-1} |h_m|^2 \tag{7}$$

The purpose is to minimize Eq. 7 under the constraints of Eq. 6. This is done using the method of Lagrange multipliers by considering Eq. 7 as the function to be minimized and the $L+1$ equations of Eq. 6 as the constraint functions. Then we can construct a Lagrange function as follows:

$$\Phi(h_m, \dots, h_{M-1}, \lambda_0, \dots, \lambda_L) = \sum_{m=0}^{M-1} |h_m|^2 + \sum_{l=0}^L \lambda_l \left[\sum_{m=0}^{M-1} h_m m^l - [-N]^l \right] \tag{8}$$

Then we get:

$$h_m = \sum_{l=0}^L c_l m^l \tag{9}$$

where:

$$\begin{bmatrix} S_0 & S_1 & \dots & S_L \\ S_1 & S_2 & \dots & S_{L+1} \\ \vdots & \vdots & \ddots & \vdots \\ S_L & S_{L+1} & \dots & S_{2L} \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_L \end{bmatrix} = \begin{bmatrix} (-N)^0 \\ (-N)^1 \\ \vdots \\ (-N)^L \end{bmatrix} \tag{10}$$

and

$$S_l = \sum_{m=0}^{M-1} m^l \tag{11}$$

For example, with $N = 1, L = 1, h_m$ becomes:

$$h_m = \frac{4M - 6m - 4}{M(M-1)} \tag{12}$$

with $N = 1, L = 2, h_m$ becomes

$$h_m = \frac{9M^2 + (-27 - 36m)M + 30m^2 + 42m + 18}{M^2 - 3M^2 + 2M} \tag{13}$$

with $N = 1, L = 3, h_m$ becomes:

$$h_m = \frac{16M^3 + (24 - 120m)M^2 + (240m^2 - 120m + 56)M - 140m^3 + 120m^2 - 100m + 24}{M^4 - 6M^3 + 11M^2 - 6M} \tag{14}$$

THE EXTENDED STATE PARTICLE FILTER

Extended model construction: Firstly we consider the process model. Suppose that the true model is as follows:

$$x_{k+1} = f_k(x_k) + n_k \tag{15}$$

where, n_k is the process noise and $f_k(\cdot)$ is unknown. By the knowledge of polynomial predictive filter reviewed in last section, we rewrite Eq. 15 as:

$$x_{k+1} = \sum_{m=0}^{M-1} h_m x_{k-m} + w_{k+1} \tag{16}$$

where, w_{k+1} is a noise term and it is zero if x_k is a strictly polynomial signal. Further more, Eq. 16 can be rewritten in an extended form:

$$\bar{X}_{k+1} = F\bar{X}_k + \bar{W}_{k+1} \tag{17}$$

where,

$$\bar{X}_k = [x_k, x_{k-1}, \dots, x_{k-M+1}]^T \tag{18}$$

$$F = \begin{bmatrix} h_0 & h_1 & \dots & h_{M-1} \\ 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 \end{bmatrix} \tag{19}$$

$$\bar{W}_k = [w_k, w_{k-1}, \dots, w_{k-M+1}]^T \tag{20}$$

Suppose that w_k is white Gaussian with mean zero and covariance Q and it is independent at different time steps. Then \bar{W}_k is also white Gaussian with mean zero and covariance \bar{Q} , i.e.,

$$\bar{Q} = \begin{bmatrix} Q & 0 & \dots & 0 \\ 0 & Q & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & Q \end{bmatrix}_{M \times M} \tag{21}$$

where, l is the dimension of the original state.

