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A Review on Selected Target Tracking Algorithms

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Abstract: An important requirement of growing research in machine/computer vision is a reliable and efficient target tracking methods. This letter is a review of selected estimation and tracking algorithms as applied to image processing, these are grouped into two namely: Deterministic based and Nondeterministic based algorithms. Various issues affecting success and/or failure of such algorithms are discussed and presented together with current useful references on the topic. With only two groups of tracking and estimation algorithms, this work is anticipated to guide tracking systems designers in deciding on a suitable algorithm for use. There is no single ideal algorithm for all tracking problems.

Key words: Deterministic and nondeterministic, sequential filter, mean square error, estimation algorithm, tracking

INTRODUCTION

Current focus of research in machine/computer vision is to find a fast, accurate, and robust method to the track real-world position and orientation (pose) of moving targets. Tracking is a process of estimating the current and future state (position, color, shape, size, velocity etc) of a target given that certain assumptions are fulfilled and is usefully because visual systems users (human/robot) must be continually provide a two-dimensional computer generated representation of the target's three-dimensional real-world position and orientation in an accurate or/and fast manner, otherwise disturbing or even harmful effects in observation may be realized (Danette *et al*., 2001).

This letter's objective is to review current target estimation and tracking algorithms grouped into two, namely; deterministic and non-deterministic and also present issues that may affect success and/or failure of such algorithms together with current references on the topic. It is hoped that, with only two broad categories, this work will be a useful guide for target tracking systems designers in selecting a suitable algorithm.

OVERVIEW ON TARGET TRACKING AND ESTIMATION

From a classical point of view, tracking can be defined as the set of algorithms which, when applied to sensor detections allows (Farina, 2006): (1) target recognition, (2) estimation of motion parameters (position, velocity and acceleration) of a target moving in a predictable or partially predictable manner, (3) extrapolation of the track parameters, (4) differentiating targets, (5) differentiate false (caused by intentional or natural interference) from true targets, (6) adaptive refinement of the threshold setting based on content map of sensor(s) and (7) Efficient track detection management in a camera network. More advanced literature defines tracking as an idea of locating the transformed (rotated as well as translated form) target in different frames (Khare and Shanker, 2007). The process starts with approximate knowledge which is corrected with subsequent observations so as to satisfy all the observations in the most accurate manner possible (Bar-Shalom *et al*., 2001).

Although there are numerous causes of visual errors in interactive human-computer systems, the tracking process tends to be dominant (Welch, 2009), these errors are characterized as static or dynamic errors, the former arising from transform inconsistencies which must be estimated prior to system use and from poor visibility that leads to random noise in measurement and hence target misrepresentation (Welch, 2009). Dynamic errors arise from latency (delay) and motion of tracking device after the tracking cycle (detect, sample, estimate, display) has started, they can be determined concurrently during system use (Welch, 2009). Zero value of latency is

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impossible to realize and the only means of obtaining measurements close to the actual target position is through prediction (Welch, 2009); it is this problem of target prediction that lead to the invention of many tracking algorithms some of which are viewed in this letter.

Choosing a tracking algorithm is a complex process and may involve try and error while taking into account the structure of the problem at hand, ie, linear or nonlinear, quadratic, least square problem, type of derivatives in the objective function, any constraints needed and if these constraints are analytically tractable or not. The next section presents nondeterministic based filter.

NON-DETERMINISTIC METHODS

Techniques based on nondeterministic model assume a random description of signal/image and can be described by probability laws (Ruhm, 2008), such signal/images are known as noisy or probabilistic and can be analyzed with statistics and stochastic models, characteristic functions of such signals are usually difficult to describe or measure and predicting their trajectories may not be possible but however, for a particular target observation x_k at instant k a statement about future values (forecast, prediction) may be made based on the past event using a dynamic model (e.g., a linear predictive model) and/or a probabilistic model (e.g., Gaussian model) of the process with accuracy depending on the available information (Vaseghi, 2000).

Conclusions made from these inferences are not certain results but estimates which provide a high degree of certainty (optimal) by application of suitable priory information, skillful procedures and careful analysis (Kevin and Stemler, 2008; Chen, 2004; Ruhm, 2008).

In practical problems, observations are sequential in nature, however, such observations must be finite and therefore a condition or rule on when to end the observation process, Bayesian methodologies are used for this purpose (Keinosuke, 1990; Chen and Jiao, 2009) with an idea to combine evidence contained in such observations signal with prior knowledge of probability distribution of the process. Bayesian methodologies include the classical estimators such as Maximum A Posteriori (MAP), Maximum-Likelihood (ML), Minimum Mean Square Error (MMSE) and minimum mean absolute value of error (MAVE) (Vaseghi, 2000). A widely used example of a Bayesian model is the hidden Markov model. The following are examples of nondeterministic estimation methods.

The Wiener filter: The filter was named after its inventor Norbert Wiener in the 1940 s and it is a theory of filtering of stationary time series for continuous time processes. The theory is characterized by the following (Rober and Huang, 1997):

- Assumption that both signal and noise are random processes with known spectral characteristics or, equivalently, known auto and cross correlation functions
- For mathematical tractability, the best performance criterion is minimum mean square error
- Based on scalar methods that lead to the optimal filter weighting function or transfer function in the stationary case

The filter belongs to Least-square filtering (also known as linear minimum mean square error filtering) known to be the best linear filters in the sense of minimizing the mean squared error.

