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Fuzzy Games under Possibility and Necessity Expectations

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Abstract: In this study, fuzzy games with fuzzy payoffs that consider the player risk preferences are studied. Based on the possibility and necessity expectations, the Shapley function for this kind of fuzzy games is researched. An axiomatic system of the given Shapley function is defined. Meantime, some properties are also discussed which coincide with the classical case. Finally, a numerical example is given to explain the player Shapley values for fuzzy games under possibility and necessity expectations.

Key words: Fuzzy game, Shapley function, possibility expectation, necessity expectation, credibility expectation

INTRODUCTION

With the social development, game theory is used in many fields. Kalliny and Gentry (2010) introduced the online games and applied it to advertising practices-advergaming and product placement. Lo (2008) and Akramizadeh *et al.* (2009) studied multi-agent model by using Nash equilibrium. Cheheltani and Ebadzadeh (2010) further researched the Nash equilibrium. Recently, Ayanzadeh *et al.* (2011) discussed the mixed Nash equilibrium of honey bees foraging optimization. Daghistani (2011) studied the thinking skills of kindergarten children by using educational games. Isin and Miran (2005) researched agriculture in Turkey by using game theory.

Since there are many uncertain factors during the cooperation of the players and they can only know the possible payoffs for the cooperation. For this problem, Mares (2000) and Mares and Vlach (2001) studied games with fuzzy payoffs, where the coalitions are crisp and the coalition values are fuzzy numbers. Fuzzy games of which the coalition and the characteristic function are both fuzzy information are researched by Borkotokey (2008). But the given Shapley function does not satisfy the efficiency. Based on the Hukuhara difference on fuzzy numbers (Banks and Jacobs, 1970). Yu and Zhang (2010) concerned games with fuzzy payoffs, where the coalitions are crisp and gave the so-called Hukuhara-Shapley value. Furthermore, the authors researched a special kind of fuzzy games with fuzzy payoffs which can be seen an extension of fuzzy games given by Tsurumi *et al.* (2005). Furthermore, the Shapley function for fuzzy games with Choquet integral and fuzzy payoffs is researched by Meng and Zhang (2010). Meng *et al.* (2012) researched the Banzhaf value for fuzzy games with fuzzy payoffs. Dhar *et al.* (2011) and Amiri *et al.* (2008) studied Utopian transport scenario and multiple attribute decision making problems by using fuzzy sets, respectively. Tan *et al.* (2011) researched IP network traffic by introducing fuzzy decision mechanism. Furthermore, Chao *et al.* (2010) researched type-2 interval fuzzy immune controller.

Possibility and necessity measures play a key role in possibility theory. The necessity and possibility constraints are introduced by Zadeh (1978) and Dubois and Prade (1987, 1988) which are very relevant to the real-life decision making problems and presented the process of defuzzification for these constraints. Later, Liu and Liu (2002, 2003) proposed the creditability theory and gave the creditability measure. These three measures respect the risk attitudes of the decision makers or the players for uncertainty information. Based on above analysis, we shall research the fuzzy games with fuzzy payoffs under possibility and necessity measures.

PRELIMINARIES

Here, we recall some basic concepts for possibility/necessity/credibility measures and give the model of fuzzy games under possibility and necessity measures.

Some concepts for possibility/necessity/credibility measures: Let us start by recalling the most general definition of a fuzzy number. Let $\mathbb{R}$ be $(-\infty, \infty)$, i.e., the set of all real numbers.

Definition 1: A fuzzy number, denoted by $\bar{u}$., is a fuzzy subset with membership function $\mu_{\bar{u}}$: $\mathbb{R}\rightarrow[0, 1]$ satisfying the following conditions:

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- $\mu_{\tilde{u}}$ is upper semi-continuous
- There exists an interval number [a, d] such that $\mu_{\tilde{u}}(x) = 0$ for any $x \notin [a, d]$
- There exist real numbers b, c such that $a \leq b \leq c \leq d$ and (i) $\mu_{\tilde{u}}(x)$ is nondecreasing on [a, b] and nonincreasing on [c, d]; (ii) $\mu_{\tilde{u}}(x) = 1$ for any $z \in [b, c]$

Definition 2: Let X be a nonempty set and P(X) be the power set of X. Pos: $P(X) \to [0, 1]$ is called possibility measure, if:

- $Pos(\varnothing) = 0$, $Pos(X) = 1$
- $Pos(\cup_{i=1}^{+\infty} A_i) = \vee_{i=1}^{+\infty} Pos(A_i)$, where, $A_i \in P(X)$ for all $1 \leq i \leq +\infty$

Definition 3: Let X be a nonempty set and P(X) be the power set of X. Nec: $P(X) \to [0, 1]$ is called necessity measure, if:

- $Nec(\varnothing) = 0$, $Nec(X) = 1$
- $Nec(\cap_{i=1}^{+\infty} A_i) = \wedge_{i=1}^{+\infty} Nec(A_i)$, where $A_i \in P(X)$ for all $1 \leq i \leq +\infty$