We now consider the correspondent measurement model. Suppose that the original measurement model is as follows:

$$y_k = g_k(x_k) + v_k \tag{22}$$

where, $h_k(\cdot)$ is known linear or nonlinear function, v_k is Gaussian white noise with mean zero and covariance R . Rewrite Eq. 21 in an extended form corresponding to Eq. 17, i.e.,

$$\bar{Y}_k = G_k(\bar{X}_k) + \bar{V}_k \tag{23}$$

where,

$$\bar{Y}_k = [y_k, y_{k-1}, \dots, y_{k-M+1}]^T \tag{24}$$

$$G_k(\bar{X}_k) = [g_k(x_k), g_{k-1}(x_{k-1}), \dots, g_{k-M+1}(x_{k-M+1})]^T \tag{25}$$

$$\bar{V}_k = [v_k, v_{k-1}, \dots, v_{k-M+1}]^T \tag{26}$$

Suppose that v_k is independent at different time steps, we have that the covariance of \bar{W}_k can be expressed as:

$$\bar{R} = \begin{bmatrix} R & 0 & \dots & 0 \\ 0 & R & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & R \end{bmatrix}_{Mm \times Mm} \tag{27}$$

where, m is the dimension of the original measurement.

Then we have the extended dynamic state space model, which is comprised of Eq. 17 and 22, where, \bar{X}_k and \bar{Y}_k are extended state and measurement, respectively, \bar{W}_k and \bar{V}_k are extended process and measurement noises, respectively.

The extended state particle recursion: We apply particle filter to Eq. 17 and 22. According to the generic particle filter (Arulampalam *et al.*, 2002) and the proposed extended dynamic state space models Eq. 17 and 22, we have the following four steps, which are similar to the particle filter and constitute the main parts of the proposed Extended State Particle Filter (ESPF).

Initialization: Initialize the extended particles according to the probability of extended initial state, i.e.,

$$\bar{X}_{10}^{(i)} \sim p(\bar{X}_0), \quad i=1, \dots, N \tag{28}$$

where, $\bar{X}_{10}^{(i)}$ is the extended particles, $p(\bar{X}_0)$ is the probability of extended initial state, N is the particle number.

Prediction: Suppose that we have the extended particles at time k , i.e., $\bar{X}_{k|k}^{(i)}$, we predict new extended particles at time $k+1$ according to:

$$\bar{X}_{k+1|k}^{(i)} \sim p(\bar{X}_{k+1} | \bar{X}_{k|k}^{(i)}), \quad i=1, \dots, N \tag{29}$$

where,

$$p(\bar{X}_{k+1|k} | \bar{X}_{k|k}^{(i)})$$

is the state transition density. From Eq. 17, we have that:

$$\bar{X}_{k+1|k}^{(i)} \sim N(F\bar{X}_{k|k}^{(i)}, \bar{Q}) \quad (30)$$

where, $N(x, \alpha, \beta)$ denotes the Gaussian distribution with mean α and covariance β , F and \bar{Q} are given in Eq. 19 and 21, respectively.

Update: Firstly evaluate the importance weights according to the measurement likelihood and normalize it, i.e.,

$$q_{k+1}^{(i)} = p(\bar{Y}_{k+1} | \bar{X}_{k+1|k}^{(i)}) \quad (31)$$

$$\tilde{q}_{k+1}^{(i)} = \frac{\tilde{q}_{k+1}^{(i)}}{\sum_{j=1}^N \tilde{q}_{k+1}^{(j)}}, \quad i = 1, \dots, N \quad (32)$$

where,

$$q_{k+1}^{(i)} \quad \text{and} \quad \tilde{q}_{k+1}^{(i)}, \quad i = 1, \dots, N$$

are the importance weights and normalized importance weights, respectively:

$$p(\bar{Y}_{k+1} | \bar{X}_{k+1|k}^{(i)})$$

is the measurement likelihood. According to Eq. 22, we have:

$$q_{k+1}^{(i)} = N(\bar{Y}_{k+1} - H_{k+1}(\bar{X}_{k+1|k}^{(i)}), \bar{R}) \quad (33)$$

where, $H_k(\cdot)$ and \bar{R} are given in Eq. 25 and 27, respectively.