A more elaborate wiener filter can be found in literature including (Rober and Huang, 1997; Kalman, 1960; Peebles, 1987; Pratt, 2007).

Methods for solving the Wiener problem are subject to many limitations render them practically useless; these include (Kalman, 1960): (1) The optimal filter is specified by its impulse response which makes the task of synthesizing the filter from such data difficult, (2) Optimal impulse response determination by numerical methods is often quite difficult and poorly suited to machine computation and worse with increasing complexity of the problem, (3) New derivations are required for important generalizations such as growing memory filters, nonstationary predictions etc, these derivations are frequently difficult even to specialists, (4) Fundamental assumptions and consequences tend to be hidden with nontransparent mathematics of derivations, and many others. Given these limitations, it is possible that a nonlinear filter could do better than a Wiener filter for highly nonlinear/nonGaussian systems; it is possible to show however that for a linear/Gaussian process, no nonlinear filter can do better than the Wiener filter.

The Kalman Filter (KF): The KF named after its inventor Rudolph E. Kalman in 1960 s, describes a recursive solution to the discrete-data linear filtering problem (Kalman, 1960). This filter is a special and highly successful family of filters that belong to sequential Bayesian filters (Kevin and Stemler, 2010). It is important to note that KF is an extension of Wiener filter to deal with both stationary and nonstationary processes in descrete and continuous time and that it is a steady state representation of the Wiener filter (Rober and Huang, 1997; Grewal and Andrews, 2008; Bar-shalom *et al*., 2001). Unlike Wiener filters, the KF has the ability to adapt itself to nonstationary environments. Although the two theories are considered in their historical order, neither is prerequisite material for the other (Rober and Huang, 1997). KF uses state-space model as basis for nearly all linear estimation methods and a convenience notation to make analysis notationally tractable (Danette *et al*., 2001).

KF prediction: The KF is a set of mathematical equations implementing a single, simple, fast and accurate predictor-corrector type estimator (Fig. 1) optimal in the sense of estimating the state and process error covariance by minimizing the mean of the squared error covariance when some presumed conditions are met; it's simple recursive nature makes it very attractive and famous for use in various fields including communications, surveillance and many more (Welch, 2009; Chen, 2004). It is also a perfect tool for multiple-input fusion or multiple-output applications filtering requiring only the "best guess" and not the entire history of the system state to calculate a new state, the filter is readily implemented in time variable (Rober and Huang, 1997; Ye and Jiao, 2008).

The KF addresses the general problem of estimating the state $x \in \Re^n$ of a discrete time controlled process that is governed by the linear stochastic difference equation (Welch, 2009; Welch and Bishop, 2007; Daviesy *et al*., 1998; Danette *et al*., 2001):

$$x_k = Ax_{k-1} + Bu_k + w_{k-1} \tag{1}$$

And observable via measurements $Z \in \Re^m$ modeled by:

$$z_k = Hx_k + v_k \tag{2}$$

Equation 1 is the process model, where subscript k denotes the time step or iteration number and superscript $(.)^k$ denotes k dimension, $A \in \Re^{n \times m}$ models the transition of the state over time in the absence of either a driving function or process noise. The matrix $B \in \Re^{n \times l}$ relates the optional control input $u \in \Re^l$ to the state x. Equation 2 is the measurement model in which $H \in \Re^{m \times n}$ models the relationship between the state and the measurements z_k. The random variables $w_k \in \Re^n$ and $v_k \in \Re^m$ respectively model the process and measurement noise, they are assumed to be independent (of each other), zero mean, spectrally white noise vectors and normally distributed (as in Eq. 3 and 4). If $w_k = 0$ for all k then the system is deterministic model (discussed in section IV below)

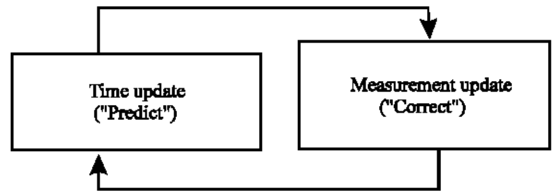

Fig. 1: Predict and correct method of KF, Welch and Bishop (2007)

and a perfect model scenario is assumed, otherwise it is nondeterministic or stochastic also called the imperfect model scenario (Kevin and Stemler, 2009):

$$p(w) \sim \mathcal{N}(0,Q), \tag{3}$$

$$p(v) \sim \mathcal{N}(0,R) \tag{4}$$

where, Q is process noise covariance matrix and is measurement noise covariance matrix.

In practice matrices A, H, as well as noise parameters Q and R are variant with time but assumed to be constant in theory (Welch and Bishop, 2007). The posterior density is assumed to be Gaussian at every time step i.e., if $P(x_{k-1}/z_{k-1})$ is Gaussian then under certain assumptions $P(x_k/z_k)$ is also Gaussian (Sanjeev *et al*., 2002) and can be parameterized by a mean and covariance.

