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Review on Selected Channel Estimation Algorithms for Orthogonal Frequency Division Multiplexing System

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Abstract: This study reports a detailed review on selected channel estimation algorithms for orthogonal frequency division multiplexing communication systems. Pilot based channel estimation algorithms such as least square, minimum mean square error, maximum likelihood and decision feedback estimators are discussed and also compared in terms of their simplicity, computational cost and suitability conditions. Moreover Subspace Based Blind channel estimation algorithm for cyclic prefix system is described along with its limitation of applicability for fast varying wireless channels. References for the report and other estimating algorithms which are not considered here are also cited.

Key words: Inter symbol interference, channel estimation, equalization, Orthogonal Frequency-Division Multiplexing (OFDM)

INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) (Li and Stuber, 2006) is a promising multicarrier digital communication technique for transmitting high bit-rate data over wireless communication channels. The indispensable obstacle for most wireless communications systems is the multipath channel that causes Inter Symbol Interference (ISI). Reliable channel estimation and tracking is a fundamental step for recovering the transmitted symbols in the presence of ISI for coherent detection. OFDM systems are also more sensitive to the frequency offset (Zhang and Lindner, 2005a, b, 2007) which results in the loss of orthogonality among sub-carriers and causes intercarrier interference (ICI). ICI degrades the performance of both the channel estimation and symbol detection. So, due consideration should be given to frequency offset along with ISI. Channel state information can be obtained in two ways: One way is to insert known symbols (pilots) into the data on several sub-carriers in frequency and time dimension. This approach is more feasible even though there is a significant bandwidth loss due to pilot tones. This has motivated development of blind channel estimation methods which possess desirable advantages of better bandwidth efficiency. In this approach need for a pilot sequence is replaced by

some knowledge of statistical characteristics of received signal. However, many blind methods suffer from several drawbacks which prevent them from widespread use.

Pilot based channel estimation in OFDM is a two-dimensional (2-D) problem which means channels needs to be estimated in time -frequency domain. Hence, 2-D methods could be applied to estimate channel from pilots. However, due to computational complexity of 2-Destimators (Sanzi et al., 2003), the scope of channel estimators can be limited to one-dimensional (1-D).The idea behind 1-D channel estimators are usually adopted in OFDM systems to achieve tradeoff between complexity and accuracy. The focus of this review report is based on 1-D methods. The two basic types of pilot arrangements used in OFDM 1-D channel estimations are block type and comp type in which pilots are inserted in frequency and time direction respectively as shown in Fig. 1a and b.

Following earlier developments of Blind equalizations algorithms for Single Input Single Output (SISO) (Shalvi and Weinstein, 1990) and Single-Input and Multiple Output (SIMO) (Moulines et al., 1995; Tong et al., 1991) systems, in recent years, block transmission systems using Linear Redundant Precoders (LRP) have become popular due to their capability to facilitate block channel equalization of frequency selective channels. Blind estimation with LRP estimation with LRP

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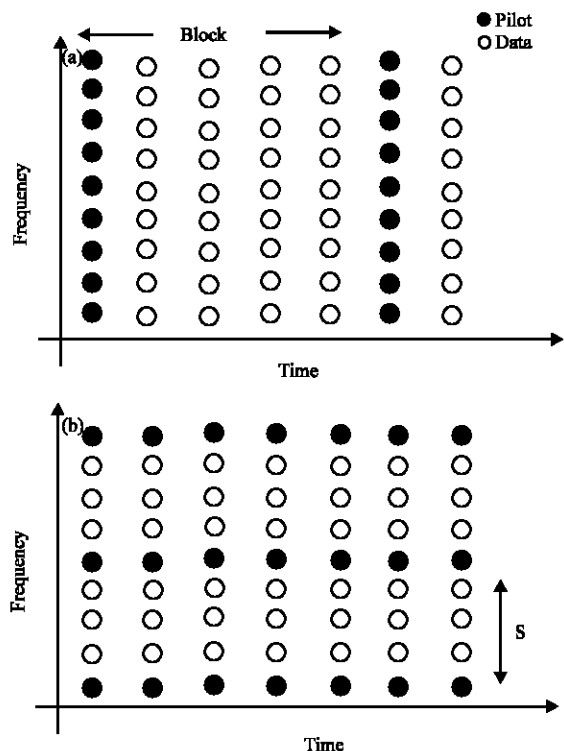Tel: +86124500168 Fax: +8682589811

Fig. 1: (a) Block-type. (b) Comb-type

has a small band width expansion factor which is asymptotically unity and is robust to channel order overestimation. Hence, blind methods developed in LRP systems are considered superior to those in SIMO systems. As block transmission systems using redundant precoding, such as Orthogonal Frequency Division Multiplexing (OFDM) systems, become progressively more popular and research on blind channel estimation is recently given much attention. Recent developmental work on block transmission systems with redundant precoding (Pham and Manton, 2003; Su, 2008) has shown that redundancy, originally introduced for purpose of eliminating interblock interference (IBI), is useful to blind channel estimation. Many blind methods with different types of redundant precoding have been developed for block transmission and proved to be free from several limitations present in conventional blind channel estimation (Moulines *et al.*, 1995). These new algorithms, however, still have several problems such as slow convergence speed due to requirement of a large amount of received data which makes them less applicable in an environment where channel status is fast varying. Computational complexity, constraints on data constellations are also some of the problems to be mentioned. Subspace based blind channel estimation

algorithms are widely considered for OFDM systems due to reasons stated by Su, (2008) and are considered for cyclic prefix (cp)system in this report.

Estimators based on Least Square (LS), Minimum Mean Square Error (MMSE) and Maximum Likelihood (ML) are considered and compared for pilot based and the widely used subspace based algorithm is treated for blind based channel estimation.

Notation: Normal letters represent scalar quantities. Bold symbols denote matrices and vectors. The Discrete Fourier Transform (DFT) matrix of size M×M is given by:

$$\mathbf{F} = \begin{bmatrix} W_M^{00} & \cdots & W_M^{0(M-1)} \\ & \ddots & \\ W_M^{(M-1)0} & \cdots & W_M^{(M-1)(M-1)} \end{bmatrix}$$

where:

$$W_M^{i,l} = \frac{1}{\sqrt{M}}\left[e^{-j\frac{2\pi}{M}il} \right], \quad i,l \in 0,1,\ldots,M-1$$

$\mathbf{F}^H$ denotes The inverse FFT, $(.)^*$, $(.)^T$, $(.)^H$ denote conjugate, transpose and transpose conjugate respectively. I_M denotes identity matrix with M size, $\Re$ denotes real part, $\Im$ imaginary part, $E\{.\}$ is statistical expectation.

OFDM SYSTEM MODEL

The main idea of Orthogonal Frequency Division Multiplexing (OFDM) transmission is to turn the channel convolutional effect in to multiplicative one. The complete base band OFDM system model is shown in Fig. 2 and described.

Let M be OFDM block length, *i* be block number and consider the processing of *i* th block. After serial to parallel conversion the data will be modulated. Then, we take the IFFT of the block as given in Eq. 1. After taking IFFT, a cyclic prefix of length P is inserted to each block. Note that in OFDM systems instead of inserting a cyclic prefix, a set of P zeros can also be added as the guard interval called Zero Padding (ZP).

