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ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSI*net*

Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Outage Capacity Region of MIMO BC Channel under Long-term Average Power Constraint

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Abstract: In this study, we derive the Outage capacity region of Multiple-Input and Multiple-Output (MIMO) Broadcasting Channel (BC) under long-term average power constraint. Assuming channel state information known both at transmitter and receivers, we use the concept of equivalent channel and water-filling method to obtain the minimum required power for given rate and given channel state. In optimal power allocation strategy, we apply the ratio of reward to the minimum required power as criterion and give a threshold. Afterwards, we give a tail power allocation strategy, which together with the threshold algorithm guarantees the maximal reward of the long-term power.

Key words: MIMO BC channel, outage capacity region, power allocation, threshold algorithm, water-filling

INTRODUCTION

By applying iterative algorithm, the Shannon capacity of MIMO BC channel is obtained (Jindal *et al.*, 2005), which is a measure of long-term achievable rate averaged over the fading channel. In this study we try to find the capacity region of MIMO BC when certain outage are allowed. The minimum outage capacity is solved for the singleuser fading channel (Caire *et al.*, 1999). The BC outage capacity region of single antenna user is derived. There is a principle in both study: The optimal power allocation is decided through a threshold rule according to the channel state information: If the decision variable is more than or less than the threshold, then the transmission is turned on or turned off. The following equivalence principle is also used (Li and Goldsmith, 2001). Finding the optimal power allocation strategy that achieves the boundary of the outage capacity region for given outage probability is equivalent to deriving the strategy that minimizes the outage probability for given rate vector, which is applicable to MIMO BC too. It's proved that the Dirty-Subspace Coding (DPC) is capacity achieving for MIMO BC (Weingarten *et al.*, 2004). We apply DPC when coding the transmitted information. Because the problem of seeking the minimum required power for MIMO BC is concave, we use the duality between MIMO Multiple Access Channels (MAC) and MIMO BC so as to solve it from the aspect of MIMO MAC. For a certain DPC order, we introduce the concept of equivalent channel and use water-filling strategy to obtain the minimum required power. We enumerate all coding orders and compare the minimum required powers to get the minimum required power for a certain coding order. Finally, we take the ratio of the reward to the minimum required power as decision variable RPR to

select which user combination and which order will be used in every channel state and then give a threshold algorithm. The transmission to user combination whose RPR is larger than and most close to this threshold would be turned on and off otherwise. Furthermore, we present a tail power (that is, the difference between the long-term power and the sum of power allocated to every channel state) allocation algorithm.

CHANNEL MODEL

We consider a MIMO BC consisting of a transmitter with n_t antennas and K users each with n_r antennas. We assume that the frequency response of channel on the signal bandwidth is constant, that is, the channel is flat fading. We use the discrete time equivalent model:

$$y_i = H_i x + n_i; i = 1, 2, \dots, K \quad (1)$$

where, y_i is the received signal for user i , $H_i \in \mathbb{C}^{n_r \times n_t}$ represents the channel of user i . $z_i \in \mathbb{C}^{n_r \times 1}$ is additive white circular Gaussian noise with zero mean. $x \in \mathbb{C}^{n_t \times 1}$ is the transmitted signal. \bar{p} is long-term average power.

DPC CODING AND MIMO MAC-MIMO BC DUALITY

An achievable set of rates for MIMO BC channel based on DPC is developed (Caire and Shamai, 2003). DPC allows a channel with interference known at the transmitter to achieve the same data rate as if the interference does not exist. Without loss of generality, assume user K coded first and the coding order is $K, K-1, \dots, 1$. Select a codeword for user K first. When selecting codeword for user $K-1$, with full knowledge of user K , we do not see codeword for user K as

interference. It's the same that user K-2 can void all the interference from user K and user K-1. This process continues for all K users and then we have the following rate R(i) for user I:

$$R_i^{bc} = \frac{1}{2} \log \frac{|I + H_i (\sum_{j \neq i} S_j) H_i^T|}{|I + H_i (\sum_{j \neq i} S_j) H_i^T|} \quad (2)$$

where, S_j is transmission correlation matrix. When given aim rate vector, we need to find the optimal transmission correlation matrix so as to minimize the required power.