Determining the process noise covariance Q is difficult because of the inability to observe directly the process being estimated; in some cases a relatively simple process model can produce acceptable results if enough uncertainty is available in the process of selecting Q Better filter performance (statistically) can be obtained by turing the filter parameters Q and R in a system identification process which often takes place off-line and quickly stabilizes when Q and R are constant, however, these quantities frequently do not remain constant (Welch and Bishop, 2007).

The KF permits the application of all information contained in a model along with any system observations through time. The theory can be found in many publications including (Welch, 2009; Welch and Bishop, 2007; Postalcloglu *et al*., 2005; Maybeck, 1979; Danette *et al*., 2001). The KF asymptotically reduces to the Wiener filter given that certain requirements are satisfied and there is steady data stream; this is an important conclusion because the Kalman formalism is generally computationally more difficult than is the Wiener one.

Pros and cons of KF: The state-space form and recursive nature makes the implementation of KF is easier compared to traditional estimation methods like Wiener filter which needs integral equations and stationarity (Welch and Bishop, 2007). In distributed systems, the KF is constrained by a large computational expense dominated by the error covariance propagation step which tends to make the implementation of the KF impractical in large scale models (Postalcloglu *et al*., 2005). The KF does not provide means to correct errors that could have been made in the past estimates and therefore these continue to propagate and spread themselves to subsequent state estimates (Kevin and Stemler, 2010). There are means of extending the KF concept to some nonlinear and nonGaussian applications or developing and using nonlinear filters directly, but these are considered only if linear models prove inadequate, these Kalman type filters (Simo, 2009) include: Extended KF (EKF), Unscented KF (UKF), Interacting Multiple Model (IMM) have been developed for estimation and the more recent Ensemble KF (EnKF), Iterative KF (IKF). This letter looks at the Extended KF as an example of Kalman type filter, because of its popularity, reliability and efficiency in nonlinear approximation.

Significant efforts have been made to implement ensemble KF for global weather forecasting but however proved impossible goal mainly because of the inability to represent high dimensional uncertainty with any reasonably sized ensemble, however, it has been implemented in meso-scale with additional assumptions (Houtekamer and Mitchell, 2005; Kevin and Stemler, 2010).

Single-Constraint-at-a-Time (SCAAT): Traditional methods of target tracking and estimation involve collecting a group of sensor measurements and attempt simultaneously solve the system of equations which constrain the solution, these methods surfer from lower estimate rate due to the need to collect multiple measurements per estimate and the lack of description for sensor movement throughout the measurements sequence, these methods instead assume a simultaneity arrival of measurement which further increases error in measurement, further more these methods cannot identify unusually noisy individual measurements. A solution to these problems is SCAAT, a type of KF in which the state estimate is incrementally improved as each new single "insufficient" sensor measurement is obtained hence more frequent estimates are generated with less latency and improved accuracy (Welch, 1996; Welch and Bishop, 2007). One unusual fact about this approach is that the state is actually not computed directly, only sensor measurements are computed leading to the state estimate (Danette *et al*., 2001).

Extended Kalman Filter (EKF): The EKF is an extension of KF to address nonlinear measurements with assumptions that the a priori distributions are Gaussian and that the nonlinear system can be linearized relative to prescribed reference values to produce a linear and Gaussian system and thus enabling utilization of KF (Alspach and Sorenson, 1972). The filter can be obtained by modifying the traditional KF as follows:

Assume a process with a state vector $x \in \Re^n$ governed by the nonlinear stochastic difference equation:

$$x_k = f(x_{k-1}, u_{k-1}, w_{k-1}) \tag{5}$$

with measurement $z \in \Re^m$ as:

$$z_k = h(x_k, v_k) \tag{6}$$

where, The random variables w_k and v_k represent the process and measurement noise as in Eq. 1 and 2 (Welch and Bishop, 2007). The nonlinear function f in the difference Eq. 5 relates the state at time step k−1 to the state at current time step k. It includes as parameters any driving function u_{k-1} and the zero-mean process noise w_k. The nonlinear function h in measurement Eq. 6 relates the state x_k to the measurement z_k (Welch and Bishop, 2007):

$$\tilde{x}_k = f(\hat{x}_{k+1}, u_{k-1}, 0) \tag{7}$$

and

$$\tilde{z}_k = h(\tilde{x}_k, 0) \tag{8}$$

where, x_k is the a posteriori estimate of the state (from a previous time step k). The State estimation of nonlinear process and measurement begins with writing new governing equations that linearize an estimate about Eq. 7 and 8 as follows (Welch and Bishop, 2007):

$$x_k \approx \tilde{x}_k + A(x_{k-1} - \hat{x}_{k-1}) + Ww_{k-1} \tag{9}$$

$$z_k \approx \tilde{z}_k + H(x_k - \tilde{x}_k) + Vv_k \tag{10}$$

Where:

- x_k and z_k are the actual and measurement vector
- $\hat{x}_k$ and $\tilde{z}_k$ are the approximate state and measurement vectors from Eq. 7 and 8
- $\tilde{x}_k$ is an a posteriori estimate of the state at step k
- Random variables w_k and v_k represent the process and measurement noise as in Eq. 4 and 5
- A, W, H and V are the Jacobian matrix of respective partial derivatives

The requirement to explicitly calculate Jacobian for the various variables explicitly at each time step imposes a disadvantage to EKF, this is in its self a difficult task and makes the general EKF computationally costly when done numerically while analytic computation requires complicated derivatives (Karlsson, 2002). More complete EKF can be found in reference (Welch and Bishop, 2007; Postalcloglu *et al*., 2005; Sanjeev *et al*., 2002, 2004; Karlsson, 2002; Hwirtha and Speer, 2004).