Secondly resample with replacement N particles according to the normalized importance weights, i.e.,

$$p(\bar{X}_{k+1|k+1}^{(i)} | \bar{X}_{k+1|k}^{(i)}) = \tilde{q}_{k+1}^{(i)}, \quad i = 1, \dots, N \quad (34)$$

where, $\bar{X}_{k+1|k+1}^{(i)}$ denote the extended particles after resampling. Then reset the importance weights, i.e.,

$$q_{k+1}^{(i)} = \frac{1}{N}, \quad i = 1, \dots, N \quad (35)$$

Estimate: Firstly we obtain the extended state estimate according to:

$$\hat{\bar{X}}_{k+1} = \sum_{i=1}^N \tilde{q}_{k+1}^{(i)} \bar{X}_{k+1|k}^{(i)} \quad (36)$$

or

$$\hat{\bar{X}}_{k+1} = \frac{1}{N} \sum_{i=1}^N \bar{X}_{k+1|k+1}^{(i)} \quad (37)$$

Then we may obtain the original state estimate at time $k+1$, \hat{x}_{k+1} , just by reserving the first component of $\hat{\bar{X}}_{k+1}$ and discarding the others, i.e., according to Eq. 18, we may rewritten $\hat{\bar{X}}_{k+1}$ as:

$$\hat{\bar{X}}_{k+1} = [\hat{x}_{k+1}, \hat{x}_k, \dots, \hat{x}_{k-M+2}]^T \quad (38)$$

and we just need to discard $\hat{x}_k, \dots, \hat{x}_{k-M+2}$.

Summary: According to the above subsections, we summarize the proposed ESPF in algorithm 1.

Algorithm 1: The extended state particle filter

Part I: Extended model construction

- (1) Obtain the extended process model as Eq. 17-21
- (2) Obtain the extended measurement model as Eq. 23-27

Part II: extended state filtering

- (3) At time $k = 1$, initialize the extended particles according to Eq. 28

For $k = 2, 3, \dots$

- (4) Predict new extended particles according to Eq. 29 or 30
- (5) Evaluate and normalize the importance weights according to Eq. 31 and 32
- (6) Resample according to Eq. 34 and reset the importance weights according to Eq. 35
- (7) Obtain the extended state estimate according to Eq. 36 or 37

Part III: original state estimate

- (8) Obtain the original state estimate by reserving the first component of the extended state estimate and discarding the others

Implementation issue: We know from Weierstrass approximation theorem that any continuous function on a closed and bounded interval can be uniformly approximated on that interval by polynomials to any degree of accuracy. Thus, any continuous signal on a closed and bounded interval can be approximated by polynomials whether we know the signal exactly or not. Suppose that the interval is short enough, the signal during the interval can be approximated by a polynomial with low degree, i.e., the parameter L in Eq. 1 maybe small, for example, 1, 2.

In this way we just need to calculate filter coefficients h_m with low degrees according to Eq. 12-14. For example if $M = 2, N = 1, L = 1$, we obtain from Eq. 13 that:

$$\begin{aligned} h_0 &= 2 \\ h_1 &= -1 \end{aligned} \quad (39)$$

then the extended process and measurement model of Eq. 17 and 23 become:

$$\begin{bmatrix} x_{k+1} \\ x_k \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-1} \end{bmatrix} + \begin{bmatrix} w_{k+1} \\ w_k \end{bmatrix} \quad (40)$$

$$\begin{bmatrix} y_k \\ y_{k-1} \end{bmatrix} = \begin{bmatrix} g(x_k) \\ g(x_{k-1}) \end{bmatrix} + \begin{bmatrix} v_k \\ v_{k-1} \end{bmatrix} \quad (41)$$

However, there appears a problem that how to deal with the signal in the connection of different intervals. We rewrite Eq. 16 as follows:

$$x_{k+1} = \sum_{m=0}^{M-1} h_m x_{k-m} + w_{k+1} \quad (42)$$

where, w_{k+1} is a noise term with covariance Q . We may give Q a low value during each intervals of the signal, where the signal can be approximated by polynomials well. In the connection of different intervals, since the signal may not be approximated by low degree polynomials probably, we have to give Q a high value. The question is how to decide when to give a high value and when to give a low value because we do not have the prior knowledge of the signal.