Let $\tilde{x} \in \tilde{R}$ with membership function $\mu_{\tilde{x}}$. In the setting of creditability theory, the creditability measure for the fuzzy event given as follows:

$$Cr(\tilde{x} \in [a,b]) = \frac{1}{2}\left(\sup_{t \in [a,b]} \mu_{\tilde{x}}(t) + 1 - \sup_{t \notin [a,b]} \mu_{\tilde{x}}(t)\right) = \frac{1}{2}(Pos(\tilde{x} \in [a,b]) + Nec(\tilde{x} \in [a,b])) \quad (1)$$

Some concepts for fuzzy games with fuzzy payoffs: Let $N = \{1, 2, \ldots, n\}$ be the set of the players. By L (N), we denote the set of all fuzzy coalitions in N. The fuzzy coalitions in L (N) are denoted by S, T, ... Let $S \in L(N)$ and player i, S (i) indicates the membership grade of i in S, i.e., the rate of the ith player in S. For any $S \in L(N)$, the support is denoted by $SuppS = \{i \in N \mid S(i) > 0\}$ and the cardinality is written as |SuppS|. We use the notation $S \subseteq T$ if and only if $S(i) = T(i)$ or $S(i) = 0$ for any $i \in N$. Let $S, T \in L(N)$, $S \vee T$ denotes the union of fuzzy coalitions S and T, namely, $i \in Supp(S \vee T)$ if and only if $i \in SuppS \cup SuppT$ and $(S \vee T)(i) = S(i) \vee T(i)$; $S \wedge T$ denotes the intersection of fuzzy coalitions S and T, namely, $i \in Supp(S \vee T)$ if and only if $i \in SuppS \cap SuppT$ and $(S \wedge T)(i) = S(i) \wedge T(i)$.

In the following, we use $S = \{S(i_1), S(i_2), \ldots, S(i_n)\}$ to denote $S \in L(N)$. A function $\tilde{v}$: $L(N) \to \tilde{R}_+ = \{\tilde{a} \in \tilde{R} \mid \tilde{a} \geq 0\}$, satisfying $\tilde{v}(\varnothing) = 0$, is called a fuzzy characteristic function. All fuzzy games with fuzzy number payoffs on L (N) are denoted by $\tilde{G}(N)$. We will omit braces for singletons, e.g., by writing S, $S \vee (\wedge) T$, S (i) instead of {S}, $(S) \vee (\wedge)(T)$, {S(i)} for any {S}, {T}, $\{S(i)\} \in L(N)$.

Definition 4: Let $\tilde{v} \in \tilde{G}(N)$ and $U \in L(N)$, $E_p(\tilde{v})$ is defined by:

$$E_P(\tilde{v}(S)) = \int_0^{+\infty} Pos(\tilde{v}(S) \geq x)\,dx - \int_{-\infty}^0 (1 - Pos(\tilde{v}(S) \geq x))\,dx \quad \forall S \subseteq U \quad (2)$$

If $E_p(\tilde{v}(S))$ exists for any $S \subseteq U$, then Eq. 2 is called Possibility Expectation (PE) for $\tilde{v}$ in U.

Definition 5: Let $\tilde{v} \in \tilde{G}(N)$ and $U \in L(N)$, $E_{ne}(\tilde{v})$ is defined by:

$$E_{Ne}(\tilde{v}(S)) = \int_0^{+\infty} Nec(\tilde{v}(S) \geq x)\,dx - \int_{-\infty}^0 (1 - Nec(\tilde{v}(S) \geq x))\,dx \quad \forall S \subseteq U \quad (3)$$

If $E_{Ne}(\tilde{v}(S))$ exists for any $S \subseteq U$, then Eq. 3 is called Necessity Expectation (NE) for $\tilde{v}$ in U.

Remark 1: In this study, without loss of generality, for any $\tilde{v} \in \tilde{G}(N)$ and $U \in L(N)$, we always mean the Necessity and Possibility Expectations (NPE) exist. Since NE and PE reflect the player risk preferences for the approximate values of fuzzy coalitions, we shall use $w_i \in [0, 1]$ to denote the player i's NE weight for the approximate value of $U \in L(N)$ and $1 - w_i$ to denote the player i's PE weight for the approximate value of $U \in L(N)$. By:

$$W_{Ne}^S = \sum_{i \in SuppS} w_i \Big/ |SuppS|$$

we denote the weight of $S \subseteq U$ with respect to $E_{Ne}(\tilde{v}(S))$ and $W_P^S = 1 - W_N^S$ indicates the weight of $S \subseteq U$ with respect to $E_P(\tilde{v}(S))$.

