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Performance Analysis of Multiuser Scheduling System Based on Block Diagonalization Zero Forcing Transmission Strategy

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Abstract: As well known, Multiple-Input Multiple-Output (MIMO) technique can be used in conjunction with scheduling to further increase the throughput of the multiuser system. This study analyzes the expected throughput of the multiuser downlink scheduling system based on block diagonalization zero-forcing transmission strategy. We present four theorems to describe the correlation between any two users' mutual information in the system and give three estimations including one close-form approximation of the throughput of the system. It turns out that the multiuser diversity gain is significant when the number of antennas becomes large.

Key words: MIMO, multiuser diversity, block diagonalization zero-forcing

INTRODUCTION

As stated by Knopp and Humblet (1995), multiuser diversity is a kind of selection diversity among users. Its essence is a dynamic allocation of system resources (time slots, transmit power and so on) by proper scheduling algorithm to the user (or users) with best channel conditions so as to improve the throughput of the multiuser system.

In existing literature (Jiang *et al*., 2006; Yoo and Goldsmith, 2006; Shen *et al*., 2006; Bayesteh and Khandani, 2007) some suboptimal scheduling algorithms are proposed to decrease the complexity of multiuser selection, which are shown to be asymptotically optimal when the number of users in the system goes to infinity. However, until now there is no related paper that gives a close-form expression describing the throughput that multiuser scheduling system based on Block Diagonalization Zero-Forcing (BD-ZF) transmission (Spencer *et al*., 2004) can achieve and all previous analysis of suboptimal ZF algorithms that reduce the complexity of scheduling are based on Monte-Carlo simulation.

It is concluded in existing literature (Hochwald *et al*., 2004) that the channel quickly hardens as the number of antennas grows in the sense that the variance of mutual information deceases rapidly relative to its mean, which results in that the gain of the multiuser diversity decreases as the number of antennas grows, which is definitely not what we expect. Transmit strategy of time sharing to the strongest user is implicitly assumed in existing literature (Hochwald *et al*., 2004).

In this study, we consider the multiuser scheduling system based on BD-ZF transmission strategy as stated in existing literature (Spencer *et al*., 2004). Qualitatively, in this system, the transmitter can communicate with multiple users simultaneously, creating more spatial sub-channels than that based on transmit strategy of time sharing so as to utilize the spatial resources more effectively and the achievable throughput could be larger. In each scheduling time, the transmitter selects some users (m, for example) out of K users in the system and communicate with them based on BD-ZF transmission strategy to achieve maximum throughput as stated in existing literature (Bayesteh and Khandani, 2007). The selected user combination is assumed to be the optimum that could be found by an exhaustive search over all possible user combinations. To obtain a closed-form description of the achievable maximum throughput, we need to obtain the distribution of the throughput achieved by arbitrary use combination. Therefore, it is necessary to derive the correlation between any two users' mutual information. We analyze the correlation and give simple approximations in the case of large antenna arrays. Then, we give three estimations including one close-form estimation of the throughput of the multiuser scheduling system based on BD-ZF transmission strategy (Hochwald *et al*., 2004). The closed-form expression shows that the channel of multiuser scheduling system based on BD-ZF transmission strategy does not harden, i.e., the mutual information fluctuation increases with the increase of the number of antennas, which means that increasing the number of antennas brings more multiuser diversity gains. As we know from existing literature

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(Ravindran and Jindal, 2008), obtaining multiuser diversity requires the receivers to feed information about channel conditions back to the transmitter so that the transmitter can adapt the transmission to the conditions. The closed-form expression of the multiuser diversity enables us to consider if the scheduling and feedback advantage justify the overhead needed for the receivers to send the channel information back to the transmitter, though the feedback, which might be completed in future research, is not the focus of this study and not included within.

CHANNEL MODEL AND BLOCK DIAGONALIZATION ZERO FORCE ALGORITHM

A multiuser downlink channel with K users is considered in this study. The transmitter has n_t antennas and the jth receiver has n_{r_j} antennas. For convenience, it is assumed that each receiver has the same number of antennas, i.e., $n_{r_1} = ... = n_{r_k} = n_r$ and $n_t = mn_r$ where m is a constant integer. At most m users could be selected in each scheduling time due to the zero-forcing constraint. Let m users be indexed by ϕ_1 , ..., ϕ .2 In this m-user scheme, the $n_t\times1$ signal at the receiver ϕ_j can be described as:

$$y_{\phi_j} = H_{\phi_j}M_{\phi_j}x_{\phi_j} + \sum_{i=1,i\neq j}^{m} H_{\phi_j}M_{\phi_i}x_{\phi_i} + n_{\phi_i} \tag{1}$$

where, j = 1, ..., m, X_{ϕ_j} is the data vector of user ϕ_j with dimension n_r and n_{ϕ_j} is a $n_r\times1$ vector of independent zero-mean complex Gaussian noise with unit variance. The channel matrix H_{ϕ_j}, with dimension $n_r\times n_t$, has i.i.d. zero-mean complex Gaussian entries with variance 1/2 per real component. In this study, our focus is on the description of the maximum throughput and multiuser diversity gain of the multiuser scheduling system based on block diagonalization zero-forcing transmission strategy, so the way that the channel state information is obtained is out of our scope and H_{ϕ_j} is assumed to be known at both transmitter and receivers. Interested reader can refer to the limited feedback system considered in existing literature (Ravindran and Jindal, 2008), where each receiver knows its channel perfectly, but the transmitter is provided with a finite number of channel feedback bits from each receiver. The information symbol x_{ϕ_j} is modulated by the $n_t\times n_r$ matrix M_{ϕ_j} before entering spatial channel. M_{ϕ_j} is used to null out all interference from other users in the system. Let $\overline{H}_{\phi_j} = [H_{\phi_1}^H \cdots H_{\phi_{j-1}}^H\ H_{\phi_{j+1}}^H \cdots H_{\phi m}^H]^H$ and decompose $\overline{H}_{\phi_j}$ by singular value decomposition as:

$$\overline{H}_{\phi_j} = \overline{U}_{\phi_j}\overline{D}_{\phi_j}\overline{V}_{\phi_j}^H \tag{2}$$

Let $\overline{V}_{\phi_{j,0}}$ stand for the range space of $\overline{V}_{\phi_j}, \overline{V}_{\phi_{j,x}}$ stands for null space of $\overline{V}_{\phi_j}$. Then $\overline{V}_{\phi_{j,x}}$ forms an orthogonal basis of the null space of H_{ϕ_j} and its columns are naturally the candidates for the modulation matrix M_{ϕ_j}.

It can be seen that the throughput of this -user BD-ZF scheme is:

$$\hat{g}_{(\phi_1,\cdots,\phi_m)} = \sum_{k=1}^{m} \hat{u}_{\phi_k,(\phi_1,\cdots,\phi_{k-1},\phi_{k+1},\cdots,\phi_m)} \tag{3}$$

where, $\hat{u}_{\phi_k,[\phi_1,\cdots,\phi_{k-1},\phi_{k+1},\cdots,\phi_m]}$ is denoted as channel mutual information of user ϕ_k when modulation matrix M_{ϕ_j} is constituted by the null space matrix of space spanned by channel matrix of user ϕ_1, ..., ϕ_{k-1}, ϕ_{k+1}, ..., ϕ_m. Here, $\phi_1 \neq ... \neq \phi_m$. It is obviously that $u_{\phi_k,\{\phi_{k-1},\ \phi_{k+1},\ ...,\ \phi_m\}}$ is independent of the order of . So, we assume:, $\phi_1 \le \cdots \le \phi_{k-1} \le \phi_{k+1} \le \cdots \le \phi_m$ to identify the subscripts exclusively.

It is stated in Theorem 5 in existing literature (Jindal and Goldsmith, 2005) that in low SNR region TDMA (it means time sharing to the strongest user therein) is equivalent to the optimal DPC strategy. Because zero-forcing algorithm is more complex than TDMA and its performance is disastrous in low SNR region, it makes no sense to use ZF transmit strategy at low SNR region. These are the reasons that we assume high transmit SNR in the analysis in this study. In high SNR region, the difference between the performance achieved by waterfilling and that achieved by equal power allocation among subchannels is small (Goldsmith *et al*., 2003). Thus, we assume equal power allocation among subchannels in this study. And $\hat{g}_{\{\phi_1,\ ...,\ \phi_m\}}$ becomes:

$$\begin{aligned}\hat{g}_{[\phi_1,\cdots,\phi_m]} &= \sum_{k=1}^{m}\log\left|I + \frac{\rho}{mn_r}\overline{V}_{\phi_{k,x}}^H H_{\phi_k}^H H_{\phi_k}\overline{V}_{\phi_{k,x}}\right| \\ &\doteq \sum_{k=1}^{m}\log\left|\frac{\rho}{mn_r}\overline{V}_{\phi_{k,x}}^H H_{\phi_k}^H H_{\phi_k}\overline{V}_{\phi_{k,x}}\right| \\ &= mn_r\log\left(\frac{\rho}{mn_r}\right) + \sum_{k=1}^{m}\log\left|\overline{V}_{\phi_{k,x}}^H H_{\phi_k}^H H_{\phi_k}\overline{V}_{\phi_{k,x}}\right|\end{aligned} \tag{4}$$

where, ρ is the aggregate transmit SNR. The approximation in the second equation is widely used in performance analysis of MIMO channels (Bliss *et al*., 2002), which becomes more accurate when transmit SNR becomes larger.

We omit the first term of Eq. 4 in following analysis because it is fixed when fixing ρ whether there is multiuser selection or not. That is:

$$\begin{aligned}u_{\phi_k,[\phi_1,\cdots,\phi_{k-1},\phi_{k+1},\phi_m]} &= \log\left|\overline{V}_{\phi_{k,x}}^H H_{\phi_k}^H H_{\phi_k}\overline{V}_{\phi_{k,x}}\right| \\ g_{(\phi_1,\cdots,\phi_m)} &= \sum_{k=1}^{m}\log\left|\overline{V}_{\phi_{k,x}}^H H_{\phi_k}^H H_{\phi_k}\overline{V}_{\phi_{k,x}}\right|\end{aligned} \tag{5}$$

Let:

$$K_m = \binom{K}{m}$$

and denote the collection of the K_m throughputs achieved by all K_m user combinations as $\hat{g}_{\{\phi_1, \ldots, \phi_m\}}$. Our focus of this study is then on the description of the mean value of:

$$G_{K,m} = \max_{1 \le \phi_1 \cdots \le \phi_m \le K} g_{[\phi_1, \cdots, \phi_m]} \tag{6}$$

Note that the maximum throughput of this BD-ZF-transmit-strategy-based multiuser scheduling system is:

$$T_h(K, m, n_r, H_1, \cdots, H_K) = mn_r \log\left(\frac{\rho}{mn_r}\right) + G_{K,m} \tag{7}$$

GAUSSIAN APPROXIMATION OF $\log|HH^H|$

To obtain the distribution of $u_{\phi_k, \{\phi_{k-1}, \phi_{k+1}, \ldots, \phi_m\}}$, we first introduce the Gaussian approximation of $\log|HH^H|$ for the channel matrix with dimension. It is argued in existing literature (Hochwald *et al*., 2004) that $\log|HH^H|$ is asymptotically Gaussian and we can observe from simulation that the mean and variance are general enough to characterize the distribution of $\log|HH^H|$ when n_r and n_t are relatively large.

As stated in the proof of Theorem 3 ii) in existing literature (Hochwald *et al*., 2004), the mean value and variance of $\log|HH^H|$ is:

$$\begin{aligned} C_{n_r,n_t} &= -n_r\gamma + n_r \sum_{i=1}^{n_t-n_r} \frac{1}{i} + \sum_{i=1}^{n_t-1} \frac{i}{n_t - i}, \prod_{n_r,n_t} \\ &= n_r\left[\frac{\pi^2}{6} - \sum_{i=1}^{n_t-1} \frac{1}{i^2}\right] + \sum_{i=1}^{n_r-1} \frac{i}{(n_t - n_r - i)^2} \end{aligned} \tag{8}$$

where, $\gamma = 0.5772\ldots$ is Euler constant. For high SNR and large n_r, n_r, fixing:

$$\frac{n_r}{n_t} = m > 1$$

the variance of can be approximated as in Eq. 26 in existing literature (Hochwald *et al*., 2004):

$$\prod_{n_r,n_t} = \log\left(\frac{m}{m-1}\right) \tag{9}$$

Moreover:

$$\prod_{n_r,n_t} = \log n_r + \gamma + 1 + o(1)$$

Two corollaries can be obtained easily.

Corollary 1:

$$\frac{\prod_{n_r,mn_r}}{\prod_{n_r,n_r}}$$

tends to zero when m tends to positive infinite and n_r is fixed.

Corollary 2:

$$\frac{\prod_{n_r,mn_r}}{\prod_{n_r,n_r}}$$

tends to zero when n_r tends to positive infinite and m keeps constant.