Suppose **x** be the M by 1 vector obtained by taking the IFFT of the symbol vector **X**:

$$\mathbf{x}^i = \mathbf{F}^H \mathbf{X}^i \tag{1}$$

where, **F** is the M×M FFT matrix $\mathbf{x}^i = [x^i(0), \cdots, x^i (M-1)]^T$ and let the channel length is L. At receiver if we consider M+P symbols received which are y^i (-P), y^i (-P+1), $\cdots$, y^i (M-1) we have:
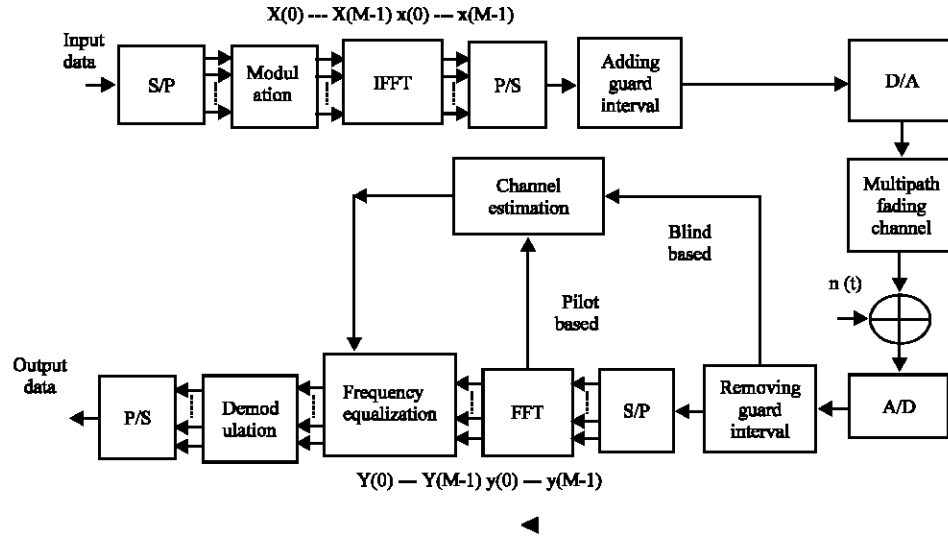
X(0) --- X(M-1) x(0) --- x(M-1)

Fig. 2: OFDM baseband model

$$
\begin{bmatrix} y^i(-P) \\ y^i(-P+1) \\ \vdots \\ y^i(-2) \\ y^i(-1) \\ y^i(0) \\ y^i(1) \\ \vdots \\ y^i(M-1) \end{bmatrix}_{(M+P)\times 1} = \underbrace{\begin{bmatrix} h_{L-1} & h_{L-2} & \cdots & h_0 & \cdots & \cdots & 0 \\ 0 & h_{L-1} & h_{L-2} & \cdots & h_0 & \cdots & 0 \\ 0 & 0 & h_{L-1} & h_{L-2} & \cdots & h_0 & 0 \\ 0 & \cdots & \cdots & h_{L-1} & h_{L-2} & \cdots & h_0 \end{bmatrix}}_{(M+P)\times(M+P+L-1)} \times \begin{bmatrix} x^{i-1}(M-L+1) \\ x^{i-1}(M-L+2) \\ \vdots \\ x^{i-1}(M-1) \\ x^i(M-P) \\ x^i(M-P+1) \\ \vdots \\ x^i(M-1) \\ x^i(0) \\ x^i(1) \\ \vdots \\ x^i(M-1) \end{bmatrix}_{(M+P+L-1)\times 1}
$$

(2)

Before any processing, the cyclic prefix of length P is discarded from each block. If P≥L by discarding first P symbols received we get:

$$
\begin{bmatrix} y^i(0) \\ y^i(1) \\ \vdots \\ y^i(M-1) \end{bmatrix} = \begin{bmatrix} h_{L-1} & h_{L-2} & \cdots & h_0 & \cdots & \cdots & 0 \\ 0 & h_{L-1} & h_{L-2} & \cdots & h_0 & \cdots & 0 \\ 0 & 0 & h_{L-1} & h_{L-2} & \cdots & h_0 & 0 \\ 0 & \cdots & \cdots & h_{L-1} & h_{L-2} & \cdots & h_0 \end{bmatrix} \times \begin{bmatrix} x^i(M-L-1) \\ x^i(M-L) \\ \vdots \\ x^i(M-1) \\ x^i(0) \\ x^i(1) \\ \vdots \\ x^i(M-1) \end{bmatrix}
$$

(3)

which can be written as:

$$
\begin{bmatrix} y^i(0) \\ y^i(1) \\ \vdots \\ y^i(M-1) \end{bmatrix} = \begin{bmatrix} h_0 & 0 & h_{L-1} & \cdots & & h_1 \\ & \ddots & & \ddots & & \\ h_{L-1} & & \ddots & & h_{L-1} & \\ & \ddots & & \ddots & & 0 \\ & & \ddots & & \ddots & \\ 0 & \cdots & h_{L-1} & \cdots & & h_0 \end{bmatrix} \times \begin{bmatrix} x^i(0) \\ x^i(1) \\ \vdots \\ x^i(M-1) \end{bmatrix}
$$

(4)

$$
\mathbf{y}^i = \mathbf{H}\mathbf{x}^i
$$

(5)

where, $\mathbf{H}$ is an M by M circulant matrix with first column being $[h_0,..., h_{L-1}, 0,...,0]^T$ Next, we take FFT of received block y^i and obtain:

$$
\mathbf{Y}^i = \mathbf{F}\mathbf{y}^i = \mathbf{F}\mathbf{H}\mathbf{F}^H\mathbf{X}^i
$$

(6)

where, $\mathbf{Y}^i = [Y^i(0),..., Y^i(N-1)]^T$ Since $\mathbf{H}$ is circulant, we have:

$$
\mathbf{F}\mathbf{H}\mathbf{F}^H = \text{diag}[H(0),..., H(N-1)]
$$

(7)

where, H_k is k th component of M point FFT of the channel:

$$
H_k = \sum_{n=0}^{L-1} h_n e^{-j2\pi\frac{kn}{M}}
$$

(8)

Due to diagonal structure of $\mathbf{H}$ we have:

$$
Y^i(k) = H_k X_k^i + N_k, \quad k \in (0, M-1)
$$

(9)

where, we have included the noise term N_k. Equation 9 demonstrates that an OFDM system is equivalent to a transmission of data over a set of parallel channels.

In regard to channel estimation block, for the pilot based one pilot signals are extracted after the FFT block to compute the channel parameters. For blind estimation information in the CP is processed to compute the channel parameters. After obtaining the channel parameters, phase and amplitude distortion is compensated before demodulation.

PILOT BASED CHANNEL ESTIMATION ALGORITHMS

Pilot based channel estimation is mostly adopted for OFDM communication systems.

Inserting known signal by receiver so that the channel can be estimated at reference value location carries it out. The entire channel can then be estimated. Different channel estimation algorithms are considered for block type and comb type pilot arrangements next.