Definition 6: Let $\tilde{v} \in \tilde{G}(N)$ and $U \in L(N)$, $\tilde{v}$ is said to be weighted NPE superadditivity if we have:

$$W_{Ne}^S E_{Ne}(\tilde{v}(S)) + W_{Ne}^T E_{Ne}(\tilde{v}(T)) \leq W_{Ne}^{S \vee T} E_{Ne}(\tilde{v}(S \vee T))$$

And:

$$W_P^S E_P(\tilde{v}(S)) + W_P^T E_P(\tilde{v}(T)) \leq W_P^{S \vee T} E_P(\tilde{v}(S \vee T))$$

for any $S, T \subseteq U$ with $S \wedge T = \varnothing$.

Remark 2: If there is no special explanation, for any $\tilde{v} \in \tilde{G}(N)$, we always mean $\tilde{v}$ is weighted NPE supperadditive.

Definition 7: Let $\tilde{v} \in \tilde{G}(N)$ and $U \in L(N)$, $\tilde{v}$ is said to be weighted NPE convex if we have:

$$W_{Ne}^S E_{Ne}(\tilde{v}(S)) + W_{Ne}^T E_{Ne}(\tilde{v}(T)) \leq W_{Ne}^{S \vee T} E_{Ne}(\tilde{v}(S \vee T)) + W_{Ne}^{S \wedge T} E_{Ne}(\tilde{v}(S \wedge T))$$

And:

$$W_P^S E_P(\tilde{v}(S)) + W_P^T E_P(\tilde{v}(T)) \leq W_P^{S\vee T} E_P(\tilde{v}(S\vee T)) + W_P^{S\wedge T} E_P(\tilde{v}(S\wedge T))$$

for any S, T⊆U.

Definition 8: Let $\tilde{v}\in\tilde{G}(N)$ and U∈L (N), T⊆U is said to be a weighted NPE carrier for $\tilde{v}$ in U, if it satisfies $W_{Ne}^{S\wedge T} E_{Ne}(\tilde{v}(S\wedge T)) = W_{Ne}^S E_{Ne}(\tilde{v}(S))$ and $W_P^{S\wedge T} E_P(\tilde{v}(S\wedge T)) = W_P^S E_P(\tilde{v}(S))$ for any S⊆U.

Definition 9: Let $\tilde{v}\in\tilde{G}(N)$ and U∈L (N), C (E_{NP} ($\tilde{v}$), U) is said to be the weighted NPE fuzzy core of $\tilde{v}$ in U which is defined by:

$$C(E_{NP}(\tilde{v}),U) = \Big\{\tilde{x}\in^{n}_{+} \Big| \sum_{i\in SuppU}\tilde{x}_i = W_{Ne}^U E_{Ne}(\tilde{v}(U)) + W_P^U E_P(\tilde{v}(U)), \sum_{i\in SuppS}\tilde{x}_i \geq W_{Ne}^S E_{Ne}(\tilde{v}(S)) + W_P^S E_P(\tilde{v}(S)), \forall S\subseteq U\Big\}$$

Definition 10: Let $\tilde{v}\in\tilde{G}(N)$ and U∈L (N), the vector y = $(y_i)_{i\in SuppU}$ is said to be a weighted NPE participation monotonic allocation schemes for $\tilde{v}$ in U if it satisfies:

- $\sum_{i\in SuppS} y_i = W_{Ne}^S E_{Ne}(\tilde{v}(S)) + W_P^S E_P(\tilde{v}(S))\ \forall S\subseteq U$
- y_i (S)≤y_i (T) ∀ i∈SuppS, ∀S, T⊆U s.t. S⊆T

Definition 11: Let $\tilde{v}\in\tilde{G}(N)$ and U∈L (N), the function is $f:\tilde{G}(N)\to\mathbb{R}_+^{SuppU}$ said to a weighted NPE Shapley function for $\tilde{v}$ in U it satisfies the following axioms:

Axiom 1 (weighted NPE efficiency): If T⊆U is a weighted NPE carrier for $\tilde{v}$ in U, then we have:

$$\sum_{i\in SuppT} f_i(E_{Ne}(\tilde{v}),U) = W_{Ne}^T E_{Ne}(\tilde{v}(T)),\ \sum_{i\in SuppT} f_i(E_P(\tilde{v}),U) = W_P^T E_P(\tilde{v}(T))$$

where, W^U_{Ne} and W^U_p are the weights of U with respect to E_{Ne} ($\tilde{v}$ (U)) and E_P ($\tilde{v}$ (U)), respectively.

Axiom 2 (weighted NPE symmetry): For any i, j∈SuppU and any given weight W^U_{Ne}, if we have:

$$W_{Ne}^{S\vee U(i)} E_{Ne}(\tilde{v}(S\vee U(i))) = W_{Ne}^{S\vee U(j)} E_{Ne}(\tilde{v}(S\vee U(j)))$$

And:

$$W_P^{S\vee U(i)} E_P(\tilde{v}(S\vee U(i))) = W_P^{S\vee U(j)} E_P(\tilde{v}(S\vee U(j)))$$

for any S⊆U with i, j∉SuppS . Then, we have:

f_i (E_{Ne} ($\tilde{v}$), U) = f_j (E_{Ne} ($\tilde{v}$), U), f_i (E_P ($\tilde{v}$), U) = f_j (E_P ($\tilde{v}$), U)

Axiom 3 (weighted NPE additivity): Let $\tilde{v}$, $\tilde{w}\in\tilde{G}(N)$ if we have:

$$W_{Ne}^S E_{Ne}((\tilde{v}+\tilde{w})(S)) = W_{Ne}^S E_{Ne}(\tilde{v}(S)) + W_{Ne}^S E_{Ne}(\tilde{w}(S))$$

And:

$$W_P^S E_P((\tilde{v}+\tilde{w})(S)) = W_P^S E_P(\tilde{v}(S)) + W_P^S E_P(\tilde{w}(S))$$

for any S⊆U.