It can also be obtained from Lemma A.2 that:

$$\begin{aligned} &E\left[\log\left|\overline{V}^H_{\phi_k,x} H^H_{\phi_k} H_{\phi_k} \overline{V}_{\phi_k,x}\right|\right] = C_{n_r,n_t}, \\ &X\left[\log\left|\overline{V}^H_{\phi_k,x} H^H_{\phi_k} H_{\phi_k} \overline{V}_{\phi_k,x}\right|\right] = \prod_{n_r,n_t} \end{aligned} \tag{10}$$

THE ANALYSIS OF THE CORRELATION

In this study, we obtain the mean value of $G_{K,M}$ by means of its distribution function. Thus, in this section, we analyze the correlation between $u_{\phi_k, \{\phi_1, \ldots, \phi_{k-1}, \phi_{k+1}, \ldots, \phi_m\}}$ and $u_{\phi_i, \{\phi, \ldots, \phi_{i-1}, \phi_{i+1}, \ldots, \phi_m\}}$ with four theorems below and give some approximations.

Theorem 1: If $k_1 = k_2$, $\phi_i = \beta_i \ne k_1$ i = 1, ..., s for i = 1, ..., s and $\phi_i \ne \beta_i \ne k_1$ for i = 1, ..., m-1, then:

$$\begin{aligned} &E\left[u_{k_1},[\phi_1, \cdots, \phi_{m-1}]u_{k_2},[\beta_1, \cdots, \beta_{m-1}]\right] \\ &= \prod_{n_r,(m-s)n_r} + \left(C_{n_r,n_r}\right)^2 \end{aligned} \tag{11}$$

Theorem 2: If $k_1 \ne k_2$, $\phi_i = \beta_i \ne k_1 \ne k_2$ for i = 1, ..., s and $\phi_i \ne \beta_i \ne k_1 \ne k_2$ for , i = s + 1, ..., m-2, then:

$$\begin{aligned} &E\left[u_{k_1},[k_2, \phi_1, \cdots, \phi_{m-2}]u_{k_2},[k_1, \beta_1, \cdots, \beta_{m-2}]\right] \\ &= \prod_{n_r,(m-s-1)n_r} - \prod_{n_r,(m-s)n_r} + \left(C_{n_r,n_r}\right)^2 \end{aligned} \tag{12}$$

Theorem 3: If $k_1 \ne k_2$ $\phi_1 \ne \ldots \ne \phi_{m-1} \ne k_1 \ne k_2$ and $\beta_1 \ne \ldots \ne \beta_{m-2} \ne k_1 \ne k_2$ then:

$$E\left[u_{k_1},[\phi_1, \cdots, \phi_{m-1}]u_{k_2},[k_1, \beta_1, \cdots, \beta_{m-2}]\right] = \left(C_{n_r,n_r}\right)^2 \tag{13}$$

Theorem 4: If $k_1 \neq k_2$, $\phi_1 \neq \ldots \neq \phi_{m-1} \neq k_1 \neq k_2$ and $\beta_1 \neq \ldots \neq \beta_{m-1} \neq k_1 \neq k_2$ then:

$$E\left[u_{k_1},[\phi_1,\cdots,\phi_{m-1}]u_{k_2},[\beta_1,\cdots,\beta_{m-1}]\right]=\left(C_{n_r,n_r}\right)^2 \tag{14}$$

The proof of the Theorem are omitted due to space limitation.

From Eq. 5 and Theorem 1-4, it can be easily obtained that:

$$E\left[g_{\{\phi_1,\cdots,\phi_m\}}\right]=mC_{n_r,n_r},X\left[g_{\{\phi_1,\cdots,\phi_m\}}\right]=m^2\prod{}_{n_r,n_r} \tag{15}$$

Approximations of the correlation: The correlation matrix of the elements in $\{g_{\{\phi_1,\ldots,\phi_m\}}\}$, with dimension $K_m \times K_m$, would be very large and complex when the number of users in the system is large. So we consider the following approximations. Denote $\theta_{uk1, \{\phi1, \ldots, \phi m\text{-}d\}uk2, \{\beta1,\ldots, \beta m\text{-}1\}}$ as correlation coefficient between u_{k1}, $[\phi_1,\ldots, \phi_{m-1}]$ and u_{k2}, $[\beta_1,\ldots, \beta_{m-1}]$.