In the ESPF, the importance weights are calculated according to Eq. 33, i.e.,

$$q_{k+1}^{(i)} = \frac{1}{(2\pi)^{n_x/2} (\det[\bar{R}])^{1/2}} \exp\left(-\frac{1}{2}(\bar{y}_{k+1} - H_{k+1}(\bar{x}_{k+1|k}^{(i)}))^T \bar{R}^{-1}(\bar{y}_{k+1} - H_{k+1}(\bar{x}_{k+1|k}^{(i)}))\right) \quad (43)$$

where, n_x denotes the dimension of the extended state, $(\cdot)^T$ denotes matrix transpose. Evidently, if the true process model can not be approximated by low degree polynomials well (abbreviated to NAW situation for concision), for example, in the connection of different intervals, then:

$$\bar{y}_{k+1} - H_{k+1}(\bar{x}_{k+1|k}^{(i)})$$

will be far from zero, thus the importance weights will tend to zero and the sum of all importance weights (SW) will also be negligible. On the other hand, if the true process model is approximated well by low degree polynomials (abbreviated to AW situation for concision), for example, during each intervals,

$$\bar{y}_{k+1} - H_{k+1}(\bar{x}_{k+1|k}^{(i)})$$

will be relatively small, thus the importance weights will not tend to zero and the sum of importance weights will not be negligible. So, we use the SW:

$$\Xi_k = \sum q_k^{(i)} \quad (44)$$

to decide whether Q should be evaluated a high value or not. If SW is below one threshold, say Λ , Q is evaluated a high value Q_h , otherwise a low value Q_l .

We now discuss how to decide the value of Λ . Define a distance:

$$\Gamma_{k+1}^{(i)} = \frac{1}{2}(\bar{y}_{k+1} - H_{k+1}(\bar{x}_{k+1|k}^{(i)}))^T \bar{R}^{-1}(\bar{y}_{k+1} - H_{k+1}(\bar{x}_{k+1|k}^{(i)})) \quad (45)$$

then Eq. 43 can be rewritten as:

$$q_{k+1}^{(i)} = \frac{1}{(2\pi)^{n_x/2} (\det[\bar{R}])^{1/2}} \exp(-\Gamma_{k+1}^{(i)}) \quad (46)$$

Suppose that:

$$0 < \Gamma_{l,k+1}^{(i)} \ll r_{\text{thre},k+1} < \Gamma_{h,k+1}^{(i)} \quad (47)$$

where, $\Gamma_{h,k+1}^{(i)}$ is the distance with high value, which is corresponding to the NAW situation, $r_{\text{thre},k+1}$ is the distance with low value, i.e., the AW situation, $\Gamma_{l,k+1}^{(i)}$ is the lower limit value of the distance in the NAW situation. Then in the AW situation, we have that:

$$q_{k+1}^{(i)} \gg \frac{1}{(2\pi)^{n_x/2} (\det[\bar{R}])^{1/2}} \exp(-r_{\text{thre},k+1}) \quad (48)$$

$$\Xi_{k+1} \gg \frac{N}{(2\pi)^{n_x/2} (\det[\bar{R}])^{1/2}} \exp(-r_{\text{thre},k+1}) \quad (49)$$

In the NAW situation, we have that:

$$q_{k+1}^{(i)} < \frac{1}{(2\pi)^{n_x/2} (\det[\bar{R}])^{1/2}} \exp(-r_{\text{thre},k+1}) \quad (50)$$

and

$$\Xi_{k+1} < \frac{N}{(2\pi)^{n_x/2} (\det[\bar{R}])^{1/2}} \exp(-r_{\text{thre},k+1}) \quad (51)$$

So, we define the threshold:

$$\Lambda_{k+1} = \frac{N}{(2\pi)^{n_x/2} (\det[\bar{R}])^{1/2}} \exp(-r_{\text{thre},k+1}) \quad (52)$$

Then, we have:

$$Q = \begin{cases} Q_l, & \Xi_{k+1} > \Lambda_{k+1} \\ Q_h, & \text{else} \end{cases} \quad (53)$$