Gaussian sum methods: Gaussian-Sum filter (GSF) also called Gaussian mixture filter is a method which takes nonGaussian and nonlinear distribution of measurements into Bayesian framework (Hwirtha and Speer, 2004; Karlsson, 2002). In this method, both distribution of measurement error and estimated quantities are modeled by a combination of Gaussian densities. The GSF is an extension of KF with the assumption that prior and posterior distributions have density functions, but generally GSF does not necessarily have a density function (Simo, 2009). The GSF is quite reasonable especially when the posterior density is multimodal (Sanjeev *et al*., 2004). Ortega (2007) considered the GSF as an extension of the EKF with the main idea of approximating the pdfs by linear combination of Gaussian densities each of which adequately fulfilling the EKF requirements on linearization. Gaussian sums filter can be applied for two reasons: (1) to represent a nonGaussian distribution; and 2) very severe nonlinearity which can never be handled by a single Gaussian distribution (Ortega, 2007).

The Gaussian sum representation $p_A(x_k/z_k)$ of a nonlinear, nonGaussian probability density function $P(x_k/z_k)$ associated with a vector valued random variable x_k is defined as (Sanjeev *et al*., 2004; Karlsson, 2002):

$$p(x_k / z_k) \approx p_A(x_x / z_k) = \sum_{i=1}^{N_k} w_k^i \mathcal{N}(x_k^i; \hat{x}_{k/k}^i, P_{k/k}^i) \qquad (11)$$

where, weights w_k^i sum to unity, i.e; $\sum_{i=1}^{N_k} w_k^i = 1, w_k^i \geq 0$ for all i. The approximation P_A can be made accurate as desired by choosing N_k, the number of mixture components (Sanjeev *et al*., 2004) and can converge uniformly to any density function of practical concern as N_k increases while covariance $P_{k/k}^i$ approaches the zero matrix (Alspach and Sorenson, 1972; Karlsson, 2002). In this estimation process however, there is a problem of formulating an algorithmic procedure for online computation of weights w_k^i, means $\hat{x}_{k/k}^i$ and covariances $P_{k/k}^i$. This problem is minor for large number of components N_k which grows exponentially with k (Sanjeev *et al*., 2004), however, the problem is significant for small N_k, Alspach and Sorenson (Alspach and Sorenson, 1972) provide a convenient method to find these parameters.

Another problem with the application of the GSF in real time implementation is the exponentially growing value of N_k which can easily explode if not reduced by some methods such as forgetting, merging, resampling, clustering or minimizing cost-function; some of these methods when applied also hinder the general convergence rate of the filter (Simo, 2009).

Although Ortega (Ortega, 2007) considers the EKF as the only nonlinear recursive candidate model for GSF, Sanjeev *et al*. (2004) presents two types of Gaussian sum filters namely; Static multiple model (MM) estimator and the dynamic MM estimator both of which can aply various Kalman type models. Other types of GSF include Box GSF and Sigma point GSF (Sanjeev *et al*., 2004).

Grid-based method: This method is applicable in recursive estimation of the filtered density $P(x_k/z_k)$ when the state space is discrete and consists of a finite number of states (Sanjeev *et al*., 2004). The approach is given credit for being the first approximate nonlinear filter in history, in this method a grid of state space and time is used (Li and Jilkov, 2010).

An elaborate derivation of the filter is given in reference (Sanjeev *et al*., 2004) and also its suboptimal algorithm variation for approximating nonlinear states known as an approximate Grid Based method is discussed. Other grid based methods include: finite difference method (FDM), finite element method (FEM), and finite volume method (FVM) (Li and Jilkov, 2010).

Grid based methods give good approximation to a continuous state space if the grid is sufficiently dense in a fixed interval and they are widely applied in the Hidden Markov model (HMM) filter (Sanjeev *et al*., 2002). Disadvantages of these Grid based methods include; the drastic increase in computation cost as the dimensionality of state space increases, the state space must be finite otherwise there is need for truncation of the state space.

For accuracy, there is also need for a predefined, finite state space which cannot be unevenly re-partitioned in high probability density regions, a prerequisite for greater resolution (Sanjeev *et al*., 2002, 2004).