Corollary 3: Under the condition in Theorem 1:

$$\theta_{u_{k_1},\{\phi_1,\cdots,\phi_{m-1}\}u_{k_2},\{\beta_1,\cdots,\beta_{m-1}\}}=\frac{\prod_{n_r,(m-s)n_r}}{\prod_{n_r,n_r}} \tag{16}$$

which tends to zero when n_r tends to positive infinite according to Corollary 2.

Corollary 4: Under the condition in Theorem 2, when $s \neq m-2$:

$$\theta_{u_{k_1},\{k_2,\phi_1,\cdots,\phi_{m-2}\}u_{k_2},\{k_1,\beta_1,\cdots,\beta_{m-2}\}}=\frac{\prod_{n_r,(m-s-1)n_r}-\prod_{n_r,(m-s)n_r}}{\prod_{n_r,n_r}} \tag{17}$$

which tends to zero when n_r tends to positive infinite according to Corollary 2.

Corollary 5: Under the condition in Theorem 2, when $s = m-2$:

$$\theta_{u_{k_1},\{k_2,\phi_1,\cdots,\phi_{m-2}\}u_{k_2},\{k_1,\beta_1,\cdots,\beta_{m-2}\}}=\frac{\prod_{n_r,n_r}-\prod_{n_r,2n_r}}{\prod_{n_r,n_r}} \tag{18}$$

which tends to 1 when n_r tends to positive infinite according to Corollary 2.

THROUGHT AND PERFORMANCE OF A MULTIUSER DIVERSITY

Based on the correlation analysis, we give descriptions on the achievable throughput of the BD-ZF-transmission-strategy-based multiuser scheduling system. Because the accurate description is quite difficult to obtain, we give three estimations based on different approximations.

Estimation I: From Corollary 3 to 5, we can see that: for any two sets $\{\alpha_1,\ldots, \alpha_m\}$ and $\{\beta_1,\ldots, \beta_m\}$ where there is at least one corresponding element is unequal, it can be approximated that $u_{\alpha k, \{\alpha_1,\ldots, \alpha_{k-1}, \alpha_{k+1},\ldots, \alpha_m\}}$ is totally correlated with $u_{\alpha n, \{\alpha_1,\ldots, \alpha_{n-1}, \alpha_{n+1},\ldots, \alpha_m\}}$ and is totally uncorrelated with $u_{\beta_n, \{\beta_1,\ldots, \beta_{n-1}, \beta_{n+1},\ldots, \beta_m\}}$, here, $1 \leq k, n \leq m$ and $k \neq n$.

Subsequently, $g_{\{\alpha_1,\ldots, \alpha_m\}}$ could be approximated by a Gaussian variable with mean $mC_{n_r n_r}$ and variance $m^2\prod_{n_r n_r}$. Moreover, $g_{\{\alpha 1,\ldots, \alpha_m\}}$ is approximatively uncorrelated with $g_{\{\beta 1,\ldots, \beta_m\}}$ as long as there is at least one corresponding element unequal to each other between sets $\{\alpha 1,\ldots, \alpha_m\}$ and $\{\beta 1,\ldots, \beta_m\}$.

Because un-correlation means independence for Gaussian variables, it can be approximated that $g_{\{\alpha 1,\ldots, \alpha_m\}}$ and $g_{\{\beta 1,\ldots, \beta_m\}}$ are independent. According to the theorem that the distribution function of m i.i.d. variables with distribution function F (x) is F^m (x), we can get an optimistic estimation of $G_{K, m}$, denoted as U (o), whose distribution function is:

$$F_{U_0(g)}=f^{K_m}(g) \tag{19}$$

where,

$$f(x)=\frac{1}{\sqrt{2\pi m^2\prod_{n_r,n_r}}}e^{-\frac{\left(x-mC_{n_r,n_r}\right)^2}{2m^2\prod_{n_r,n_r}}} \tag{20}$$

The mean value of U_o can be obtained by means of numerical integration.

Estimation II: Here, the correlation between the elements of $\{g_{\{\phi 1,\ldots, \phi m\}}\}$ is omitted totally. One may ask, how much does the correlation influences the final multiuser diversity gain. We begin with a lemma that describes the relationship between two set of variables, one with correlation and one without correlation.