BLOCK TYPE CHANNEL ESTIMATION

The block type pilot arrangement is shown in Fig. 1a and developed for channel, which is slow fading. Pilot signals are transmitted periodically on all subcarriers. Given pilot signals $\mathbf{X}$ and received signals $\mathbf{Y}$, channel frequency response $\mathbf{H}$ will be estimated. The estimated channel shall be used to decode received data inside the block till next pilot symbols are received. Appropriate methods will be applied as described below:

LS estimator: relation between transmitted signal X_k and received signal Y_k is already stated in Eq. 9 where H_k is channel transfer function at kth subcarrier and N_k is noise for an OFDM system with M carriers, the observations can be computed as follows:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_M \end{bmatrix} = \begin{bmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \cdots & \vdots \\ \vdots & \cdots & \ddots & 0 \\ 0 & \cdots & 0 & X_M \end{bmatrix} \cdot \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_M \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_M \end{bmatrix}$$

$$\mathbf{Y} = \mathbf{X}.\mathbf{H}+\mathbf{N} \qquad (10)$$

The least square estimate of such system is obtained by minimizing square distance between the received signal $\mathbf{Y}$ and the transmitted signal $\mathbf{X}$ as (Manolakis *et al.*, 2000). LS estimator minimizes the parameter:

$$\min J\ (\mathbf{H}) = \min\ \{|\mathbf{Y}\text{-}\mathbf{X}\mathbf{H}|^2\}$$

$$= \min\ \{(\mathbf{Y}\text{-}\mathbf{X}\mathbf{H})^H(\mathbf{Y}\text{-}\mathbf{X}\mathbf{H})\}$$

$$= \min\ \{\mathbf{Y}^H\mathbf{Y}\text{-}\mathbf{Y}^H\mathbf{X}\mathbf{H}\text{-}\mathbf{H}^H\mathbf{X}^H\mathbf{Y}+\mathbf{H}^H\mathbf{X}^H\mathbf{X}\mathbf{H}\}$$

Differentiating cost function with respect to $\mathbf{H}^H$ and finding the minima, we get:

$$\frac{\partial}{\partial \mathbf{H}^H}J(\mathbf{H}) = -\mathbf{X}^H\mathbf{Y} + \mathbf{X}^H\mathbf{X}\mathbf{H} = 0$$

$$= \mathbf{X}^{-1}\,(\mathbf{X}\mathbf{H})^{-1}.\ \mathbf{X}^H\mathbf{Y}$$

$$= \mathbf{X}^{-1}.\ \mathbf{Y}$$

Hence the LS estimate of the channel is:

$$\widehat{\mathbf{H}}_{LS} = \begin{bmatrix} \dfrac{Y_1}{X_1} & \dfrac{Y_2}{X_2} & \cdots & \dfrac{Y_M}{X_M} \end{bmatrix}^T \qquad (11)$$

Without using knowledge of the statistics of the channels, LS estimators can be computed with very low complexity but it suffers from high mean square error (Van-de-Beek *et al.*, 1995).

LMMSE estimator: LMMSE estimator utilizes second order statistics of the channel conditions to minimize mean square error. The estimated channel can be computed as:

$$\widehat{\mathbf{H}}_{LMMSE} = \mathbf{R}_h\left(\mathbf{R}_h + \sigma_n^2\left(\mathbf{X}\mathbf{X}^H\right)^{-1}\right)\widehat{\mathbf{H}}_{LS} \qquad (12)$$

It is assumed that autocorrelation matrix of the channel $\mathbf{R}_h$ and noise variance σ_n^2 are known in advance by the receiver and can be computed as:

$$\mathbf{R}_h = E\left\{\widehat{\mathbf{H}}\widehat{\mathbf{H}}^H\right\}$$

and:

$$\mathbf{R}_n = E\left[\mathbf{N}\mathbf{N}^H\right] = \sigma_n^2\mathbf{I}$$

$\widehat{\mathbf{H}}_{LS}$ is a rough LS estimate of the channel computed in previous section. Much better performance can be achieved using LMMSE estimator, especially under low SNR scenarios (Van-de-Beek *et al.*, 1995). LMMSE estimator has considerable complexity since a matrix inversion is needed every time data in $\mathbf{X}$ changes. Complexity of this estimator can be reduced by replacing $(\mathbf{X}\mathbf{X}^H)^{-1}$ with its expectation $E\{(\mathbf{X}\mathbf{X}^H)^{-1}\}$. Assuming the same signal constellation on all tones and equal probability on all constellation points, we have:

$$E\left\{\left(\mathbf{X}\mathbf{X}^H\right)^{-1}\right\} = E\left\{\left|\frac{1}{X_k}\right|^2\right\}\mathbf{I}\,.$$

where, $\mathbf{I}$ is the identity matrix. Defining average signal to noise ratio as:

$$SNR = E\left\{\left|X_k\right|^2\right\}\Big/\sigma_n^2$$

We get a simplified LMMSE estimator as:

$$\widehat{H}_{LMMSE} = R_h\left(R_h + \frac{\gamma}{SNR}I\right)^{-1}\widehat{H}_{LS} \qquad (13)$$

where:

$$\gamma = E\left\{\left|X_k\right|^2\right\}\cdot E\left\{\left|\frac{1}{X_k}\right|^2\right\}$$

is a constant depending on signal constellation. For instance for 16 QAM transmission $\gamma = 17/9$. Note that **X** is no longer a factor in the matrix calculation. Estimator can be further simplified by using low rank approximations (Edfors *et al.*, 1998; Coleri *et al.*, 2002).

Maximum Likelihood (ML) estimators: ML algorithm regards the channel as deterministic but unknown vector (Jiang and Weiling, 2003). The ML algorithm achieves Cramer Rao Lower Bound (CRLB) (Kay, 1993).Therefore, it is a minimum variance unbiased estimator. Minimum mean square error is achieved on condition the channel state information is considered deterministic and unbiased. The simple relationship between Y(k) and X(k) for an OFDM system is:

$$Y(k) = \left(\sum_{l=0}^{L-1}h_l\exp^{-j2\pi\frac{kl}{M}}\right)X(k) + N(k) \qquad (14)$$

$$= H(k)\,X\,(k) + N(k),\ 0 \le k \le M\text{-}1$$

Channel estimation problem can be solved using the above expression. The channel frequency response parameters H(0),...,H (M-1) are generally correlated among each other, whereas impulse response parameters h_0,..., h_{L-1} may be independently specified, thus number of parameters in time domain is smaller than that of frequency domain. Hence, it is more suitable to apply ML algorithm to the above relation to get ML estimate of the channel in time domain. Using matrix notation, the likelihood function (Chen and Kobayashi, 2002; Jiang and Weiling, 2003) of **Y** given **X** and **h** is:

$$f\left(Y\,|\,X,h\right) = \frac{1}{\left(2\pi\sigma^2\right)^M}\exp\left\{-\frac{D(h,X)}{2\sigma^2}\right\} \qquad (15)$$

where, σ^2 is variance of both real and imaginary components of the noise n(k) and is equivalent to:

$$\frac{1}{2}E\left[\left|n(k)\right|^2\right]$$

and the function, usually called the "distance" function and is defined as:

$$D(h,X) = \sum_{k=0}^{M-1}\left|Y(k) - \sum_{l=0}^{L-1}h_l\exp^{-j2\pi\frac{kl}{M}}X(k)\right|^2 \qquad (16)$$