Particle filter: In real world applications characterized by strong clutter, the posterior is multimodal and cannot be approached by traditional methods like KF, such cases require more general sequential filtering, and the particle filter has become a popular class of algorithms to solve such problem of optimal estimation for nonlinear, nonGaussian state space models. Particle filter is an example of Sequential Monte Carlo (SMC) algorithms and it has become normal to present them as being the same thing in much literature (Doucet and Johansen, 2009). The particle filter takes on many names in literature including but not restricted to the following: Sequential importance sampling (SIS), Bayesian Bootstrap, JetStream, Condensation algorithm, interacting particle approximations and survival of the fittest (Sanjeev *et al.*, 2002). The filter is an optimal approximation algorithm to the nonlinear and nonGaussian estimation problem and it can outperform traditional methods in many ways, given availability of sufficient computational resources (Sanjeev *et al.*, 2004; Karlsson, 2002). The filter is characterized by a combination of importance sampling and Monte Carlo Schemes in order for consistent exploration of a sequence of multiple distributions of interest (Kevin and Stemler, 2009). The particles can either generate other particles or terminate, depending on their ability to represent the distribution of the state basing on the given observations. Ortega (2007) considered the particle filter as a special type of GSF in a sense that a Dirac function is no less than a Gaussian as shown in Eq. 12 with zero variance and affirms that particle filtering is a method in which a sum of Dirac functions or particles with weights are used:

$$\delta(x-a) \triangleq \lim_{\sigma \to 0} \mathcal{N}(x-a;\sigma^2) \tag{12}$$

More complete Particle filter algorithm can be found in literature including the study of Bergman (1999), Doucet *et al.* (2000), Pitt and Shephard (1999) Qing-Bo *et al.* (2010).

Particle filters are indeed robust in case of Clutter (noise) or partial occlusions and very successful for nonlinear and/or nonGaussian estimation of target state x_k at time t_k, conditionally to measurements z_k from first time to time t_k. The key idea in SIS algorithm is to represent a required posterior density function by a set of random samples (also called particles) $\{x_k^{(i)}, i = I...N\}$ with associated weights $\{\pi_k^{(i)}, I = I...N\}$ such that all weights sum to unity, the location and weight of particles represents the value of density in the region of state space, these weights and location are updated recursively with each new observation. A representation of the density function:

$$p(x_k|z_{0:k}) \approx \sum_{i=1}^{N} \pi_{k-1}^{(i)} \delta(x_k - x_{k-1}^{(i)}) \tag{13}$$

where, $x_k^{(i)}$ is a hypothetical state of target and $\pi_k^{(i)}$ is proportional to the probability of the state $x_k^{(i)}$. The set of samples is propagated through time in two steps: (1)samples are selected proportional to their weights: samples with high weights are duplicated, whereas those with low weights are eliminated (re-sampling step), (2) Sample are then propagated and weighted according to an importance density that describes regions of interest in the state space. As the number of samples becomes very large, this characterization becomes an equivalent representation of the usual functional description of the posterior probability density function $p(X_k| Z_k)$ and the particle filter approaches the optimal Bayesian estimate. This procedure is only an ideal case in which resampling is not required and it is quite a narrow case, but without resampling step, after a number of iterations, only one particle will have a significant normalized weight while others are negligible, this state leads to occurrence of the degeneracy phenomenon (Simo, 2009).

Particle filter implementation will fail unless the ensemble is sufficiently large and the Monte Carlo simulation sufficiently good, failure of the filter is a result of the ensemble collapsing onto a few ensemble members, which are not a good representation of the true uncertainty. This kind of failure is potentially increased with small ensembles, strongly nonlinear systems, poor implementation of simulations, high dimension of system and systems close to being deterministic (Sanjeev *et al.*, 2002; Kevin and Stemler, 2010). Another deficiency of particle filter is that a poor state estimate in the past such as caused by an unusually large observational error could propagate to subsequent state estimates and could even mix with several updates because it provides no possibility of correcting the inevitable past mistakes (Kevin and Stemler, 2010).

The SIS filter forms basis for many other Monte Carlo methods in literature (Sanjeev *et al.*, 2004; Chen, 2004; Kalos and Whitlock, 2004; Karlsson, 2002), these various versions can be regarded as special cases of the SIS whose only difference is the way population of the particles evolves with time (Cheung-Mon-Chan and Moulines, 2004), in other words difference is in the resampling technique and the choice of the importance function (Renaud and Siler, 2009). These special case filters include but notlimited to: sampling importance re-sampling (SIR) filter, Auxiliary sampling importance re-sampling (ASIR) filter, regularized PF (RPF), Gaussian PF, Gauss-Hernite PF, Cost Reference PF, Rao-Blackwellized PF, Unscented PF, ICondensation PF, Hybrid bootstrap, Quasi Monte Carlo, Fast weighted bootstrap, annealed PF, Local linearization PF, Multiple model PF.

Given sufficient samples and the right importance density, Particle filters approach Bayesian optimal estimate and therefore they are often substitutes and canbe made more accurate than either the EKF or UKF. Thechoice of importance density suited to a given problem requires careful thought because success or failure in a particular optimization process depends on it.

Wavelet filter for target tracking: Current advancement in wavelet theory has led to research interest in its application to image processing, the technique has been applied to implement digital filters where it has been effective as a data analysis tool in many fields such as approximation (representation), estimation, classification, enhancement and compression (Postalcloglu *et al.*, 2005; Figueiredo *et al.*, 1999; Graps, 1995; Qing-Bo *et al.*, 2007).