While the above problem is not convex, we introduce duality between MIMO-BC and MIMO MAC. According to theorem (Caire and Shamai, 2003), the dirty coding region of a MIMO BC channel with power constraint 'p is equal to the capacity region of the dual MIMO MAC with sum power constraint \bar{p} :

$$C_{bc}(\bar{p}, H) = C_{mac}(\bar{p}, H^T) \quad (3)$$

The dual MAC channel is Hy and the dual decoding order is 1, 2, ..., K, so the problem becomes:

$$\begin{aligned} \min_{Q_i} \sum_{k=1}^K \text{tr}(Q_k) \\ \text{s.t. } \frac{1}{2} \log |I + \frac{H_i^T Q_i H_i}{I + \sum_{m=i+1}^K H_m^T Q_m H_m}| = R_i \end{aligned} \quad (4)$$

where, Q_i is the MAC transmission correlation matrix we will find.

EQUIVALENT CHANNEL AND WATERFILLING METHOD

Because $I + \sum_{m=i+1}^K H_m^T Q_m H_m$ is conjugate and symmetrical, without loss of generality, we can write Eq. 4 as:

$$\frac{1}{2} \log |I + (\sum_{m=i+1}^K H_m^T Q_m H_m)^{-1} H_i^T Q_i H_i (\sum_{m=i+1}^K H_m^T Q_m H_m)^{-1}| = R_i \quad (5)$$

We can see the equivalent channel as:

$$H_i (\sum_{m=i+1}^K H_m^T Q_m H_m)^{-1/2}$$

then find the minimum required power to support given rate vector using waterfilling method.

When channel state information is known at both transmitter and receiver, the channel capacity using waterfilling method (Chen *et al.*, 2002):

$$C_n = \sum_{i=1}^n \log(\lambda_i \mu^{\dagger}) \quad (6)$$

where, μ satisfies:

$$\sum_i (\mu - \frac{1}{\lambda_i})^{\dagger} = \bar{p} \quad (7)$$

and

$$HH^T = U \text{diag}(\lambda_1 \dots \lambda_n) U^T$$

where, λ_i are the eigenvalues of HH^T and U is the eigenvector vector matrix. For the channel with nonzero eigenvalues equal to n, after rate r is given, we should judge how many sub-channels would be used. If the first k subchannels that correspond to smaller eigenvalues are used while the other n-k subchannels are not used, the channel capacity is:

$$\sum_{i=1}^k \log(\frac{\lambda_i}{\lambda_k})$$

then the first k subchannels would be used. If:

$$\sum_{i=1}^{k-1} \log(\frac{\lambda_i}{\lambda_k}) \leq r \leq \sum_{i=1}^k \log(\frac{\lambda_i}{\lambda_k})$$

all subchannels would be used. So we can find how many subchannels would be used firstly. After k is found, we can use Eq. 6 to get:

$$r \geq \sum_{i=1}^k \log(\frac{\lambda_i}{\lambda_n}) \quad (8)$$

and the minimum required power is:

$$p = k \cdot \mu - \sum_{i=1}^k \frac{1}{\lambda_i} \quad (9)$$

and the corresponding transmission correlation matrix is:

$$Q = U \text{diag}([\mu - \frac{1}{\lambda_1} \mu - \frac{1}{\lambda_2} \dots \mu - \frac{1}{\lambda_k} \underbrace{0 \dots 0}_{n-k}]) U^T \quad (10)$$

OUTAGE CAPACITY REGION

Defination and optimization object: For outage probability $Pr = [Pr_1, Pr_2, \dots, Pr_K]$, under the requirement that the outage probability of user i is less than Pr_i, the set of rate vectors that long-term average power 'p can maintain is defined as Outage capacity regio $C_{out}(\bar{p}, Pr)$ n.

Assuming that the transmission to all users is turned on or turned off simultaneously and the maximum permitted outage probability is p_r , the set of rate vectors that μ can maintain is defined as common outage probability capacity region $C_{\text{comm}}(\bar{p}, p_r)$. At this time, the system is degraded to a single user system.