Then we have:

$$f(E_{Ne}(\tilde{v}+\tilde{w}),U) = f(E_{Ne}(\tilde{v}),U) + f(E_{Ne}(\tilde{w}),U)$$

And:

$$f(E_P(\tilde{v}+\tilde{w}),U) = f(E_P(\tilde{v}),U) + f(E_P(\tilde{w}),U)$$

THE WEIGHTED NPE SHAPLEY FUNCTION FOR FUZZY GAMES WITH FUZZY PAYOFFS

Let $\tilde{v}\in\tilde{G}(N)$ and U∈L (N), we give the weighted NPE Shapley function for $\tilde{v}$ in U as follows:

$$\varphi_i(E_{NP}(\tilde{v}),U) = \varphi^N_i(E_{Ne}(\tilde{v}),U) + \varphi^P_i(E_P(\tilde{v}),U)\ \forall\ i\in SuppU \quad (4)$$

Where:

$$\varphi_i^N(E_{Ne}(\tilde{v}),U) = \sum_{\substack{S\subseteq U,\\ i\notin SuppS}} \alpha_U^S (W_{Ne}^{S\vee U(i)} E_{Ne}(\tilde{v}(S\vee U(i))) - W_{Ne}^S E_{Ne}(\tilde{v}(S))) \quad (5)$$

And:

$$\varphi_i^P(E_P(\tilde{v}),U) = \sum_{\substack{S\subseteq U,\\ i\notin SuppS}} \alpha_U^S (W_P^{S\vee U(i)} E_P(\tilde{v}(S\vee U(i))) - W_P^S E_P(\tilde{v}(S))) \quad (6)$$

for any i∈SuppU and:

$$\alpha_U^S = \frac{|SuppS|!(|SuppU|-|SuppS|-1)!}{|SuppU|!}$$

Theorem 1: Let $\tilde{v}\in\tilde{G}(N)$, U∈L(N) and any given weight W^U_N, then the function $\tilde{G}(N)\to\mathbb{R}_+^{|SuppU|}$, given in Eq. 4, is the unique weighted NPE Shapley function for $\tilde{v}$ in U.

Proof

⇒Axiom 1: Since T⊆U is a weighted NPE carrier, we have:

$$\begin{aligned} & W_{Ne}^{S\vee U(i)} E_{Ne}(\tilde{v}(S\vee U(i))) \\ &= W_{Ne}^{(S\vee U(i))\wedge T} E_{Ne}(\tilde{v}((S\vee U(i))\wedge T)) \\ &= W_{Ne}^{(S\wedge T)\vee(U(i)\wedge T)} E_{Ne}(\tilde{v}((S\wedge T)\vee(U(i)\wedge T))) \\ &= W_{Ne}^{S\wedge T} E_{Ne}(\tilde{v}(S\wedge T)) \\ &= W_{Ne}^S E_{Ne}(\tilde{v}(S)) \end{aligned}$$

where, $i \notin SuppT$.

From Eq. 5, we get $\varphi^N_i(E_N(\tilde{v}), U) = 0$ for any $i \in SuppU \backslash SuppT$. Thus, we have:

$$\begin{aligned} W^T_{Ne} E_{Ne}(\tilde{v}(T)) &= W^{T \wedge U}_{Ne} E_{Ne}(\tilde{v}(T \wedge U)) \\ &= W^U_{Ne} E_{Ne}(\tilde{v}(U)) \\ &= \sum_{i \in SuppU} \varphi^N_i(E_{Ne}(\tilde{v}), U) \\ &= \sum_{i \in SuppT} \varphi^N_i(E_{Ne}(\tilde{v}), U) \end{aligned}$$

Similarly, we obtain:

$$W^T_P E_P(\tilde{v}(T)) = \sum_{i \in SuppT} \varphi^P_i(E_P(\tilde{v}), U)$$

From Eq. 4, we get:

$$\begin{aligned} &\sum_{i \in SuppT} \varphi_i(E_{NP}(\tilde{v}), U) \\ &= \sum_{i \in SuppT} \varphi^N_i(E_{Ne}(\tilde{v}), U) + \sum_{i \in SuppT} \varphi^P_i(E_P(\tilde{v}), U) \\ &= W^T_N E_{Ne}(\tilde{v}(T)) + W^T_P E_P(\tilde{v}(T)) \end{aligned}$$