We need to find h that maximize $f(Y|X,\ h)$, or equivalently, minimize distance function D(**h**, **X**). Let $h_l = a_l + jb_l$ for $0 \le l \le L\text{-}1$.If we know **X**, we can solve for h_l by:

$$\frac{\partial D(h,X)}{\partial a_l}\Big|_{h=\hat{h}} = 0,$$

$$\frac{\partial D(h,X)}{\partial b_l}\Big|_{h=\hat{h}} = 0, \qquad (17)$$

which will lead to:

$$\sum_{l=0}^{L-1}\hat{a}_l\Re\{s(k-l)\}$$
$$-\sum_{l}^{L-1}\hat{b}_l\Im\{s(k-l)\} = \Re\{z(k)\}$$
$$\sum_{l=0}^{L-1}\hat{a}_l\Im\{s(k-l)\}$$
$$-\sum_{l}^{L-1}\hat{b}_l\Re\{s(k-l)\} = \Im\{z(k)\} \qquad (18)$$

For $0 \le k \le L\text{-}1$ or equivalently:

$$\sum_{l=0}^{L-1}\hat{h}_l\,s(k-l) = z(k),\quad 0 \le k \le L-1 \qquad (19)$$

where, z(k) and s(k) are defined as IDFT of Z (k) = X* (k) Y (k) and S (k) = $|X(k)|^2$, $0 \le k \le M\text{-}1$, respectively. If we take DFT of size L on both sides of Eq. 19, we have:

$$\widehat{H}^{(L)}(l)S^{(L)}(l) = Z^{(L)}(l),\quad 0 \le l \le L-1 \qquad (20)$$

where, superscript (L) denotes size of DFT to distinguish from previous DFT and IDFT, which are all of size M. Thus $\hat{h}_l$ can be obtained as the size L IDFT of $Z^{(L)}$ (l)/$S^{(L)}$ (l) for, $0 \le l \le L\text{-}1$. That is:

$$\hat{h} = IDFT\left\{\frac{Z^{(L)}}{S^{(L)}}\right\} \qquad (21)$$

For constant modulus signals, we have $|X(k)|^2 = C$ for all k, where C is a constant. Therefore:

$$s(k) = \begin{cases} C, & k = 0 \\ 0, & k \neq 0 \end{cases} \qquad (22)$$

In this case, from Eq. 19 we can directly obtain:

$$\hat{h}_k = \frac{z(k)}{C}, \quad 0 \leq k \leq L - 1 \qquad (23)$$

Hence, for given X, ML estimate of the channel $\hat{h}$ is the solution given by Eq. 21 or 23. One problem with the above algorithm is unknown channel memory length L. However, since the system requires that channel memory be no greater than guard interval P we can satisfy this requirement by setting L = P. ML estimator has comparable performance at intermediate and high signal to noise ratio (SNR) compared to LMMSE. But at Low SNR LMMSE performs better than ML (Morelli and Mengali, 2001). ML algorithm is simple for implementation. Note that large estimation error is inevitable in case of model mismatch.

Estimation with decision feedback: This estimator is proposed to enhance performance for block type scheme in which estimation is performed once per block till next pilot symbols are received. The idea here is updating the estimator inside the block using decision feedback equalizer at each subcarrier (Coleri *et al.*, 2002). First estimate of the channel $\hat{H} = \{\hat{H}_k\}$ (k=0,....... M-1) in the block can be computed using LS or other methods. Within the block (Fig. 1a) for each symbol and its subcarriers, estimation of the transmitted signal is obtained by the previously computed $\hat{H}_k$ as:

$$\hat{X}_k = \frac{Y_k}{\hat{H}_k}$$

The estimated transmitted signal $\tilde{X}_k$ is mapped to binary data through demodulation process in accordance with "signal demapper" and then obtained back through "signal mapper" as $\tilde{X}_k$. Then estimated channel $\hat{H}_k$ is updated by:

$$\hat{H}_k = \frac{Y_k}{\tilde{X}_k} \qquad (24)$$

and is used in the next symbol within the block. Application of this scheme is limited to slow fading channels only.

COMB TYPE BASED CHANNEL ESTIMATION

In comb type based channel estimation, pilots are inserted uniformly for each transmitted symbol as shown in Fig. 1b. This arrangement is proposed for intermediate and fast fading channels. Different methods, which are described next section, can be implemented to estimate the channel in frequency domain.

LS estimator based on 1-D interpolation: Equation 9 shows relationship between transmitted signals X_k and received Y_k. To estimate the channel, pilots symbols are needed. We assume that every p-th subcarrier contains known pilot symbols (X_{pk}). Using known pilots symbols (X_{pk}) and received symbols (Y_{pk}) at those pilot sub-carriers, we can calculate raw channel estimate ($\hat{H}_{pk}$) at pilots as:

$$\hat{H}_{pk} = \frac{Y_{pk}}{X_{pk}} + \frac{N_{pk}}{X_{pk}} = H_{pk} + N'_{pk} \qquad (25)$$

where, N_{pk} is the noise contribution at pk -th sub-carrier, N'_{pk} is a scaled noise contribution at that sub-carrier. 1-D linear interpolation method estimates the channel by interpolating channel transfer function between $\hat{H}_{pk,l}$ and $\hat{H}_{pm,l}$ (fixed time). Where $\hat{H}_{pk,l}$ is raw channel estimate at pk-th subcarrier frequency and time 1 and $\hat{H}_{pm,l}$ is raw channel estimate at pm-th subcarrier frequency and time 1 (Coleri *et al.*, 2002; Hsieh and Wei, 1998). This estimator works well for a channel with high coherence bandwidth. But fails for a channel with low coherence bandwidth (Akram, 2007). It is simple channel estimation. The following interpolation methods can be employed:

- Linear Interpolation (LI)
- Spline Interpolation (SI)
- Cubic Interpolation (CI)
- Low pass Interpolation (LPI)

The performance of the above interpolation techniques is given in (Arshad and Sheikh, 2004). Their performance in decreasing order is LPI, CI, SI, LI.

LS estimator based 1-D general linear models: The channel estimation problem can also be solved using 1-D generalized linear model framework (Chang and Su, 2002, 2000; Wang and Liu, 2002; Tang *et al.*, 2002). The channel transfer function H_k can be modeled as a linear weighed sum of some basis function evaluated at k-th sub-carrier frequency as:

$$H_k = \sum_{i=1}^{M-1} \alpha_i . \Psi_i(f_k) \qquad (26)$$

Ψ_i (f_k) is i-th basis function evaluated at that k-th subcarrier frequency.

α_i is weighing factor of the basis function. M is number of basis functions used in the linear model.