If the strategy that achieves the boundary of outage capacity region is found, we would get outage capacity region. For a certain rate vector R , define the set of outage probability vectors given μ can sustain as outage probability vector region $\Psi(\bar{p}, R)$. Then finding the strategy that reaches the boundary of $C_{\text{out}}(\bar{p}, p_r)$ is equivalent to finding the strategy that reaches the boundary of $\Psi(\bar{p}, R)$. Because $\forall R$, if $\text{Pr} \in \Psi(\bar{p}, R)$, then it's sure that $R \in C_{\text{out}}(\bar{p}, p_r)$ and $R \notin C_{\text{out}}(\bar{p}, p_r)$ otherwise.

We continue introducing the concept of usage probability vector region, which is the complementary region of the outage probability vector region, as defined by Li and Goldsmith (2001). So, finding the boundary of $\Psi(\bar{p}, R)$ is equal to finding the boundary of $\bar{\Psi}(\bar{p}, R)$. We can easily prove that it is convex using time-sharing technique. Define usage reward:

$$W(R) = \sum_{i=1}^K \mu_i P_i^{\text{out}}(R) \tag{11}$$

Where,

$$\sum_{i=1}^K \mu_i = 1$$

Our aim is maximizing usage reward under the long-term average power constraint.

Threshold algorithm: Define the ratio of the reward to the minimum required power as Reward Power Ratio RPR. When the channel state is known at both ends, optimization concludes: 1. How much power should be allocated to every state? 2. To which user combination and in what DPC order the transmission should be performed in every state? We use RPR as decision variable. There are $N = 2K + 1$ possible user combinations for K user system. For a certain combination consist of i users, assuming the usage reward of user n is μ_n , when the transmission to this combination is turned on, the whole usage reward is:

$$\sum_{n=1}^i \mu_n$$

There are $i!$ DPC coding order for this combination, with minimum required power p_{pk} for each order in a certain channel state. here, $k = 1, \dots, i!$

Then, $\min p_{pk}$ is the minimum required power of this combination and the corresponding order is the selected DPC order.

In given channel state, we first select permutation $\pi(\bullet)$ so that the usage reward:

$$0 \leq \mu\pi(1) \leq \mu\pi(2) \leq \dots \leq \mu\pi(N)$$

Write the minimum required power of combination $\pi(i)$ as:

$$P_{\pi(i)}^{\text{min}}(R, H)$$

Without loss of generality, let:

$$\begin{aligned} \omega_i &= \mu_{\pi(i)} \\ p_i &= P_{\pi(i)}^{\text{min}}(R, H) \end{aligned} \tag{12}$$

where, $1 \leq i \leq N$.

Preparing: Before allocating power, average the minimum required power of every channel state. if the mean value is less than given long-term power μ , then the outage probability is zero. If not, for every channel state:

Note the set of N user combinations as:

Step 1: Find from-the user combination $\mu(1)$ so as to

$$\frac{\omega_{q(1)}}{P_{q(1)}} = \max_{m \in \Omega} \left(\frac{\omega_m}{P_m} \right) \tag{13}$$

Let $z1(H) = \mu(1)/p\mu(1)$. If the power that can be allocated to state H is more than $p\mu(1)$, we consider user combination $\mu(1)$ firstly, because it can bring $p\mu(1)$ most reward.

Delete $\mu(1)$ from-firstly. As for the rest combination, if:

$$j < \mu(1); p_j > p\mu(1)$$

for user combination j , delete it from-, because comparing with $p\mu(1)$, it consume more power while bringing less reward.

If:

$$j < \mu(1); p_j < p\mu(1); j/p_j < z1(H)$$

transfer it from-into label 1, because if the power that can be allocated to state H is less than $\mu(1)$, we would have to compromise and select one combination bringing less reward while consuming less power.

Step 2: if the allocated power is more than $p\mu(1)$, we continue to seek such a combination from changed-that brings additional power most reward. Assume:

$$\theta(2) = \arg \dots \max_{m \in \Omega} \left(\frac{\omega_m - \omega_{q(1)}}{P_m - P_{q(1)}} \right)$$

that is, $\mu(2)$ brings $p\mu(2)$ i $p\mu(1)$ the largest possible reward.

Let:

$$z_2(H) = \frac{\omega_{\alpha(2)} - \omega_{\alpha(1)}}{P_{\alpha(2)} - P_{\alpha(1)}} \quad (14)$$

where, $z_2(H) < z_1(H)$.