SIMULATION RESULTS IN TARGET TRACKING

To demonstrate the versatility of the proposed ESPF on nonlinear problems, consider a bearing and range tracking application. The measurement model is as follows (Vo *et al.*, 2005, 2009):

$$z_k = \begin{bmatrix} \theta_k \\ r_k \end{bmatrix} = \begin{bmatrix} \arctan\left(\frac{x_{k,1}}{x_{k,3}}\right) \\ \sqrt{(x_{k,1})^2 + (x_{k,3})^2} \end{bmatrix} + \begin{bmatrix} v_{k,1} \\ v_{k,2} \end{bmatrix} \quad (54)$$

where, θ_k and r_k denote the bearing and range measurements respectively at time k ; $x_{k,1}$ and $x_{k,3}$ denote the x and y position, respectively; $x_{k,2}$ and $x_{k,4}$ denote the x and y velocity. v_k is the Gaussian measurement noise and

$$v_k \sim N(0, R) \quad (55)$$

$$R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}$$

Assume that we do not know the target dynamics, this is particularly true in applications such as maneuver target tracking. As in the previous discussion, we construct the extended process model according to:

$$\bar{x}_{k+1} = F\bar{x}_k + \bar{w}_{k+1} \quad (56)$$

Suppose, that the parameter we use is $N=1$, $L=2$, $M=3$, then according to Eq. 14, we obtain:

$$\begin{aligned} h_0 &= 3 \\ h_1 &= -3 \\ h_2 &= 1 \end{aligned} \quad (57)$$

thus:

$$\bar{x}_k = \begin{bmatrix} x_k \\ x_{k-1} \\ x_{k-2} \end{bmatrix} \quad (58)$$

$$F = \begin{bmatrix} 3 & -3 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (59)$$

$$\bar{w}_{k+1} = \begin{bmatrix} w_{k+1} \\ w_k \\ w_{k-1} \end{bmatrix} \quad (60)$$

Thus according to Eq. 21 and 53, the covariance of the extended process noise is:

$$\bar{Q} = \begin{bmatrix} Q & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & Q \end{bmatrix} \quad (61)$$

where:

$$Q = \begin{cases} Q_1, & \Xi_{k+1} > \Lambda_{k+1} \\ \gamma Q_1, & \text{else} \end{cases} \quad (62)$$

Ξ_{k+1} and Λ_{k+1} are given in Eq. 44 and 52, respectively. Then the extended measurement model corresponding to Eq. 54 becomes:

$$\bar{z}_k = G_k(\bar{x}_k) + \bar{v}_k \quad (63)$$

where,

$$\bar{z}_k = \begin{bmatrix} z_k \\ z_{k-1} \\ z_{k-2} \end{bmatrix} \quad (64)$$

$$G_k(\bar{x}_k) = \begin{bmatrix} \arctan\left(\frac{x_{k,1}}{x_{k,3}}\right) \\ \sqrt{(x_{k,1})^2 + (x_{k,3})^2} \\ \arctan\left(\frac{x_{k-1,1}}{x_{k-1,3}}\right) \\ \sqrt{(x_{k-1,1})^2 + (x_{k-1,3})^2} \\ \arctan\left(\frac{x_{k-2,1}}{x_{k-2,3}}\right) \\ \sqrt{(x_{k-2,1})^2 + (x_{k-2,3})^2} \end{bmatrix} \quad (65)$$

$$\bar{v}_k = [v_{k,1} \ v_{k,2} \ v_{k-1,1} \ v_{k-1,2} \ v_{k-2,1} \ v_{k-2,2}] \quad (66)$$

We consider two scenarios in simulation.

The White-Noise Acceleration (WNA) model: Suppose that the true process model is the WNA model as follows (Vo *et al.*, 2009; Panta *et al.*, 2009; Zhang *et al.*, 2010).