The generalized linear model can be used to rewrite raw channel estimates, $\widehat{H}_{pk}$ as:

$$\widehat{H}_{pk} = H_{pk} + N_{pk} = \sum_{i=0}^{M-1} \alpha_i . \Psi_i(f_k) + N_{pk} \qquad (27)$$

where, every p-th sub-carrier is a pilot, H_{pk} is actual channel transfer function at pk -th carrier, N_{pk} is noise at pk-th carrier, $\psi_i (f_{pk})$ is I-th basis function evaluated at that pk-th sub-carrier frequency f_k:

$$\begin{bmatrix} \widehat{H}_0 \\ \widehat{H}_p \\ \vdots \\ \widehat{H}_{P(M_p-1)} \end{bmatrix} = \underbrace{\begin{bmatrix} \psi_0 & \psi_1(f_0) & \cdots & \psi_{M-1}(f_0) \\ \psi_0 & \psi_1(f_p) & \cdots & \psi_{M-1}(f_p) \\ \vdots & \vdots & \ddots & \vdots \\ \psi_0(f_{P(M_p-1)}) & \psi_1(f_{P(M_p-1)}) & \cdots & \psi_1(f_{P(M_p-1)}) \end{bmatrix}}_{G} \cdot \underbrace{\begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{M-1} \end{bmatrix}}_{\alpha} + \underbrace{\begin{bmatrix} N_0 \\ N_p \\ \vdots \\ N_{P(M_p-1)} \end{bmatrix}}_{N}$$

$$(28)$$

where, M_p is the number of pilot carriers. Using matrix notation, we can write it as:

$$\widehat{H} = G \cdot \alpha + N$$

The least square estimate of weighing matrix is calculated by minimizing the squared distance between actual channel **H** and modeled channel $\widehat{H}$ as:

$$J(\alpha) = \left|\widehat{H} - H\right|^2 = \left(\widehat{H} - H\right)^H \left(\widehat{H} - H\right) \qquad (29)$$

Minimizing we get:

$$\frac{\partial}{\partial \alpha^H} J\{\alpha\} = -G^H \widehat{H} + G^H G \alpha = 0$$
$$\widehat{\alpha}_{LS} = \left(G^H G\right)^{-1} G^H \widehat{H} \qquad (30)$$
$$\widehat{\alpha}_{LS} = \begin{bmatrix} \widehat{\alpha}_{LS\ 0} & \cdots & \widehat{\alpha}_{LS\ N-1} \end{bmatrix}^T$$

The least-square estimate $\widehat{\alpha}_{LS}$ can then be used to estimate the channel $\widehat{H}_{LS,k}$ at regular sub-carrier frequencies f_k as:

$$\widehat{H}_{LS,k} = \sum_{i=0}^{M-1} \widehat{\alpha}_{LS_i} \cdot \psi_i(f_k) \qquad (31)$$

Basis functions such as polynomials (Orthogonal or, Legendre polynomials), Fourier series and others can be implemented. For polynomial basis functions, orthogonal polynomials have following merits over legendre polynomials.

- The calculation of $\widehat{\alpha}_{LS}$ remains numerically stable since the product $G^H G$ is a diagonal matrix for orthogonal polynomials
- Since $G^H G$ is a diagonal matrix, the inverse $(G^H G)^{-1}$ can be calculated at low computational complexity
- The degree of orthogonal polynomials can be increased without changes in any previous calculation results of $\widehat{\alpha}_{LS}$

This estimator works well as long as the right selection of polynomial order is done. It is more computational complex as it requires a matrix inversion and multiplication for each set of pilots. Since same matrix is inverted for each set of pilot, saving polynomial matrix inverse and using the inverse matrix for channel estimation for each set of pilots can reduce complexity. It is effective in reducing LS estimation error (Chang and Su, 2002).

LMMSE estimator based on 1-D winner filtering: Theoretical framework for 1-D Wiener filtering is presented (Akram, 2007; Scharf, 1991; Edfors *et al.*, 1998; Sandell and Edfors, 1996). LMMSE method uses knowledge of channel properties to estimate the unknown channel transfer function at non-pilot sub-carriers. These properties are assumed to be known at receiver for the estimator to perform optimally.

Where, $\widehat{H}_{pi}$ is ith raw channel estimate (at the pilot signal) and c_i is ith filter coefficients. Let us see how to determine filter coefficients. Consider M_p set of raw channel estimate:

$$\widehat{H}_{pk} = H_{pk} + N_{pk} \qquad (32)$$

$\widehat{H}_{pk}$ is a raw channel estimate at pk-th subcarrier frequency, H_{pk} is actual channel value at pk-th subcarrier frequency and N_{pk} is noise contribution at pk-th subcarrier frequency.

Given a set of observations on the raw channel estimate ($\widehat{H}_{pk}$), LMMSE estimate of the channel can be written ($\widehat{H}_{LMMSE,k}$) at k-th subcarrier as:

$$\widehat{H}_{LMMSE,k} = \sum_{i=0}^{M_p-1} c_i \widehat{H}_{pi}$$
$$= \sum_{i=0}^{M_p-1} c_i (H_{pi} + N_{pi}) \qquad (33)$$

where, c_i is i-th Wiener filter coefficient and $\widehat{H}_{pi}$ is i- th raw channel estimate. The LMMSE $H_{LMMSE,\ k}$ estimate at k-th sub-carrier is calculated by filtering raw channel estimate vector $\widehat{H} = [\widehat{H}_0 \quad \widehat{H}_p \quad \cdots \quad \widehat{H}_{p(M_p-1)}]^T$ by a Wiener filter $c_{LMMSE} = [c_0 \ c_1 \ ... \ c_{mp-1}]^T$ and can be written as:

$$\widehat{H}_{LMMSE,k} = \mathbf{c}_{LMMSE}^{H} \widehat{H} \qquad (34)$$

where, $\mathbf{c}$ and $\widehat{H}$ are column vectors, containing M_p Wiener filter coefficients and M_p raw channel estimates, respectively. Filter coefficients can be found by minimizing expected mean square error (Manolakis *et al.*, 2000). Given as:

$$
\begin{aligned}
& E\left\{\left|H_k - \widehat{H}_{LMMSE,k}\right|^2\right\} \\
& = E\left\{\left(H_k - \widehat{H}_{LMMSE,k}\right) \cdot \left(H_k - \widehat{H}_{LMMSE,k}\right)^H\right\} \\
& = E\left\{H_k H_k^*\right\} - \mathbf{c}^H E\left\{\widehat{H}H_k^*\right\} \\
& \quad - E\left\{H_k \widehat{H}^H\right\}\mathbf{c} + \mathbf{c}^H E\left\{\widehat{H}\widehat{H}^H\right\}\mathbf{c}
\end{aligned} \qquad (35)
$$

Minimizing the above expression we can get:

$$
\frac{\partial}{\partial \mathbf{c}^H} E\left\{\left|H_k - \widehat{H}_{LMMSE,k}\right|^2\right\} = -E\left\{\widehat{H}H_k^*\right\}
$$
$$
+ E\left\{\widehat{H}\widehat{H}^H\right\}\mathbf{c} = 0
$$

$$\mathbf{c}_{LMMSE} = \left(E\left\{\widehat{H}\widehat{H}^H\right\}\right)^{-1} E\left\{\widehat{H}H_k^*\right\} \qquad (36)$$

Let us consider first expression $E\left\{\widehat{H}\widehat{H}^H\right\}$:

$$E\left\{\widehat{H}\widehat{H}^H\right\} = E\left\{\left(\widehat{H} + N\right) \cdot \left(\widehat{H} + N\right)^H\right\}$$

$$
= \underbrace{E\left\{\widehat{H}\widehat{H}^H\right\}}_{R_h} + \underbrace{E\left\{\widehat{H}N^H\right\}}_{0} + \underbrace{E\left\{N\widehat{H}^H\right\}}_{0}
$$
$$
+ \underbrace{E\left\{NN^H\right\}}_{R_n}
$$

The additive white noise (AWGN) and channel are uncorrelated. Hence the above expression can be written as:

$$E\left\{\widehat{H}\widehat{H}^H\right\} = R_h + R_n = R_h + \sigma^2 I \qquad (37)$$

where, R_h is autocorrelation matrix of the channel and σ^2 is noise-variance per subcarrier. Note that:

$$
E\left\{\widehat{H}H_k^*\right\} = E\left\{(H + N) \cdot H_k^*\right\}
$$
$$
= \underbrace{E\left\{\widehat{H}H_k^*\right\}}_{r} + \underbrace{E\left\{N \cdot H_k^*\right\}}_{0}
$$