Axiom 2: From Eq. 5, we obtain:

$$\begin{aligned} &\varphi^N_i(E_{Ne}(\tilde{v}), U) = \sum_{\substack{S \subseteq U, \\ i \notin SuppS}} \alpha^S_U (W^{S \vee U(i)}_{Ne} E_{Ne}(\tilde{v}(S \vee U(i))) - W^S_{Ne} E_{Ne}(\tilde{v}(S))) \\ &= \sum_{\substack{S \subseteq U, \\ i,j \notin SuppS}} \alpha^S_U (W^{S \vee U(i)}_{Ne} E_{Ne}(\tilde{v}(S \vee U(i))) - W^S_{Ne} E_{Ne}(\tilde{v}(S))) \\ &+ \sum_{\substack{S \subseteq U, \\ i,j \notin SuppS}} \alpha^{S+1}_U (W^{S \vee U(i) \vee U(j)}_{Ne} E_{Ne}(\tilde{v}(S \vee U(i) \vee U(j))) \\ &- W^{S \vee U(j)}_N E_N(\tilde{v}(S \vee U(j))) \\ &= \sum_{\substack{S \subseteq U, \\ i,j \notin SuppS}} \alpha^S_U (W^{S \vee U(j)}_{Ne} E_{Ne}(\tilde{v}(S \vee U(j))) - W^S_{Ne} E_{Ne}(\tilde{v}(S))) \\ &+ \sum_{\substack{S \subseteq U, \\ i,j \notin SuppS}} \alpha^{S+1}_U (W^{S \vee U(i) \vee U(j)}_{Ne} E_{Ne}(\tilde{v}(S \vee U(i) \vee U(j))) \\ &- W^{S \vee U(i)}_{Ne} E_{Ne}(\tilde{v}(S \vee U(i))) \\ &= \sum_{\substack{S \subseteq U, \\ j \notin SuppS}} \alpha^S_U (W^{S \vee U(j)}_{Ne} E_{Ne}(\tilde{v}(S \vee U(j))) - W^S_{Ne} E_{Ne}(\tilde{v}(S))) = \varphi^N_j(E_{Ne}(\tilde{v}), U) \end{aligned}$$

where:

$$\alpha^{S+1}_U = \frac{(|SuppS|+1)!(|SuppU|-|SuppS|-2)!}{|SuppU|!}$$

Similarly, we have $\varphi^P_i(E_P(\tilde{v}), U)$. Thus, $\varphi_i(E_{NP}(\tilde{v}), U) = \varphi_j(E_{NP}(\tilde{v}), U)$.

From Eq. 3-5, we can easily get Axiom 3.

Uniqueness: Let $\tilde{v} \in \tilde{G}(N)$ and $U \in L(N)$, it is not difficult to show $E_{Ne} = (\tilde{v})$ can be expressed by:

$$E_{Ne}(\tilde{v}) = \sum_{\varnothing \neq S \subseteq U} E_{Ne}(a_S) u_S$$

where:

$$a_S = \sum_{T \subseteq S} (-1)^{|SuppS|-|SuppT|} E_N(\tilde{v}(S))$$

and u_s is a weighted NE unanimity game defined by:

$$E_{Ne}(u_S(T)) = \begin{cases} W^S_{Ne} & S \subseteq T \\ 0 & \text{otherwise} \end{cases}$$

and $E_P(\tilde{v})$ can be expressed by:

$$E_P(\tilde{v}) = \sum_{\varnothing \neq S \subseteq U} b_S E_P(u_S)$$

where:

$$b_S = \sum_{T \subseteq S} (-1)^{|SuppS|-|SuppT|} E_P(\tilde{v}(S))$$

and $E_P(u_s)$ is a weighted PE unanimity game given as:

$$E_P(u_S(T)) = \begin{cases} W^S_P & S \subseteq T \\ 0 & \text{otherwise} \end{cases}$$

From Axiom 3 and Eq. 3, we only need to show the uniqueness of φ^N and φ^P on unanimity games, respective.

For any u_T with $\varnothing \neq T \subseteq U$, since T is a weighted NPE carrier for u_T, from Axiom 1, we have:

$$\sum_{i \in SuppT} \varphi^N_i(E_{Ne}(u_T), U) = 1$$

And:

$$\sum_{i \in SuppT} \varphi^P_i(E_P(u_T), U) = 1$$

From Axiom 2, we get:

$$\varphi^N_i(E_{Ne}(u_T), U) = \begin{cases} \dfrac{W^T_{Ne}}{|SuppT|} & i \in SuppT \\ 0 & \text{otherwise} \end{cases}$$

And:

$$\varphi^P_i(E_P(u_T), U) = \begin{cases} \dfrac{W^T_P}{|SuppT|} & i \in SuppT \\ 0 & \text{otherwise} \end{cases}$$

Namely, φ^N and φ^P are unique in the unanimity games and the proof is finished.