$$x_k = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} x_{k-1} + \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \end{bmatrix} n_k \quad (67)$$

where, T denotes the sampling interval, n_k is the process noise and

$$w_k \sim N(0, Q) \quad (68)$$

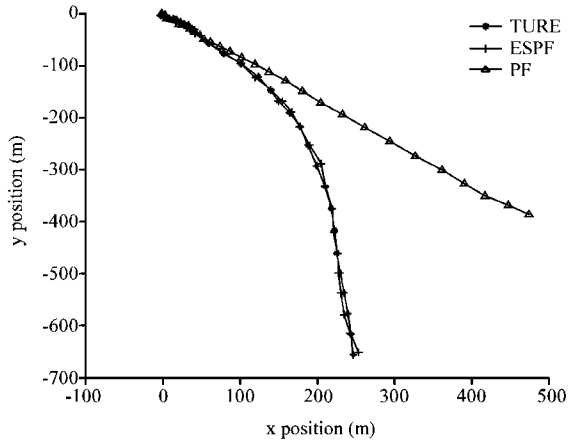


Fig. 1: Target tracks in x-y plane

$$Q = \begin{cases} \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_1^2 \end{bmatrix}, & k \leq k_1 \\ \begin{bmatrix} \sigma_2^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}, & k_1 < k \leq k_2 \\ \begin{bmatrix} \sigma_3^2 & 0 \\ 0 & \sigma_3^2 \end{bmatrix}, & k > k_2 \end{cases} \quad (69)$$

which means the covariance of process noise is different in various time intervals. We use both the proposed ESPF and the existed PF in the following situation: since we do not know the covariance of process noise well, we just use a simple description, i.e.,

$$\Phi = \begin{bmatrix} \sigma_p^2 & 0 \\ 0 & \sigma_p^2 \end{bmatrix} \quad (70)$$

The following data are used in simulations, $T = 1$, $R_1 = 0.05^2$, $R_2 = 6^2$,

$$Q_1 = \begin{bmatrix} 0.1^2 & 0 \\ 0 & 0.1^2 \end{bmatrix},$$

$\gamma = 300$, $\Lambda_{k+1} = 10^{-20}$, $\sigma_1 = 0.3$, $\sigma_2 = 9$, $\sigma_3 = 3$, $\sigma_p = \sigma_1$, $k_1 = 30$, $k_2 = 60$. The particle number is 300 in both the ESPF and the PF.

Figure 1 shows the target trajectories in x-y plane, where solid line with asterisks represents the true tracks, the solid lines with plus signs and triangles represent the ESPF and PF estimates respectively. The absolute errors of both estimates in x and y coordinates are given in Fig. 2 separately, where the solid lines with plus signs are for the results of the ESPF and those with triangles are the results of the PF. It can be seen from Fig. 1 and 2 that the proposed ESPF is capable of providing accurate tracking performances while the PF is divergent.

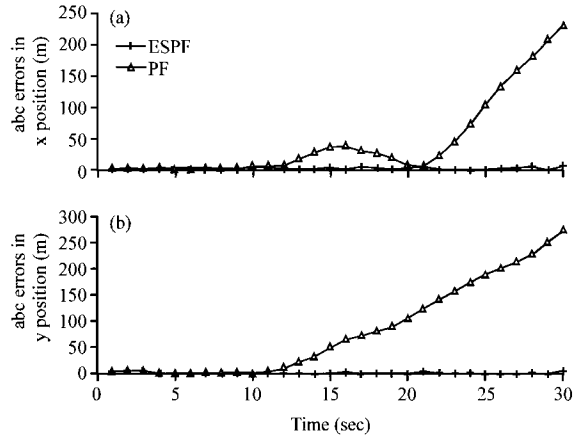


Fig. 2: Absolute errors in x and y coordinates

Mixed WNA and turn model: Suppose that the true Process Model (PM) is the mixture of the WNA model and the turn model as follows:

$$PM = \begin{cases} \text{WNA}, & k \leq k_1 \\ \text{turn} & k > k_1 \end{cases} \quad (71)$$

i.e., it's the WNA model before time k_1 and the turn model else. The WMA model is given in Eq. 67 and the turn model is as follows (Vo *et al.*, 2009).