In relation to Fourier Transform, the Wavelet Transform (WT) is a means of obtaining a representation of both the time and frequency content of a signal but unlike Fourier transform where the same time window is used over the whole frequency domain and turns out to be unnecessarily large for signals with high power at high frequencies and too small for dominantly low frequency signals, the window function width for WT varies depending on the central scale, this functionality is achieved by translation and dilation of a single function called the wavelet which provides an amount of simultaneous localization of individual wavelet functions in time and scale domain which conforms to uncertainty principle (Random motion) (Figueiredo *et al.*, 1999), this means that the wave properties can be analyzed both in time and scale domain simultaneously (Hsuan and Chen, 2006). Wavelet expansion tends to concentrate the target energy into a relatively small number of coefficients with larger values which makes wavelet analysis appropriate for trajectory estimation. Important to note that the term "frequency" often applies to functions with an oscillating behavior and it is not used for wavelet decompositions, the term "scale"' is more appropriate in describing such phenomena for wavelets (Figueiredo *et al.*, 1999; Golubev *et al.*, 2004; Misiti *et al.*, 2010). Another important property that makes wavelet transforms appropriate for signal estimation is that coefficients are less correlated than the original image even though they have dependency on each other (Figueiredo *et al.*, 1999), this enables the Wavelet filter have less estimation error than traditional filters (Postalcloglu *et al.*, 2005).

Many more advantages make the use of wavelet transform desirable such as computational speed and the infinite set of possible basis functions, thus filters designed using wavelets provides immediate access to information which may not be available with the use of traditional methods. For all these advantages, many scholars prefer to opt for wavelet filtering.

Wavelet expansion of a function f is a special case of orthonormal series expansion which can serve as a basis for nonparametric estimators.

For a function $f \in L_2(R)$, there is a homogeneous wavelet expansion:

$$f = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \langle f, \psi_{j,k} \rangle \psi_{j,k}, \tag{14}$$

where $\langle f, \psi_{j,k} \rangle = \int f(t) \psi_{j,k}(t) dt \psi$.

and can be decomposed as a wavelet series:

$$f(x) = \sum_{k} \alpha_k \varphi_{j_0 k}(x) + \sum_{j=j_0}^{\infty} \sum_{k} \beta_{jk} \psi_{jk}(x) \tag{15}$$

where, α, β are arbitrary, φ_{jk} is father and ψ_{jk} mother wavelets.

A complete derivation and implementation of the wavelet expansion can be found in reference including (Daubechies, 1988; Golubev *et al.*, 2004).

Wavelet transforms do not use low-pass filtering when reducing noise because it approaches a linear time invariant system which can blur the sharp features in an image making it difficult to separate noise from the target (Postalcloglu *et al.*, 2005), however, in a method to detect and track moving objects. Hsuan and Chen (2006), consider only low frequency component of discrete wavelet transform and justify their stunt to be of low computing costs and noise reduction. Their method uses motion information to detect and track multiple objects over the video stream based on discrete wavelet transform with the help of low frequency sub-image of third level DWT. Results of their work prove that the wavelet tracker can track a moving object from much occluded background situations, for example wavering trees where most tracking methods cannot work. Their findings also show that Wavelet transform use hierarchical features and an image can be decomposed into four sub-images which preserve not only the frequency features but also spatial features (Hsuan and Chen, 2006).

Symmetric Daubechies complex wavelet coefficients of the target have been used as relative shift invariant features to segment and track it in a sequence of frames (Khare and Shanker, 2007) with clarification that even though real-valued wavelet transform can be used for tracking applications, it suffers from shift-sensitivity. A shift insensitive transform as one in which an input signal shift causes an unpredictable change in transform coefficients. Generally, complex Daubechies wavelet transform is believed to be more suitable for tracking due to approximate shift-invariance nature and avoidance of redundancy that plagues other approximate complex wavelet trans forms like dual tree complex wavelet transform (DTCWT), Projection based complex wavelet transform, steerable pyramid complex wavelet transform etc which are proposed in literature (Khare and Shanker, 2007). A Bayesian wavelet based image estimation using noninformative priors has been formulated in (Figueiredo *et al*., 1999), the sparseness and decorrelation properties of the discrete wavelet transform are exploited in the same reference.

DETERMINISTIC BASED METHODS

A deterministic model is one in which random processes such as random noises in dynamics are assumed to be negligible and therefore cannot govern the system dynamics (Kevin and Stemler, 2009). Whereas stochastic models do estimate the probability measure on the model states and maximum likelihood state can be identified, state estimation can only make sense in the perfect model scenario because in these cases, the system and the model are identical (Kevin and Stemler, 2008). Besides, the stochastic component of the model is not important in tracking, however, this components is useful in representing uncertainty in forecasts due to model error as is the case of target estimation (Kevin and Stemler, 2010). Two significant deficiencies of sequential Bayesian filters include the attempt to represent the model error by random processes alone and they are restricted to prescribed nature of model errors. The Gaussian assumption alone greatly reduces the amount of information that is contained in the true density, particularly when handling multimodal (Alspach and Sorenson, 1972). Stochastic methods require complete characterization of the process, and observation noise of the system, this task is difficult, costly or may not be suitable for the task at hand (Pascoal *et al*., 2000). Deterministic techniques provide all the information the erroneous nondeterministic methods obtain, but much more efficiently and also provide a wealth of additional useful information (Kevin, 2003; Kevin and Stemler, 2009, 2010). All subsections herewith are examples of tracking techniques that apply deterministic model of filtering.