Delete $\mu(2)$ from changed-firstly. As for the rest combinations nr, if:

$$\omega_{nr} - \omega_{\alpha(1)} < \omega_{\alpha(2)} - \omega_{\alpha(1)} \cdot P_{nr} - P_{\alpha(1)} < P_{\alpha(2)} - P_{\alpha(1)}$$

for combination nr, delete combination nr from changed Ω . If:

$$\omega_{nr} - \omega_{\alpha(1)} < \omega_{\alpha(2)} - \omega_{\alpha(1)} \cdot P_{nr} - P_{\alpha(1)} < P_{\alpha(2)} - P_{\alpha(1)}$$

and:

$$\frac{\omega_{nr} - \omega_{\alpha(1)}}{P_{nr} - P_{\alpha(1)}} < z_2(H)$$

then transfer it from into label 2 of state H. The reason is the same as above. If the power that can be allocated to state is less $p_{\theta(2)} - p_{\theta(1)}$ than after this channel state is allocated $p_{\theta(1)}$, we have to compromise and select one combination that bring less reward while consuming less power.

Step 3: if the number of remaining sets is more than 1, then continue; and stop otherwise. After m_0 iterative, we can get:

$$\begin{aligned} \omega_{\theta(1)}(H) &\leq \omega_{\theta(2)}(H) \leq \dots \leq \omega_{\theta(m_0)}(H) \\ P_{\theta(1)}(H) &\leq P_{\theta(2)}(H) \leq \dots \leq P_{\theta(m_0)}(H) \\ z_{\theta(1)}(H) &\leq z_{\theta(2)}(H) \leq \dots \leq z_{\theta(m_0)}(H) \end{aligned}$$

where,

$$z_{\theta(j)}(H) = \begin{cases} \frac{\omega_{\theta(j)}}{P_{\theta(j)}} \dots \dots \dots j=1 \\ \frac{\omega_{\theta(j)} - \omega_{\theta(j-1)}}{P_{\theta(j)} - P_{\theta(j-1)}} \dots \dots \dots 1 < j \leq m_0 \end{cases}$$

After afore iterative algorithm, our aim is to compute how much power should be allocated to every channel state according to the realistic given long-term average power. We compute the threshold of RPR.

Let:

$$z_{\theta(m_0+1)}(H) = 0, z_{\theta(0)}(H) = N_{lim}$$

where, N_{lim} is a constant far larger than $z\mu(1)(H)$.

If:

$$z_{\theta(0)}(H) > s > z_{\theta(j+1)}(H) \quad (15)$$

then the transmission to user combination $\theta(j)$ would be turned on in channel state H and the allocated power to channel state H is $p_{\theta(j)}(H)$.

Seeking s through dichotomy: Assume the number of long-term channel state is N. Set a relative large upper bound sup and take 0 as low bound sdown.

let:

$$s = (\text{sup} + \text{sdown})/2$$

Step 1: For state H, if Eq. 15 is satisfied, the transmission to user combination $\theta(j)$ is selected to been turned on and the allocated power is $p_{\theta(j)}(H)$.

If:

$$\frac{1}{N} \sum_H p_{\theta(j)}(H) > \bar{p} \quad (16)$$

then current s is larger than aim threshold and given long-term average power cannot sustain such strategy. So let $\text{sup} = s$; and $\text{sdown} = s$ otherwise, that is, given power is not used completely. Compute s again and continue.

Step 2: If:

$$1 - \frac{\sum_H p_{\theta(j)}(H)}{N_F}$$

arrives to given precision, then stop; and continue otherwise. Note that when dichotomy ends, it should be ensured that:

$$\sum_H p_{\theta(j)}(H) < N_F$$

Tail power allocation algorithm: Tail power is the difference between long-term sum power and the sum of power allocated to every state:

$$N_F - \sum_H p_{\theta(j)}(H)$$

Tail power allocation algorithm is importantly complementary to threshold algorithm. For example, assume that tail power is pdf, the threshold is s , channel state is H_1, H_2 and that after threshold algorithm we have:

$$\begin{aligned} z_{\theta(j)}(H_1) &> s \geq z_{\theta(j+1)}(H_1) \\ z_{\theta(j)}(H_2) &> s \geq z_{\theta(j+1)}(H_2) \\ z_{\theta(j+1)}(H_1) &> z_{\theta(j+1)}(H_2) \\ P_{\theta(j+1)}(H_1) - P_{\theta(j)}(H_1) &> P_{\theta(j+1)}(H_2) - z_{\theta(j)}(H_2) \end{aligned}$$