$$\begin{aligned} y_k &= F(\omega_{k-1})y_{k-1} + Gv_k \\ \omega_k &= \omega_{k-1} + Tu_{k-1} \end{aligned} \quad (72)$$

where, the target state takes the form:

$$x_k = [y_k^T \ \omega_k^T]^T \text{ and } y_k = [p_{x,k}, \dot{p}_{x,k}, p_{y,k}, \dot{p}_{y,k}]^T$$

consisting of position $(p_{x,k}, p_{y,k})$ and velocity $(\dot{p}_{x,k}, \dot{p}_{y,k})$. Also,

$$F(\omega) = \begin{bmatrix} 1 & \frac{\sin \omega T}{\omega} & 0 & -\frac{1 - \cos \omega T}{\omega} \\ 0 & \cos \omega T & 0 & -\sin \omega T \\ 0 & \frac{1 - \cos \omega T}{\omega} & 1 & \frac{\sin \omega T}{\omega} \\ 0 & \sin \omega T & 0 & \cos \omega T \end{bmatrix} \quad (73)$$

$$G = \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \end{bmatrix} \quad (74)$$

$$v_k \sim N(\cdot; 0, \sigma_v^2) \quad (75)$$

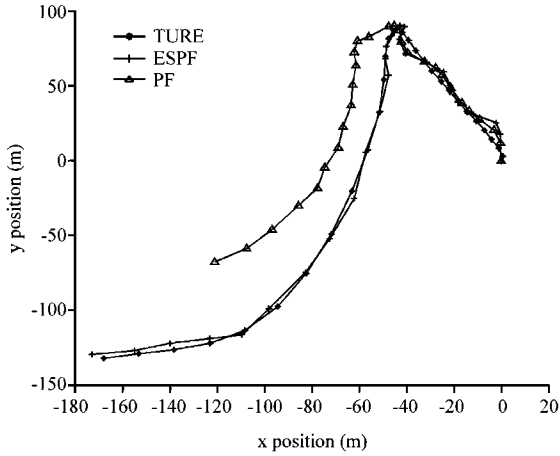


Fig. 3: Target tracks in x-y plane

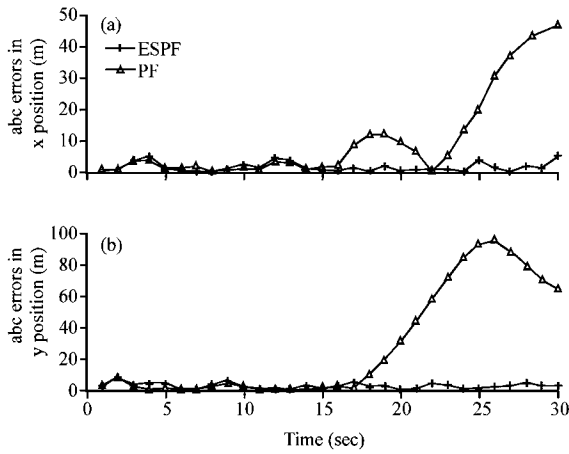


Fig. 4: Absolute errors in x and y coordinates

$$u_k \sim N(\cdot; 0, \sigma_u^2) \quad (76)$$

Similar to earlier, we use both the proposed ESPF and the existed PF in the following situation: since we do not know the model well, we only use the same model described earlier.

The following data are used in simulations,

$$\sigma_v^2 = \begin{bmatrix} 1^2 & 0; & 0 & 1^2 \end{bmatrix}, \sigma_w^2 = (\pi/180)^2, k_1 = 10$$

the other parameters are the same as described earlier.

Similar to Fig. 1-4, respectively illustrate the track results in x-y plane and the absolute errors separately in x and y coordinates, where the solid lines with plus signs and triangles represent the ESPF and PF results, respectively and solid line with asterisks represents the true tracks. Also, we may see obviously from

Fig. 3 and 4 that the proposed ESPF can still provide accurate tracking performances while the PF can not.

CONCLUSIONS

We proposed a new dynamic state space model and a new filtering method called the extended state space model and the Extended State Particle Filter (ESPF) respectively, where the system state dimension is firstly extended by the combination of the original state at current time step and those of backward time steps and finally reduced by wiping off the components corresponding to the backward time steps. By using the ESPF we may get reasonable accuracy of the estimate even in the situation that the state dynamics is not known well. The simulation results demonstrate the good performance of the proposed ESPF when the state dynamics is not known well.

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