Similarly $E\left\{\widehat{H}H_k^*\right\}$ is given as:

$$
E\left\{\widehat{H}H_k^*\right\} = E\left\{(H + N) \cdot H_k^*\right\}
$$
$$
= \underbrace{E\left\{\widehat{H}H_k^*\right\}}_{r} + \underbrace{E\left\{N \cdot H_k^*\right\}}_{0}
$$

where, $\mathbf{r}$ is cross-correlation vector of the channel at k th sub-carrier and the channel at pilot locations:

$$
\mathbf{r} = \begin{bmatrix}
E\left\{\widehat{H}_0 \widehat{H}_k^*\right\} \\
E\left\{\widehat{H}_p \widehat{H}_k^*\right\} \\
\vdots \\
E\left\{\widehat{H}_{p(M_p-1)} \widehat{H}_k^*\right\}
\end{bmatrix} \qquad (38)
$$

Hence Wiener filter coefficients that provide the LMMSE channel estimate is:

$$\mathbf{c}_{LMMSE} = \left(R_h + \sigma^2 I\right)^{-1} \mathbf{r} \qquad (39)$$

Once we have calculated Wiener filter coefficients, we can estimate channel at k -th subcarrier as:

$$
\begin{aligned}
\widehat{H}_{LMMSE,k} &= \mathbf{c}_{LMMSE}^H \widehat{H} \\
&= \left(\left(R_h + \sigma^2 I\right)^{-1} \mathbf{r}\right) \cdot \widehat{H}
\end{aligned} \qquad (40)
$$

This estimator outperforms both estimators above especially at high SNR and low coherence bandwidth. In other words, LMMSE can be applied in sparse multipath channel (Jiang and Weiling, 2003). But, its performance will degrade if mismatch between statistical characteristic and real environment arises. This method is most complex method for channel estimation as it requires matrix inversion and matrix multiplication for each set of pilots. Moreover, its high complexity prevent it from widespread application when number of paths in wireless channel is large (>10).The complexity can be reduced by low rank approximations (Edfors *et al.*, 1998).

Note that ML estimator can also be applied for comb type pilot estimation (Chen and Kobayashi, 2002). There are also other channel estimators for OFDM system such as 2D estimators (Hou *et al.*, 2004), Iterative channel estimators (Sanzi *et al.*, 2003), estimators which employ Multiple Antenna (Auer, 2003) and estimators based on parametric channel modeling (Yang *et al.*, 2001). Moreover, a channel estimation algorithm which employs wavelet denoising filters can be applied to obtain more accurate channel estimation by reducing the effect of the noise on estimated channel (Alnuaimy *et al.*, 2009).

BLIND CHANNEL ESTIMATION

Finite alphabet and non-finite alphabet based algorithms are two major classifications of blind channel estimation algorithms in Linear Redundant Precoding (LRP) systems. Algorithms that employ knowledge of finite alphabet of source data generally have a shorter convergence time but computationally expenssive when the constellation size is large (Zhou and Giannakis, 2001; Chotikakamthorn and Suzuki, 1999). Most non-finite alphabet based algorithms make use of second order statistics of received data (Heath and Giannakis, 1999; Petropulu *et al.*, 2004). These methods obviously need a longer convergence time than finite alphabet counterparts before a true channel estimate can be found due to use of statistics. Another important category of non-finite alphabet based algorithms uses subspace decomposition and can also be implemented deterministically (Pham and Manton, 2003; Cai and Akansu, 2000; Li and Roy, 2003; Muquet *et al.*, 2002). Subspace based algorithms are applicable for any kind of constellation, even though they require a longer convergence time. First subspace based blind channel estimation algorithm was proposed by Scaglione *et al.* (1999) for Zero Padding (ZP) systems. Subspace algorithms in Cyclic Prefix (CP) unlike in ZP systems the received block contains Inter Block Interference (IBI) which makes blind algorithms more difficult. These methods all need persistency of excitation property of the input signal that is signal richness to offer the data covariance matrix to have full rank. This requirement demands the receiver to collect number of blocks at least equal to block size for one channel estimate and hence makes the approach less applicable when the channel is fast varying.

Recently pointed out by Manton and Neumann (2003) that blind channel estimation without knowledge of finite alphabet in ZP systems is possible with only two received blocks. An algorithm that views the channel estimation problem as computing greatest common divisors (GCD) of polynomials representing received blocks was proposed (Pham and Manton, 2003). Even though many blind algorithms in LRP systems have been developed, they have limitations such as slow convergence speed, high complexity, poorer performance as compared to pilot assisted methods.

Subspace based blind channel estimation: Subspace based blind channel estimation methods for CP systems based on literature (Muquet *et al.*, 2002; Li and Roy, 2003; Cai and Akansu, 2000) is discussed below. Let source vector $\mathbf{X}_M$ (k) is precoded by M×M constant IDFT matrix

$\mathbf{F}_M^H$ resulted in precoded data. Subscript M shows size of matrix or vectors. A cp of length L, taking from last L elements of $\mathbf{x}_M$ (k), is defined as:

$$\mathbf{x}_{cp}(k) = \begin{bmatrix} 0_{L \times (M-L)} & I_L \end{bmatrix} \mathbf{x}_M(k) \tag{41}$$

Note that it is assumed L+1<M. The cp is appended to $\mathbf{x}_M$ (k), forming a vector:

$$\mathbf{x}(k) = \begin{bmatrix} \mathbf{x}_{cp}(k) \\ \mathbf{x}_M(k) \end{bmatrix} = \begin{bmatrix} \mathbf{x}_M(k) \end{bmatrix}_{L+1:M} \tag{42}$$

Length of $\mathbf{x}$ (k) is D = M+L. The channel is considered to be an FIR filter with a maximum order of L. The channel response $\mathbf{h}$ is L+1 column vector $[h_0 \ h_1 \dots h_L]^T$ and can be defined as:

$$H(z) = \sum_{k=0}^{L} h_k z^{-k} \tag{43}$$

The received symbols y (k) are blocked into D×1 vectors $\mathbf{y}$ (k). Let blocked version of the noise is $\mathbf{n}$ (k), $\mathbf{y}_{cp}$ (k) as first L entries and $\mathbf{y}_M$ (k) as last Mentries of $\mathbf{y}$ (k) so that:

$$\mathbf{y}(k) = \begin{bmatrix} \mathbf{y}_{cp}(k)^T & \mathbf{y}_M(k)^T \end{bmatrix}^T \tag{44}$$

It can be shown that:

$$\mathbf{y}_M (k) = H\mathbf{x}_M (k) + \mathbf{n}_M (k) \tag{45}$$

where:

$$\mathbf{H} = \begin{bmatrix} h_0 & 0 & h_L & \cdots & h_1 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ h_L & & \ddots & \ddots & h_L \\ & \ddots & & \ddots & 0 \\ 0 & & h_L & \cdots & h_0 \end{bmatrix}$$