Theorem 2: Let $\tilde{v}\in\tilde{G}(N)$ and U∈L (N), if $\tilde{v}$ is NPE convex, then we have φ (E_{NP} ($\tilde{v}$), U)∈C (E_{NP} ($\tilde{v}$), U).

Proof: From Theorem 1, we only need to show:

$$\sum_{i\in SuppS}\varphi_i(E_{NP}(\tilde{v}),U)\geq W_{Ne}^S E_{Ne}(\tilde{v}(S))+W_P^S E_P(\tilde{v}(S))$$

for any S⊆U.

From the NPE convexity of $\tilde{v}$, we get:

$$W_{Ne}^{S\vee U(i)}E_{Ne}(\tilde{v}(S\vee U(i)))-W_{Ne}^S E_{Ne}(\tilde{v}(S))\leq W_{Ne}^{T\vee U(i)}E_{Ne}(\tilde{v}(T\vee U(i)))-W_{Ne}^T E_{Ne}(\tilde{v}(T))$$

And:

$$W_P^{S\vee U(i)}E_P(\tilde{v}(S\vee U(i)))-W_P^S E_P(\tilde{v}(S))\leq W_P^{T\vee U(i)}E_P(\tilde{v}(T\vee U(i)))-W_P^T E_P(\tilde{v}(T))$$

for any S, T⊆U with S⊆T, where i∉SuppT.

From Eq. 5, 6, we obtain:

$$\varphi_i^N(E_{Ne}(\tilde{v}),U)\geq\varphi_i^N(E_{Ne}(\tilde{v}),S)$$

And:

$$\varphi_i^P(E_P(\tilde{v}),U)\geq\varphi_i^P(E_P(\tilde{v}),S)$$

for any i∈SuppS.

Thus:

$$\varphi_i(E_{NP}(\tilde{v}),U)\geq\varphi_i(E_{NP}(\tilde{v}),S)\ \ \forall i\in SuppS$$

From Theorem 1, we obtain:

$$\sum_{i\in SuppS}\varphi_i(E_{NP}(\tilde{v}),U)\geq\sum_{i\in SuppS}\varphi_i(E_{NP}(\tilde{v}),S)=W_{Ne}^S E_{Ne}(\tilde{v}(S))+W_P^S E_P(\tilde{v}(S))$$

Property 1: Let $\tilde{v}\in\tilde{G}(N)$ and U∈L(N), if $\tilde{v}$ is NPE convex, then φ (E_{NP} ($\tilde{v}$), U) is a NPE participation monotonic allocation schemes.

Proof: From Theorem 1 and 2, we can easily get the result.

Property 2: Let $\tilde{v}\in\tilde{G}(N)$ and U∈L (N), if T⊆U is a weighted NPE carrier for $\tilde{v}$ in U, then we have φ_i (E_{NP} ($\tilde{v}$), U) = φ_i (E_{NP} ($\tilde{v}$), T) for any i∈SuppU.

Proof: From Theorem 1, we get:

$$\varphi_i(E_{NP}(\tilde{v}),U)=\varphi_i(E_{NP}(\tilde{v}),T)=0$$

for any i∈SuppU\SuppT.

When i∈SuppT. The property is proved by induction on |SuppT|.

Case 1: When |SuppU| = |SuppT|+1, without loss of generality, suppose SuppU = SuppT∪k. From Eq. 5, we get:

$$\begin{aligned}\varphi_i^N(E_{Ne}(\tilde{v}),U)&=\sum_{\substack{S\subseteq U,\\ i\notin SuppS}}\alpha_U^S(W_{Ne}^{S\vee U(i)}E_{Ne}(\tilde{v}(S\vee U(i)))-W_{Ne}^S E_{Ne}(\tilde{v}(S)))\\&=\sum_{\substack{S\subseteq U,\\ i,k\notin SuppS}}\Big(\alpha_U^S(W_{Ne}^{S\vee U(i)}E_{Ne}(\tilde{v}(S\vee U(i)))-W_{Ne}^S E_{Ne}(\tilde{v}(S)))\\&+\alpha_U^{S+1}(W_{Ne}^{S\vee U(i)\vee U(k)}E_{Ne}(\tilde{v}(S\vee U(i)\vee U(k)))\\&-W_{Ne}^{S\vee U(k)}E_{Ne}(\tilde{v}(S\vee U(k))))\Big)\\&=\sum_{\substack{S\subseteq U,\\ i,k\notin SuppS}}(\alpha_U^S+\alpha_U^{S+1})(W_{Ne}^{S\vee U(i)}E_{Ne}(\tilde{v}(S\vee U(i)))-W_{Ne}^S E_{Ne}(\tilde{v}(S)))\\&=\sum_{\substack{S\subseteq T,\\ i\notin SuppS}}\alpha_T^S(W_{Ne}^{S\vee U(i)}E_{Ne}(\tilde{v}(S\vee U(i)))-W_{Ne}^S E_{Ne}(\tilde{v}(S)))=\varphi_i^N(E_{Ne}(\tilde{v}),T)\end{aligned}$$

where, α_U^{S} and α_U^{S+1} as given in Theorem 1 and:

$$\alpha_T^S=\frac{|SuppS|!(|SuppT|-|SuppS|-1)!}{|SuppT|!}$$

When, |SuppU| = |SuppT|+q, without loss of generality, suppose SuppU = SuppT∪{k_1, k_2,..., k_q}. Let $SuppT_1$ = SuppT∪{k_1}, $SuppT_2$ = $SuppT_1$∪{k_2},..., $SuppT_q$ = $SuppT_{q-1}$∪{k_q}.