At this time, threshold algorithm cannot make s less than $z_{\theta(j+1)}(H_1)$. But obviously, if the transmission to user combination $\theta(j+1)$ is turned on in channel state H_2 , we can get reward for tail power. For another example, assume that the usage reward of user m is far more than that of the others and that sub-rate for user m is far more than the others too for given R , so as that the minimum required power for every user combination consist of user m is more than given long-term sum power.

The following algorithm makes the power allocation strategy optimal.

Assume that we get threshold s from the threshold algorithm and:

$$z_{\theta_H(j)}(H) > s \geq z_{\theta_H(j+1)}(H)$$

for every channel state. Here adding subscript is to emphasize that these data depend on state H . Note the set of channel states included into one operation as Φ and note label $j + 1$ of state H as Lb_H . If $z_{\theta_H(j)} = 0$, Lb_H would be null.

Step 1: For every channel state, delete from Lb_H user combination uc , for which $P_{uc}(H) - P_{\theta(j)} > pdf$. And select $\zeta_{H_1}(1)$ so as to:

$$\zeta_{H_1}(1) = \arg \max_{\substack{H \in \Phi \\ m_H \in Lb_H}} \frac{\omega_m(H) - \omega_{\theta_H(j)}(H)}{P_m(H) - P_{\theta_H(j)}(H)} \quad m \in Lb_H$$

Let:

$$z_{\zeta_{H_1}(1)} = \frac{\omega_{\zeta_{H_1}(1)}(H_1) - \omega_{\theta_{H_1}(1)}(H_1)}{P_{\zeta_{H_1}(1)}(H_1) - P_{\theta_{H_1}(1)}(H_1)} \quad (17)$$

And then:

- Change tail power from pdf to:

$$P_{\text{tar}} - (P_{\theta_{H_1}(1)}(H_1) - P_{\theta_{H_1}(1)}(H_1))$$

- Change selected user combination of H_1 as $\zeta_{H_1}(1)$. Now, we allocate power $P_{\zeta_{H_1}(1)}(H_1)$ to channel state H_1

- Let:

$$P_{\mu_{H_1(j)}}(H_1) = P^3_{H_1}(1)(H_1)$$

So as to facilitate the following iterative. Afterwards, delete from set Lb_{H_1} user combination $\zeta_{H_1}(1)$ and user combination nr for which:

$$(P_{nr}(H_1) - P_{\theta_{H_1}(1)}(H_1)) > P_{\text{tar}}$$

or:

$$(P_{nr}(H_1) - P_{\theta_{H_1}(1)}(H_1)) < 0$$

Step 2: Judge if:

$$P_{\text{tar}} < \max_{\substack{H \in \Phi \\ m_H \in Lb_H}} (P_{m_H}(H) - P_{\theta_H(j)}(H))$$

is right or not. Continue according to step 1 if not. And stop otherwise, which means that tail power is enough small and can bring no reward. Now, we get the optimal strategy. C. The argument of optimization

Assume after threshold algorithm, we allocate power $P_{\theta_H(j)}$ to state H . We can think like that, because $z_1(H)$ is the largest in state H , user combination $\theta(1)$ bring power $P_{\theta_H(j)}$ the largest reward and on this base, from the maximum of $z_2(H)$, power $(P_{\theta(2)}(H) - P_{\theta(1)}(H))$ get the largest reward possible and so on. It's the same for other channel states. Thus, threshold algorithm brings the largest reward for power:

$$\sum_H P_{\theta_H(j)}(H)$$

As we see, tail power is not used in threshold algorithm, so in tail power allocation algorithm, we allocate it further. Assume that $m1$ iterative performed before it stop. The maximum of:

$$z^3_{H_1}(1), z^3_{H_2}(2), \dots, z^{3Hm1}(m1)$$

promise the largest reward after threshold algorithm. It's shown that this two algorithms together bring long-term average power P the largest reward and they are optimal.