Shadowing filters for estimation: The Shadowing filter is a state estimation algorithm applicable to low-dimensional nonlinear systems and other high-dimensional complex systems where the nonlinear dynamics of the system is important, it is equally applicable to stochastic as well as deterministic systems, this section focuses on its applicability to deterministic dynamics model (Kevin and Stemler, 2008).

The shadowing filter attempts to find a trajectory of a deterministic model in close proximity to a sequence of observations. It is generally effective in making such estimations and rarely fails. Shadowing filters are nonsequential because they apply a long sequence of past observations simultaneously, not just the most recent observation as assumed by the sequential filters.

This filter achieves its goal by exploiting information in the observations revealed by model's dynamics while the mode statistics is secondary to the dynamics. A consideration of all past information for nonlinear systems can provide better filtering which is a typical characteristic of the nonsequential shadowing filters based on a deterministic approach to filtering. Deterministic models have advantages for tracking and forecasting even when a stochastic model might seem to be more appropriate (Kevin and Stemler, 2010).

Kevin and Stemler (2009) showed that shadowing filter's strength is its focus on dynamic aspects rather than stochastic aspects of the tracking problem. The filter works on principle of least square methods and its theory is based on Laplace's and others which went out of use with the introduction of probabilistic methods in 1970s. A unique advantage of shadowing filters is that model errors are discovered, not prescribed (Kevin and Stemler 2009, 2010). Further case study by Kevin (2003) shows that a simple gradient-descent shadowing filters results obtained from tracking and ensemble forecasting outperforms Kalman-Bucy filters and particle filters.

Shadowing filters can be easily implemented by gradient descent of indeterminism (GDI) method, well described in Kevin and Stemler (2008 and 2009) which guarantees to obtain a shadowing trajectory, even with limited gradient information (Kevin and Stemler, 2010).

Clearly described by Kevin and Stemler (2008, 2009), the method of GDI is used to implement shadowing filter by defining indeterminism of a sequence of states $X = (x_1,..., x_n)$ as:

$$I(X) = \frac{1}{n-1}\sum_{i=1}^{n-1}\|x_{i+1} - f(x_i)\|^2 \qquad (16)$$

where, this sequence is considered to be a point in R^{nd}, I(X) is a scalar function on (n×d) dimensional space, f is assumed to be a differentiable and perfect model of the system. Among the properties of GDI include: It will always converge to a shadowing trajectory of the model, and I(X) converges monotonically to zero (Kevin and Stemler, 2008). For bounded noise and increasing length n of the observation sequence, the GDI shadowing filter converges to the true trajectory for the perfect model of a hyperbolic system.

The iterative Gradient Descent of Indeterminism (GDI) shadowing filer: A straightforward implementation of the shadowing filter is by Euler integration which provides an iterative algorithm that gives an approximate solution to the GDI and is easier to implement especially in high-dimensional systems (Kevin and Stemler, 2008).

An iterative GDI shadowing filter can be defined by letting $X_k = (x_{1,K},\ldots,x_{n,k})$ where $X_o = S$ and:

$$x_{i,k+1} = x_{i,k} - \frac{2\Delta}{n-1} \times \begin{cases} -H(x_{i,k})(x_{i+1,k} - f(x_{i,k})), i=1 \\ x_{i,k} - f(x_{i-1,k}) - H(x_{i,k}) \\ (x_{i+1,k} - f(x_{i,k})), 1<i<n \\ x_{i,k} - f(x_{i-1,k}), i=n. \end{cases} \quad (17)$$

where, subscript k denotes the iteration number,H(x) the adjoint of f (transpose of the Jacobian matrix) evaluated at x, and Δ is the arbitrary step size. In practice this formula is iterated until X_k converges sufficiently.

A more complete clarification of the GDI and how it achieves results can be found by Kevin and Stemler (2008, 2009). A potential difficulty is in calculation of the Jiacobian matrix H(x), ad$\times$d matrix which cannot be computed conveniently when d is large.

Robustness of the filter comes from the fact that the gradient of indeterminism is only descended and there is no need to take the direction of steepest descent and f need not be strictly differentiable (Kevin and Stemler, 2008), besides that, the exploitation of nonlinearity and simultaneous use of many past observations make the Shadowing filter superior enabling it to overcome propagation of unavoidable past errors.

Unlike in particle filtering, the ensemble size of shadowing filter does not affected its performance, it is arbitrary and can be varied at any time as required (the ensemble can be as small as necessary or as large as practical) without a possibility of collapse, there is also no need to regenerate the ensemble at each forecast time step, because many useful ensemble members are simply extensions of existing shadowing trajectories. Shadowing filters are valid at large and small noise levels. The shadowing filter derived from a deterministic model works best even for nondeterministic models with the exception of cases where the random elements of the system have larger variance than the observational errors (Kevin and Stemler, 2010). A question still remains on the convergence speed to the solution.