From above, we have:

$$\begin{aligned}\varphi_i^N(E_{Ne}(\tilde{v}),T)&=\varphi_i^N(E_{Ne}(\tilde{v}),T_1)\\&=\cdots\\&=\varphi_i^N(E_{Ne}(\tilde{v}),T_q)\\&=\varphi_i^N(E_{Ne}(\tilde{v}),U)\end{aligned}$$

for any i∈SuppT.

Similarly, we get:

$$\varphi_i^P(E_P(\tilde{v}),T)=\varphi_i^P(E_P(\tilde{v}),U)\ \forall i\in SuppT$$

Thus, we obtain:

$$\varphi_i(E_{NP}(\tilde{v}),U)=\varphi_i(E_{NP}(\tilde{v}),T)\ \forall i\in SuppU$$

Property 3: Let $\tilde{v}\in\tilde{G}(N)$ and U∈L (N), if we have:

$$W_{Ne}^{S\vee U(i)}E_{Ne}(\tilde{v}(S\vee U(i)))-W_{Ne}^S E_{Ne}(\tilde{v}(S))=W_{Ne}^{U(i)}E_{Ne}(\tilde{v}(U(i)))$$

And:

$$W_P^{S\vee U(i)}E_P(\tilde{v}(S\vee U(i)))-W_P^S E_P(\tilde{v}(S))=W_P^{U(i)}E_P(\tilde{v}(U(i)))$$

for any S⊆U with i∉SuppS.

Then:

$$\varphi_i(E_{NP}(\tilde{v}),U)=W_{Ne}^{U(i)}E_{Ne}(\tilde{v}(U(i)))+W_P^{U(i)}E_P(\tilde{v}(U(i)))$$

Proof: From Eq. 5, we get:

$$\begin{aligned}&\varphi_i^N(E_N(\tilde{v}),U)\\&=\sum_{\substack{S\subseteq U,\\ i\notin SuppS}}\alpha_U^S(W_{Ne}^{S\vee U(i)}E_{Ne}(\tilde{v}(S\vee U(i)))-W_{Ne}^S E_{Ne}(\tilde{v}(S)))\\&=\sum_{S\subseteq U,i\notin SuppS}\alpha_U^S W_{Ne}^{U(i)}E_{Ne}(\tilde{v}(U(i)))=W_{Ne}^{U(i)}E_{Ne}(\tilde{v}(U(i)))\end{aligned}$$

Similarly, we have:

$$\varphi_i^P(E_P(\tilde{v}),U)=W_P^{U(i)}E_P(\tilde{v}(U(i)))$$

Thus:

$$\varphi_i(E_{NP}(\tilde{v}),U)=W_{Ne}^{U(i)}E_{Ne}(\tilde{v}(U(i)))+W_P^{U(i)}E_P(\tilde{v}(U(i)))$$

Corollary 1: Let $\tilde{v}\in\tilde{G}(N)$ and U∈L (N), if we have:

$$W_{Ne}^{S\vee U(i)}E_{Ne}(\tilde{v}(S\vee U(i)))=W_{Ne}^S E_{Ne}(\tilde{v}(S))$$

And:

$$W_P^{S\vee U(i)}E_P(\tilde{v}(S\vee U(i)))=W_P^S E_P(\tilde{v}(S))$$

for any S⊆U with i∉SuppS. Then:

$$\varphi_i(E_{NP}(\tilde{v}),U)=0$$

From Eq. 1-3, we know the Credibility Expectation (CE) is equal to:

$$E_{Cr}(\tilde{v})=\frac{1}{2}(E_{Ne}(\tilde{v})+E_P(\tilde{v}))\qquad(7)$$

When each player's weight for the approximate value of U∈L (N) is 0.5, then we have $W^S_N = W^S_P = 0.5$ for any S⊆U with respect to $E_{Ne}(\tilde{v}(S))$ and $E_P(\tilde{v}(S))$. From Eq. 4, we get the weighted CE Shapley function for $\tilde{v}$ in U as follows:

$$\varphi_i(E_{Cr}(\tilde{v}),U)=\frac{1}{2}(\varphi_i^N(E_{Ne}(\tilde{v}),U)+\varphi_i^P(E_P(\tilde{v}),U))\ \forall i\in \text{SuppU}\qquad(8)$$

where:

$$\varphi_i^N(E_{Ne}(\tilde{v}),U)=\sum_{\substack{S\subseteq U,\\ i\notin SuppS}}\alpha_U^S(E_{Ne}(\tilde{v}(S\vee U(i)))-E_{Ne}(\tilde{v}(S)))\qquad(9)$$

And:

$$\varphi_i^P(E_P(\tilde{v}),U)=\sum_{\substack{S\subseteq U,\\ i\notin SuppS}}\alpha_U^S(E_P(\tilde{v}(S\vee U(i)))-E_P(\tilde{v}(S)))\qquad(10)$$

for any i∈SuppU and α^S_U as given in Eq. 5, 6.