Lemma 1 (Owen and Steckm, 1962): Let Z_1, ..., Z_n be jointly normal distributed random variables with E $[Z_i] = 0$, $E[Z_i^2]=1$, $E[Z_iZ_j] = \rho$ and $-1/n-1 \leq p \leq 1$ for $i \neq j$. Further, the collection of $\{Z_i\}$ is ordered so that $Z^{(1)} \geq \ldots \geq Z^{(n)}$. Let X_1, ..., X_n be jointly normal distributed random variables with same moments and product moments with Z_1, ..., Z_n except that E $[X_i X_j] = 0$ for $i \neq j$ and $X^{(1)} \geq \ldots \geq X^{(n)}$. Then, the first order moment of Z^i and X^i is related by:

$$E\left[Z^{(i)}\right]=(1-\rho)^{\frac{1}{2}}E\left[X^{(i)}\right] \tag{21}$$

Consider a set of jointly normal distributed variables $\Delta_1, ..., \Delta_n$ that have the same moments and product moments with $Z_1, ..., Z_n$ except that $E[\Delta_i \Delta_j] = \rho_{ij}$ for $i \neq j$ and $\Delta_1 \geq ... \geq \Delta_j$, here, $\rho_{ij} \leq \rho$ for any $i \neq j$. As we know, correlation decreases the fluctuations among variables $\Delta_1, ..., \Delta_n$. Larger correlation leads to smaller fluctuations. Thus:

$$E\left[Z^{(1)}\right] \leq E\left[\Delta^{(1)}\right] \quad (22)$$

Returning back to our problem considered now, if the elements of $\{g_{(\phi 1, ..., \phi m)}\}$ are indeed Gaussian variables (not approximately), then the maximum correlation coefficient between any two elements in $\{g_{(\phi 1, ..., \phi m)}\}$ is equal to $\theta_{g[1,...,m-1,\alpha_m]g[1,...,m-1,\beta_m]}$, here, $\alpha_m \neq \beta_m$. Let:

$$\begin{aligned}\rho &= \theta_{g[1,\cdots,m-1,\alpha_m]g[1,\cdots,m-1,\beta_m]} \\ &= \frac{E\left[g[1,\cdots,m-1,\alpha_m]g[1,\cdots,m-1,\beta_m]\right]}{X\left[g[1,\cdots,m-1,\alpha_m]\right]X\left[g[1,\cdots,m-1,\beta_m]\right]} - \\ &\frac{E\left[g[1,\cdots,m-1,\alpha_m]\right]E\left[g[1,\cdots,m-1,\beta_m]\right]}{X\left[g[1,\cdots,m-1,\alpha_m]\right]X\left[g[1,\cdots,m-1,\beta_m]\right]} \\ &= \frac{\frac{m(m-1)}{2}\prod_{n_r,n_r} - \frac{(m-2)(m-1)}{2}\prod_{n_r,3n_r}}{m^2\prod_{n_r,n_r} - (m^2-m)\prod_{n_r,2n_r}}\end{aligned} \quad (23)$$

Let $Y_1, ..., Y_{KM}$ be jointly normal distributed random variables with same moments and product moments with the elements of $\{g_{(\phi 1, ..., \phi m)}\}$ except that $E[Y_i Y_j] = \rho$ for $i \neq j$ and $Y^{(1)} \geq ... \geq Y^{(K_m)}$. Denote $A_p = Y^{(1)}$. Then:

$$E\left[G_{K,m}\right] \geq E\left[A_p\right] = (1-\rho)\frac{1}{2}E[U_0] \quad (24)$$

Because the Gaussian behavior of $\{g_{(\phi 1, ..., \phi m)}\}$ is only an approximation, so Eq. 24 is not always true in fact, which is the reason that we do not call $E[A_p]$ as a low bound. $E[A_p]$, however, provides us an explicit expression to understand the influence of correlation between the elements of $\{g_{(\phi 1, ..., \phi m)}\}$. As we can see, $\rho \to 0$ when n_r tends to infinite. So the estimation $E[A_p]$ and estimation I $E[U_0]$ converge when n_r tends to infinite. It turns out that the influence of correlation between the elements $\{g_{(\phi_1 ... \phi_m)}\}$ of can be ignored with the increase of the antenna number.

Estimation III: A close-form approximation: To get a close-form expression of the relationship between the maximum throughput and m, K and n, from above distribution function, we further consider the following.