In which $\mathbf{H}$ is an M×M circulant matrix and $\mathbf{n}_M$ (k) = $[\mathbf{n} (k)]_{L+1:D}$ is noise vector. The L×1 vector $\mathbf{y}_{cp}$ (k) contains inter-block interference (IBI) and can be expressed as:

$$\mathbf{y}_{cp}(k) = H_l \mathbf{x}_{cp}(k) + H_u \mathbf{x}_{cp}(k-1) + \mathbf{n}_{cp}(k) \tag{46}$$

where:

$$H_l \triangleq \begin{bmatrix} h_0 & & 0 \\ \vdots & \ddots & \\ h_{L-1} & \cdots & h_0 \end{bmatrix}$$

and:

$$\mathbf{H}_u \triangleq \begin{bmatrix} h_L & & h_l \\ \vdots & \ddots & \\ 0 & \cdots & h_L \end{bmatrix}$$

are L×L matrices. $\mathbf{n}_{cp}(k) = [\mathbf{n}(k)]_{1:L}$ is the noise component. For channel equalization $\mathbf{y}_{cp}(k)$ is usually dropped and only $\mathbf{y}_M(k)$ passes to an equalizer $\mathbf{T}$ with dimension of M×M and results in recovered symbol $\mathbf{Y}_M(k)$. When channel coefficients are identified, optimum value for equalizer $\mathbf{T}$ can be computed to minimize mean square error of equalized symbols.

Let us see now how to estimate the channel blindly: While $\mathbf{y}_{cp}(k)$ is often dropped before equalization, information in $\mathbf{y}_{cp}(k)$ is useful to estimate channel coefficients (Pham and Manton, 2003; Su and Vaidyanathan, 2007). First ignore noise term $\mathbf{n}(k)$ for simplicity and define a composite block $\bar{\mathbf{y}}(k)$ with a length of 2M+L and contains information from two consecutive blocks as follows:

$$\bar{\mathbf{y}}(k) = \begin{bmatrix} \mathbf{y}_M(k-1)^T & \mathbf{y}_{cp}(k)^T & \mathbf{y}_M(k)^T \end{bmatrix}^T \qquad (47)$$

Then, from Eq. 45 and 46 we have:

$$\bar{\mathbf{y}}(k) = \begin{bmatrix} \mathbf{H}\mathbf{x}_M(k-1) \\ \mathbf{H}_l\mathbf{x}_{cp}(k) + \mathbf{H}_u\mathbf{x}_{cp}(k-1) \\ \mathbf{H}\mathbf{x}_M(k) \end{bmatrix} \qquad (48)$$
$$= \bar{\mathbf{H}}\tilde{\mathbf{x}}(k)$$

where:

$$\bar{\mathbf{H}} = \begin{bmatrix} \mathbf{H} & | & 0_{M \times M} \\ \left[0_{L \times (M-L)} | \mathbf{H}_u \right] & \left[0_{L \times (M-L)} | \mathbf{H}_l \right] \\ 0_{M \times M} & | & \mathbf{H} \end{bmatrix}$$

and:

$$\tilde{\mathbf{x}}_k = \begin{bmatrix} \mathbf{x}_M(k-1)^T & \mathbf{x}_M(k)^T \end{bmatrix}^T$$

Note that $\tilde{\mathbf{H}}$ is (2M+L)×2M matrix.

- If:

$$H(z) = \sum_{k=0}^{L} h_k z^{-k}$$

does not have any non zero on unit circle grid $W_M^l, 0 \le l \le M-1$, then $\tilde{\mathbf{H}}$ has full column rank 2M.

Assume J consecutive received blocks $\mathbf{y}(0)$, $\mathbf{y}(1)$, ..., $\mathbf{y}(J-1)$ are gathered at receiver. Subsequently we shall have J-1 composite blocks $\bar{\mathbf{y}}(k)$ defined in Eq. 41 for k = 1,2,..., j-1. (2M+L)×(J-1) matrix can be constructed by placing these composite blocks together as:

$$\mathbf{y}_{cobk}^{(J)} = \begin{bmatrix} \bar{\mathbf{y}}(1) & \bar{\mathbf{y}}(2) & \cdots & \bar{\mathbf{y}}(J-1) \end{bmatrix} \qquad (49)$$

Note that subscript cobk stands for composite block. Next, we shall have:

$$\mathbf{y}_{cobk}^{(J)} = \tilde{\mathbf{H}}\mathbf{x}_{cobk}^{(J)} \qquad (50)$$

where, $\mathbf{x}_{cobk}^{(J)} = \begin{bmatrix} \tilde{\mathbf{x}}(1) & \tilde{\mathbf{x}}(2) & \cdots & \tilde{\mathbf{x}}(J-1) \end{bmatrix}$ is a 2M×(J-1) matrix. Suppose there exists an integer J≥2M+1 such that $\mathbf{x}_{cobk}^{(J)}$ has full row rank 2M. Then rank of $\left(\mathbf{y}_{cobk}^{(J)}\right) = 2M$ which means $\mathbf{y}_{cobk}^{(J)}$ has L linearly independent left annihilators. Let $\mathbf{g}_k$ be kth annihilator of $\mathbf{y}_{cobk}^{(J)}$, for $1 \le k \le L$ that is $\mathbf{g}_k^H \mathbf{y}_{cobk}^{(J)} = 0$ then, $\mathbf{g}_k^H \tilde{\mathbf{H}} = 0$ since $\mathbf{x}_{cobk}^{(J)}$ has full rank, we can write $\mathbf{g}_k^H$ as $\mathbf{g}_k^H = [g_{01} \cdots g_{0M} | g_{cl} \cdots g_{cL} | g_{11} \cdots g_{1M}]$ we can ignore index k in the contents of $\mathbf{g}_k^H$ for simplicity. By looking at columns of $\tilde{\mathbf{H}}$, a 2M×(L+1) matrix $\mathbf{G}_k$ can be produced as shown next:

$$\mathbf{G}_k = \begin{bmatrix} g_{01} & g_{02} & \cdots & g_{0,1+L} \\ g_{02} & g_{03} & \cdots & g_{0,2+L} \\ \vdots & \vdots & & \vdots \\ g_{0,M-L} & g_{0,M-L+1} & \cdots & g_{0M} \\ g_{0,M-L+1} & \cdots & g_{0M} & g_{01} + g_{cl} \\ \vdots & \ddots & & \vdots \\ g_{0M} & g_{01} + g_{cl} & \cdots & g_{0L} + g_{cL} \\ g_{11} & g_{12} & \cdots & g_{1,1+L} \\ g_{12} & g_{13} & \cdots & g_{1,2+L} \\ \vdots & \vdots & & \vdots \\ g_{1,M-L} & g_{1,M-L+1} & \cdots & g_{1M} \\ g_{1,M-L+1} + g_{cl} & \cdots & g_{1M} + g_{cL} & g_{11} \\ \vdots & \ddots & \ddots & \vdots \\ g_{1M} + g_{cL} & g_{11} & \cdots & g_{1L} \end{bmatrix} \qquad (51)$$

$$\mathbf{G}_k \mathbf{h} = 0$$

Define $\mathbf{G} = \begin{bmatrix} \mathbf{G}_1^T & \mathbf{G}_2^T & \cdots & \mathbf{G}_L^T \end{bmatrix}^T$. Then, channel coefficients $\mathbf{h}$ can be recovered within a scalar ambiguity (Cui and Tellambura, 2005) by finding only right annihilating vector of G. When $\tilde{\mathbf{H}}$ has no full column rank, the above algorithm can be made applicable by making some modifications (Muquet *et al.*, 2002). In presence of noise, Eq. 50 can be expressed as:

$$\mathbf{y}_{\text{cobk}}^{(J)} = \overline{\mathbf{H}} \mathbf{x}_{\text{cobk}}^{(J)} + \mathbf{n}_{\text{cobk}}^{(J)}, \qquad (52)$$

where, the noise component $\mathbf{n}_{\text{cobk}}^{(J)}$ comes accordingly from Eq. 45 and 46. In this case, $\mathbf{y}_{\text{cobk}}^{(J)}$ often becomes full rank and no longer has L left annihilators. By taking singular value decomposition(SVD) of $\mathbf{y}_{\text{cobk}}^{(J)}$, the left annihilators $\widetilde{\mathbf{H}}$ of which is the noise space can be estimated. In the Equation $\overline{\mathbf{y}}$ (k):

$$\mathbf{y}_{\text{cobk}}^{(J)} = \begin{bmatrix} \mathbf{U}_s & \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \sum_s & 0 \\ 0 & \sum_n \end{bmatrix} \begin{bmatrix} \mathbf{V}_s & \mathbf{v}_n \end{bmatrix} \qquad (53)$$

$\mathbf{U}_n$ contains singular vectors associated with the smallest L singular values $\mathbf{y}_{\text{cobk}}^{(J)}$ of and $\mathbf{g}_k$ is chosen as kth column of $\mathbf{U}_n$. Note that in Eq. 53 if matrix $\mathbf{y}_{\text{cobk}}^{(J)}$ is replaced with estimated autocorrelation matrix which is:

$$\mathbf{R}_{\overline{yy}} = \mathbf{y}_{\text{cobk}}^{(J)} \left[\mathbf{y}_{\text{cobk}}^{(J)} \right]^H \qquad (54)$$

Then, null space $\mathbf{U}_n$ found by singular value decomposition (SVD) will remain unchanged. Since size of $\mathbf{R}_{\overline{yy}}$ is usually smaller than $\mathbf{y}_{\text{cobk}}^{(J)}$, especially when J is large, taking SVD on $\mathbf{R}_{\overline{yy}}$ rather than on $\mathbf{y}_{\text{cobk}}^{(J)}$ actually saves computational complexity, although an additional computation will be needed for creating the autocorrelation matrix $\mathbf{R}_{\overline{yy}}$. Once matrix $\mathbf{R}_{\overline{yy}}$ is created whenever a new block is received, it can be updated easily (Muquet *et al.*, 2002). The idea of maintaining $\mathbf{R}_{\overline{yy}}$ can lead to a method in which newer blocks can be weighted more than older ones. If an initial estimate of $\mathbf{R}_{\overline{yy}}$ is made first, It can be updated each time a new composite block $\overline{\mathbf{y}}$ (k) is obtained using:

$$\widehat{\mathbf{R}}_{\overline{yy}}^{(K)} = \alpha \widehat{\mathbf{R}}_{\overline{yy}}^{(K-1)} + (1-\alpha) \overline{\mathbf{y}}(K) \overline{\mathbf{y}}(K)^H \qquad (55)$$

The parameter $\alpha \in [0,1]$ is a forgetting factor. The forgetting factor is used especially in time varying channel environments.

A matrix $\mathbf{x}_{\text{cobk}}^{(J)}$ with 2M×(J-1) should have full row rank of 2M (property of persistency of excitation) so that above method work properly. Obviously, $\mathbf{x}_{\text{cobk}}^{(J)}$ has full row rank only when the number of columns is not smaller than number of rows, that is J-1 ≥ 2M. This requires at least (2M+1).D symbol durations for the receiver to wait before channel estimation can be performed. This drawback makes the algorithm not applicable for fast fading channels since the channel parameters experience change during this time of collecting data. A forgetting factor can be utilized to give more weight to newer blocks. But use

of older blocks as old as (2M+1) earlier is unavoidable. This fundamental limitation can be overcome by the method proposed (Su and Vaidyanathan, 2007; Pham and Manton, 2003) for Zero Padding (ZP) systems.

The other important issue is to deal with the scalar ambiguity in the estimated channel coefficients (Su, 2008). The frequency response of the estimated channel is:

$$\widehat{\mathbf{H}}\left(\mathbf{W}_M^k \right) = \sum_{l=0}^{L} \widehat{\mathbf{h}}_l \mathbf{W}_M^{-kl} \qquad (56)$$

due to scalar ambiguity all equalized symbols has to be scaled by unknown complex valued scalar β. One way of resolving this ambiguity is by introducing one extra pilot symbol and comparing it with corresponding received symbol. If several blocks are using same channel estimate $\widehat{\mathbf{h}}$, the scalar ambiguity can be estimated as follows:

$$\widehat{\beta} = \arg \min_{\beta \in \mathbb{C}} \sum_i \left\| \mathbf{y}_{\text{rec}}(i) - \beta \cdot \mathbf{y}_{\text{pil}}(i) \right\|^2$$
$$= \frac{\sum_i \mathbf{y}_{\text{pil}}^*(i) \mathbf{y}_{\text{rec}}(i)}{\sum_i \left| \mathbf{y}_{\text{pil}}(i) \right|^2} \qquad (57)$$

where, C is complex domain, $\mathbf{y}_{\text{pil}}(i)$ is pilot symbol of the ith block and $\mathbf{y}_{\text{rec}}(i)$ is the corresponding received pilots. There could be different way of pilot arrangements but attention should be given so that received blocks should not be rank deficient.

DISCUSSION

A review of selected channel estimation algorithms for OFDM communication systems has been presented along with their strength and limitations in this study. Generally, they can be classified as pilot and blind channel estimation algorithms.

Block type pilot channel estimators are suitable for slow fading channels. The LS algorithm is simple but suffers from high mean square error. The LMMSE algorithm has superior performance over LS by giving 10-15dB gain in signal to noise ratio (SNR) for the same mean square error (Van-de-Beek *et al.*, 1995). However, LMMSE is computationally costly but it can be simplified by using low rank approximations (Edfors *et al.*, 1998). ML estimator does not require knowledge of the channel statistics and therefore it is simpler to implement and has comparable performance at intermediate and high SNR with that of LMMSE. But at Low SNR LMMSE performs better than ML (Morelli and Mengali, 2001). Estimation with decision feedback is proposed to improve the performance of the LS (Coleri *et al.*, 2002), LMMSE and ML estimator algorithms.

Comb type channel estimators usually out perform for middle and fast fading channel environments. LS interpolation based estimator is simple and performs well for a channel with high coherence band width. But its performance is not good for low coherence bandwidth (Akram, 2007). LS linear model works well as long as the right polynomial order is selected. The LS model based is very effective in reducing LS estimation error but computationally complex. It requires less computation compared to LMMSE winner filtering (Chang and Su, 2002), since no information about the channel and noise power level is needed. The LMMSE winner filtering based estimator outperforms both estimators especially at high SNR and low coherence bandwidth (Akram, 2007). But it is the most complex method. The complexity can be reduced by deriving an optimal low rank estimator with singular value decomposition (Hsieh and Wei, 1998).

Subspace based Blind Channel Estimation (BCE) algorithms are widely used for OFDM system. Generally BCE algorithms are band width efficient compared to pilot based ones but they suffer from low convergence and high computational cost which makes them unsuitable for fast fading channel environments at this stage (Su, 2008).

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