Equation 9 and 10 are called the NE Shapley function and the PE Shapley function for $\tilde{v}$ in U, respectively.

Similar to the above given definitions, we can get their concepts with respect to credibility expectation. Furthermore, all above given theorems and properties still hold for the weighted CE Shapley function.

NUMERICAL EXAMPLE

With the increasing competition among manufacturers, there are three electrical appliances enterprises, named 1, 2 and 3, decide to cooperate with their resources. They can cooperate freely. For example, S_0 = {2, 3} denotes the cooperation of the enterprises 2 and 3. Since there are many uncertain factors during the process of cooperation, it is impossible for the player to know the accurate payoffs of the coalitions. Here, we use the trapezoidal fuzzy numbers to denote the possible payoffs (millions RMB) of the crisp coalitions which are given by Table 1.

From Table 1, we know when the companies 1 and 2 cooperates with all their resources, their fuzzy payoff is (6, 9, 13, 18) millions RMB.

As above pointed, since there are many uncertain factors during the process of cooperation, each company is not willing to offer all its resources to a particular cooperation. Thus, we have to consider a fuzzy game. For example, the company 1 has 1000 unit resources and it supplies only 100 units to cooperate, then we think the 1th player's participation level is 0.1 = 100/1000. In such a way, a fuzzy coalition is explained. Consider a fuzzy coalition U defined by U (1) = 0.6, U(2) = 0.8 and U(3) = 0.3.

When the fuzzy coalition values and that of their associated crisp coalitions have the relationship:

$$\tilde{v}(S)=\sum_{T_0\subseteq SuppS}\left\{\Pi_{i\in T_0}U(i)\Pi_{i\in SuppS\backslash T_0}\left(1-U(i)\right)\right\}\tilde{v}_0\ (T_0)\quad \forall S\subseteq U\qquad(11)$$

Namely, this is a fuzzy game with multilinear extension form and fuzzy payoffs.

Table 1: The fuzzy payoffs of the crisp coalitions

S_0	$\tilde{v}_0(S_0)$	S_0	$\tilde{v}_0(S_0)$
{1}	(2,3,6,8)	{1,3}	(5,12,15,20)
{2}	(2,4,5,7)	{2,3}	(4,11,16,18)
{3}	(2,5,6,7)	{1,2,3}	(15,25,36,42)
{1,2}	(6,9,13,18)		

Table 2: The possibility expectations of the fuzzy coalitions

S	$E_P(\tilde{v}(S))$	S	$E_P(\tilde{v}(S))$
{1}	4.20	{1,3}	6.87
{2}	4.80	{2,3}	7.83
{3}	1.95	{1,2,3}	15.174
{1,2}	10.20		

Table 3: The necessity expectations of the fuzzy coalitions

S	$E_N(\tilde{v}(S))$	S	$E_N(\tilde{v}(S))$
{1}	1.50	{1,3}	2.94
{2}	2.40	{2,3}	3.69
{3}	1.05	{1,2,3}	7.392
{1,2}	4.86		

From Eq. 2 and 11, we get the possibility expectations of the fuzzy coalitions as given in Table 2.

From Eq. 3 and 11, we get the necessity expectations of the fuzzy coalitions as given in table 3.

Let:

$$W_N^S = \sum_{i \in SuppS} w_i \Big/ |SuppS|$$

for any $S \subseteq U$, where w_i denotes the weight of the i's with respect to NE. If the players 1, 2 and 3 are risk averse, risk pursuit and risk neutral, respectively. Namely, $w_1 = 1$, $w_2 = 0$ and $w_3 = 0.5$. From Eq. 4, we get the player weighted NPE Shapley values are:

- $\varphi_1(E_{NP}(\tilde{v}), U) = 2.85$
- $\varphi_2(E_{NP}(\tilde{v}), U) = 5.94$
- $\varphi_3(E_{NP}(\tilde{v}), U) = 2.49$

CONCLUSIONS

We have researched a general case of fuzzy games with fuzzy payoffs under possibility and necessity measures which can be used in all kinds of games with fuzzy payoffs, where the possibility and necessity expectations exist. Since the possibility and necessity measures reflect the players' risk attitudes, we give the weighted *NPE* Shapley function for fuzzy games with fuzzy payoffs. Some properties are also discussed.

However, we only research the weighted NPE Shapley function for fuzzy games with fuzzy payoffs and it will be interesting to study other payoff indices for this kind of fuzzy games under possibility and necessity measures.

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