Fuzzy predictive filters: Fuzzy predictive filter is a set of incremental controls and a rule base of simple fuzzy prediction rules (logic) for scaling these controls, the error between the system's output and the desired reference is considered in order to determine the scaling factor or gain and just as with other predictive filters, the gain is decreased when the system is close to the steady state situation and increased when the error is big or the output moves away from the reference (De Costa Sousa and Setnes, 1999). Fuzzy predictive filter is applied to nonlinear motion prediction of a moving target where the dynamics is varying over time (Khoo and Bin, 2006).

Proponents of the fuzzy methods hold the view that the filter improves the performance of motion tracking systems with the replacement of the target's dynamic model with a set of fuzzy inference rules to link measured information (input) and predicted information (output) without the necessity of complex mathematical models (Danette *et al.*, 2001; Khoo and Bin, 2005, 2006; De Costa Sousa and Setnes, 1999). The filter can be used in cases where there is no fixed motion pattern and handling incomplete and noisy data measurements.

Khoo and Kok Bin (2005) successfully replaced the dynamic model with fuzzy logic inference rules in order to track an object moving in two dimension space. Castro *et al.* (2003) use the fuzzy logic control for target tracking, in their work they use two loop fuzzy logic controller, one for range control to the target and the other for orientation correction, the two loops have independent output in such a way that a linear acceleration resulting from variables related with distance between the sensor and the target is output through the range control loop while angular acceleration resulting from processing angle inputs is output by the orientation controller. They clarify that target tracking as addressed by fuzzy controller is a set of simple rules working on range and orientation concepts to achieve the desired behavior of a target in which case the distance and orientation angle to an object are considered as two separate phenomena, a case which is not completely true in the real world but gives a good approximation under certain problems. The block diagram in Fig. 2 shows a setup of fuzzy controller.

Fuzzy predictive filters solve problems introduced by the discretization mainly those concerning oscillations around non-varying references (chattering) and slow step responses. It uses simple fuzzy criteria considering the current state of the system and the predicted error, the filter scales the gain of an adaptive set of possible control actions.

An improvement of the fuzzy predictive filter is its implementation with adaptive mechanism to enable feedback and control (De Costa Sousa and Setnes, 1999) and implement for multichannel image processing. A disadvantage of fuzzy predictive filtering method is the

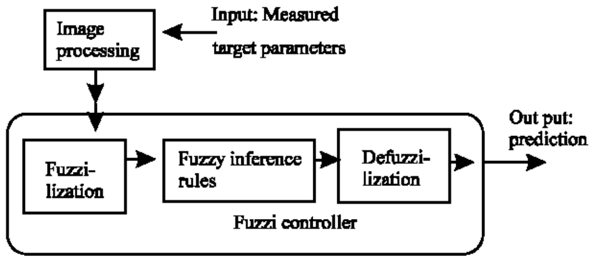

Fig. 2: A general structure of a fuzzy predictor

undetermined number and type of fuzzy rules required for the fuzzy image operation which usually necessitates a large number of rules (De Costa Sousa and Setnes, 1999).

Complementary filter as an estimator: A Complementary filter is a technique used for estimation by combination of measurements especially where sensor bandwidth is a constraint of consideration, it is applied in reconstruction of the target motion where there are two different measurement sources estimating one variable with different noise properties such that one source gives good information only in low frequency region while the other is good only in high frequency region (Pascoal *et al*., 2000).

Mathematically, the filter is obtained by a simple analysis in the frequency domain where it is defined two or more transfer functions complementing each other, in fact the filter name is derived from blending data such as low resolution and high resolution from sensors to achieve a robust estimate of a single state variable.

Various ways of implementing complementary filter can be found by (Higgins, 1975) where the complementary filter is shown to be simpler and involves less computation than KF and although the filter is usually designed without reference to Wiener or KF, it is related to both in a way that for a certain class of filtering problems, it is reduced to a steady state KF (i.e., a Wiener filter), many users of either complementary or KF do not appear to know this relationship well (Higgins, 1975) and in fact its principle is a basis for KF used in navigation in which case it is referred to as the complementary constraint. Proponents of the Kalman filtering approach work in the time domain and not much attention is given to the more specific transfer function part of the filtering problem (Higgins, 1975), complementary filters lend themselves to frequency domain interpretations that provide valuable insight into the filtering design process (Pascoal *et al*., 2000). There filter can also be found in literature of (Baldwin *et al*., 2007; Almeida and Chaparr, 1997; Williams, 2009; Fliege and Zolzer, 1993) and the various references therein.

DISCUSSION

A review of the target tracking algorithms has been presented in this letter, grouped into deterministic and nondeterministic together with pros and cons of applying such algorithms, as well as current references on the topic.

On one hand estimation algorithms make the best utilization of the available data and knowledge of the system while on the other hand, they are possibly sensitive to model errors and might be computationally expensive, it is therefore necessary to have clear understanding of the model assumptions under which a particular algorithm is optimal and how it relate to the real world prior to its use. There is no single estimation algorithm that can be successful in all estimation problems, however, algorithms can be applied individually or in combination depending on which case can give better results. A good filter can be able to produce correct solutions within given tolerance which depends heavily on the measurement for which it (the filter) is candidate. The algorithm's speed of convergence (number of iterations) to the solution must also be put in consideration.

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