Lemma 2: Let $Z_1, ..., Z_K$ be a sequence of independent Gaussian random variables with mean μ and variance δ^2. Define $M_K = \max(Z_1, ..., Z_K)$. Then:

$$\lim_{K\to\infty} \frac{M_K - \mu}{\sqrt{2\delta^2 \log(K)}} = 1 \quad (25)$$

The lemma is used (Hochwald *et al*., 2004), it says that:

$$M_K \sim \mu + \sqrt{2\delta^2 \log(K)}$$

for large K. In a Round-Robin based BD-ZF multiuser scheduling system, the mean throughput can be easily obtained as $mC_{n_t n_r}$. Similar to that in Estimation I, we approximate that the mutual information of any two users are independent. Applying Lemma 2 to the scheduling algorithm described in this paper, we have the following close-form expression:

$$\lim_{\substack{K\to\infty \\ n_r\to\infty}} E\left[G_{K,m}\right] = mC_{n_r,n_r} + \sqrt{2m^2\prod_{n_r,n_r}\log(K_m)} \quad (26)$$

which means that, compared with Round-Robin based BD-ZF system, the multiuser diversity gain that can be obtained from scheduling is:

$$d_{mu} = \sqrt{2m^2\prod_{n_r,n_r}\log(K_m)} \quad (27)$$

Assume that each channel is Gaussian and eventually becomes the best. We see that the multiuser diversity is significant if either $m^2\prod_{n_r n_r}$ or K is large.

SIMULATIONS AND DISCUSSION

Here, theoretical analysis is compared with simulation results. Moreover, we compare the performance of the multiuser scheduling system based on BD-ZF transmission strategy with the system presented in existing literature (Hochwald *et al*., 2004) and get a new knowledge of the relationship between multiuser diversity and the size of antenna arrays.

In Fig. 1, transmit SNR is 10dB, the number of users is 10 and the number of receiver antennas in each receiver is 4. The horizontal axis is the ratio of the number of transmit antennas and that of receive antennas (because receive antenna number is fixed in this simulation, the throughput can also be seen as a function of the number of transmit antennas). Our analysis gives relative accurate estimations of practical throughput. We can see that the throughput increases with the increase of the number of transmit antennas, which is opposite with the conclusion in existing literature (Hochwald *et al*., 2004) that the channels harden and the multiuser diversity gain decreases with the increase of the number of transmit antennas in multiuser scheduling system based on time-sharing transmission strategy. Thus, it turns out that BD-ZF would be a better choice than time-sharing in high SNR region.

In Fig. 2 and 3, transmit SNR is set as 10 dB and the number of transmit antennas and receive antennas are 8 and 4 in Fig. 2, the number of transmit antennas and receive antennas are 12 and 6 in Fig. 3, respectively. In both figures the throughput are plotted as the function of the number of users in the system. As we can see, the throughput of the multiuser scheduling system increases on the base of round-robin system. It is stated in existing literature (Hochwald *et al*., 2004) that the multiuser gain as a function of K is only as the square-root logarithm of K, which is very small. However, as we can see from Eq. 26, in the BD-ZF-transmission-strategy-based multiuser scheduling system, the multiuser gain as a function of K is as the square-root logarithm of K_m, which is far larger than that in existing literature (Hochwald *et al*., 2004) even if m is moderate. Moreover, the scaling factor:

$$\sqrt{2m^2\prod_{n_T,n_r}}$$

is larger than:

$$\sqrt{2\prod_{n_T,n_r}}$$

in existing literature (Hochwald *et al*., 2004). It verifies the superiority of BD-ZF transmit strategy to time-sharing transmit strategy in high SNR region.

As can be seen from Fig. 2 and 3 that, with the increase of k the difference between the theoretical approximations and the simulation results become larger, the reason of which is the same as that explained in existing literature (Hochwald *et al*., 2004). Since Gaussian approximations improve when the antenna array at the transmitter and the users become larger, the close-form estimation can be applied with increasing confidence as n_T and n_r grow. And it would become inaccurate if only increasing k without increasing n_T and n_r.

Since in high SNR region, ZF is asymptotically optimal compared with DPC, we conjecture that for a multiuser scheduling system with multiple antennas at each user, it would be better to use time-sharing transmit strategy in low SNR region and BD-ZF transmit strategy in high SNR region which is verified in Fig. 4.