SIMULATION RESULTS

Simulation results are presented here. In simulation, the total average power in the figures is demoted as p . If no specifying, the number of antenna at both ends is 2. We assume that the noise variance of user 1 is 1 and that of user 2 is 2.

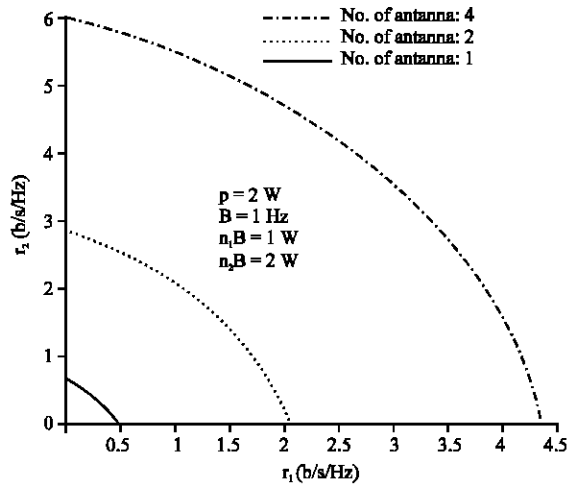


Fig. 1: Compare of different antenna at both ends

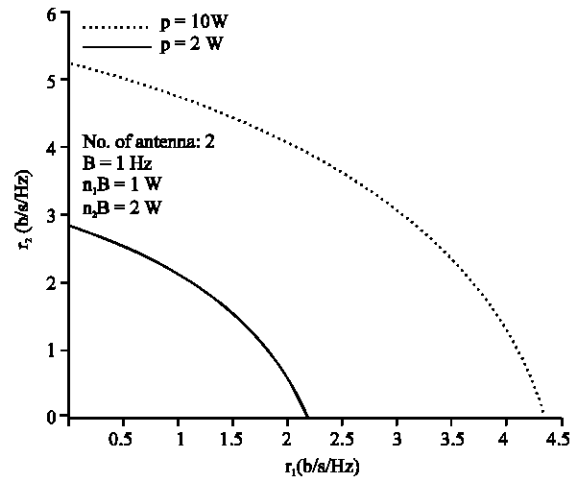


Fig. 2: Compare of different transmission power

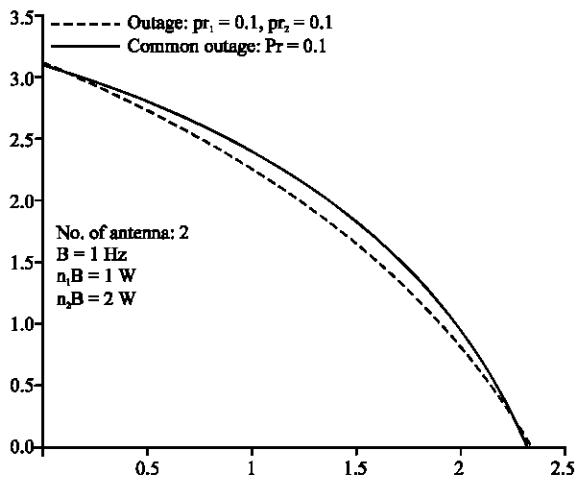


Fig. 3: Compare of common and independent outage probability

In Fig. 1, we illustrate the two-user capacity region when the number of antenna at both ends is 1, 2 and 4, from which, we can see the advantage multiple antenna brings obviously.

In Fig. 2, we compare the two-user capacity region under different power constraint when the number of antenna at both ends is 2. With more power, we can get larger rate vector.

In Fig. 3, The number of antenna at both ends is 2. We show the minimum common outage probability capacity region of two users BC system, as well as the outage capacity region when the outage probability of both users is equal to the common outage probability. We can see that the former is included in the latter.

CONCLUSION

We have obtained the outage capacity region of MIMO BC channel under long-term average power constraint. Firstly, we use the duality between MIMO-BC and MIMO-MAC to solve the optimization problem from the angle of MIMO MAC, then use waterfilling to derive the minimum required power for a certain rate vector and use threshold algorithm and tail power allocation algorithm to get the optimal power allocation strategy. When we get the boundary of the outage capacity region of MIMO BC channel, we get the outage capacity region of MIMO BC channel.

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