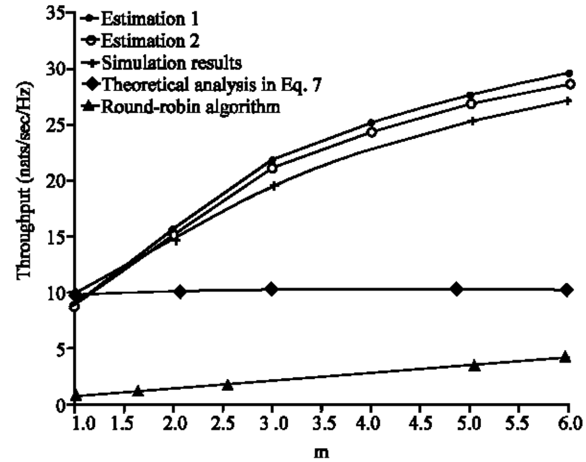

Fig. 1: Throughput as a function of the number of transmit antennas in the system

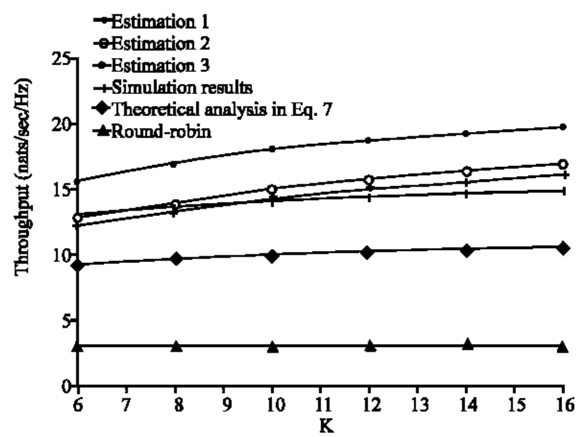

Fig. 2: Throughput as a function of the number of users in the system when $n_r = 4$

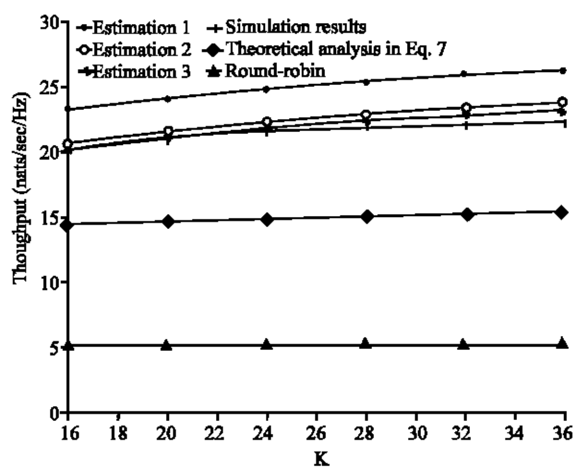

Fig. 3: Throughput as a function of the number of users in the system when $n_r = 6$

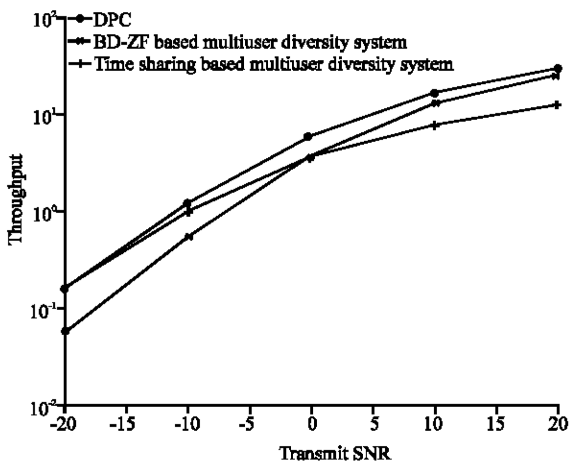

Fig. 4: Throughput comparison of multiuser system based on different transmission strategy as a function of transmit SNR

CONCLUSION

In this study, we analyze the throughput of the multiuser scheduling system based on BD-ZF transmit strategy, obtaining some relatively accurate estimations including one closed-form expression. We conclude that in high SNR region, the throughput of the multiuser downlink system could grow with the increase of antenna array, which is opposite to the conclusion of channel hardening with the increase of the number of transmit and receive antennas, presented in existing literature (Hochwald *et al.*, 2004) where time-sharing transmit strategy is used even in high SNR region. Due to the low complexity and good performance of time-sharing in low SNR region, we think that it would be better to use time-sharing transmit strategy in low SNR region and use BD-ZF transmit strategy in high SNR region.

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NOTATIONS

we use lower case letters v to denote vectors, capital letters M to denote matrices. E[·] and X[·] stand for the expectation and variance of the variables, respectively. $[\cdot]^T$ and $[\cdot]^H$ denote the matrix transposition and conjugated transposition operations. $|\cdot|$ is the matrix determinant.

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