

<http://ansinet.com/itj>

ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSI*net*

Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Carrier Frequency Offset Estimation Algorithm Based on Compressive Sensing Theory for Interleaved OFDMA Uplink Systems

¹Ruiyan Du, ¹Jinkuan Wang, ²Yanli Bo and ³Liang Cui

¹School of Information Science and Engineering, Northeastern University, Shenyang, 110819, China

²Environmental Management College of China, Qinhuangdao, 066004, China

³Hebei Vocational and Technical College of Building Materials, Qinhuangdao, 066004, China

Abstract: Compressive sensing is an emerging area which uses a relatively small number of non-traditional samples in the form of randomized projections to reconstruct sparse or compressible signals. This study considered the carrier frequency offset estimation problem for interleaved orthogonal frequency-division multiple-access uplink systems. A new carrier frequency offset estimation method based on the compressive sensing theory is proposed to estimate the carrier frequency offsets in interleaved OFDMA uplink systems. The presented method can effectively estimate all carrier frequency offsets of the active users by finding the sparse coefficients. Simulation results are presented to verify the efficiency of the proposed approach.

Key words: Compressive sensing, carrier frequency offset (CFO), orthogonal frequency division multiple access (OFDMA), sparse representation

INTRODUCTION

Orthogonal frequency-division multiple access (OFDMA) has been or is being considered for various wireless communication systems, such as IEEE 802.16, 802.20 (Bolton *et al.*, 2007), 802.22 (Cordeiro *et al.*, 2006). However, similar to OFDM, OFDMA is extremely sensitive to the carrier frequency offsets (CFOs) caused by oscillator mismatch and Doppler shift. The uncompensated CFOs will result in loss of orthogonality among subcarriers and introduce intercarrier interference (ICI) (Abdellaoui *et al.*, 2006; Kumar *et al.*, 2008; Jiang *et al.*, 2010) as well as Multiple Access Interference (MAI) (Venkatesan and Ravichandran, 2007; Ren *et al.*, 2008; Wu *et al.*, 2010), thus leading to severe performance degradation. Frequency synchronization is critically important for the OFDMA system. With imperfect synchronization, ICI is generated due to the loss of subcarrier orthogonality, contributing both co-channel and interchannel interference. In OFDMA downlink systems, each user could perform CFO estimation by exploiting the preamble from the Base Station (BS) at the beginning of each downlink time slot. This operation can be easily accomplished by using the algorithms employed in OFDM systems (Schmidl and Cox, 1997; Zhao *et al.*, 2011). However, in OFDMA uplink systems, distinct users pass through different channels and experience distinct

CFOs. All CFOs have to be simultaneously estimated via the received signal which is the superposition of signals from all active users. Thus, CFO estimation for uplink systems is a multiple-parameter estimation problem and hence, it is much more difficult than the downlink case. Therefore, reliable CFO estimation has been a challenging task and is critical for the data detection for OFDMA uplink systems.

CFO estimation schemes for uplink OFDMA systems can be categorized as data-aided and non-data-aided. Pun *et al.* (2006) proposed an effective iterative scheme which is referred to as the alternating-projection frequency estimator (APFE). However, it requires an exhaustive grid searching to estimate each CFO which results in unattractive complexity. To reduce the complexity, Sezginer and Bianchi (2008) reported a suboptimal approach by replacing the matrix inversion in APFE with an approximated matrix extension. However, it requires a large number of subcarriers. The estimator (Wu *et al.*, 2011) utilizes constant amplitude zero autocorrelation (CAZAC) training sequences to obtain the CFOs. However, it causes an irreducible error floor. A simple iterative algorithm (Sun and Zhang, 2011) is derived via exploiting the fact that the tile structure in 802.16e provides inherent MAI power compression in the frequency domain.

When training sequences are not available, the CFO for each user cannot easily be obtained. Fortunately, this problem can be overcome with a proper Carrier Assignment Scheme (CAS), such as interleaved CAS. With the interleaved-based CAS, the CFO estimators (Cao *et al.*, 2004; Lee *et al.*, 2007) exploit the inherent periodic structure of each user's signal. Due to the spectrum peak searching involved, the computational complexity of the MUSIC estimator (Cao *et al.*, 2004) is not attractive. The ESPRIT estimator (Lee *et al.*, 2007) can avoid the complex peak searching at the expense of the loss of estimation performance. Based on interleaved CAS, Hsieh and Wu (2011) proposed an optimum CFO estimator by truncating the series expansion of the correlation matrix in maximum likelihood function.

In most previous works, the CFO estimation is based on the Nyquist sampling. Compressed Sensing (CS) theory (Donoho, 2006; Candes *et al.*, 2006; Wu *et al.*, 2010) is a novel data collection and coding theory under the condition that signal is sparse or compressible. It has been shown that when the measurement matrix satisfies certain random properties, the original signal can be reconstructed even when the number of observation is far less than the number of Nyquist rate samples. It is noted that the CS theory quickly gains intensive attention in many areas including imaging, communications, analog-to-digital conversion, direction estimation as well as text classification, etc. (Berger *et al.*, 2010; Deng *et al.*, 2011; Cevher *et al.*, 2008; Liu *et al.*, 2011; Deng *et al.*, 2012).

Motivated by the rich research results using compressive sensing in different areas, this paper focused on the CFO estimation problem for interleaved OFDMA uplink systems based on CS theory.

DATA MODEL FOR INTERLEAVED OFDMA UPLINK SYSTEMS

In this section, we introduce the uplink signal model of OFDMA (Fig. 1). Consider a K-user OFDMA uplink

system with N subcarriers. Assume that the N sub-carriers are interleaved into Q subchannels, each of which has P = N/Q uniformly spaced subcarriers. The qth sub-channel consists of subcarriers with the index set {q, Q+q, ..., (P-1)Q+q}, (q = 0, 1, 2, ..., Q-1). Suppose that each user has only one subchannel and the P subcarriers of the qth subchannel are assigned to the kth user. After the Cyclic Prefix (CP) is removed, the signal sample of the nth subcarrier of one OFDMA block at the uplink receiver can be described as:

$$r(n) = \sum_{k=1}^K r_k(n) + z(n), (n = 0, 1, \dots, N-1) \tag{1}$$

where, K denotes the total number of active users in the system. z(n) presents the spatially and temporally white additive Gaussian noise with zero-mean and equal variance σ^2 . $r_k(n)$ stands for the signal sample of the nth subcarrier from the kth user at the uplink receiver, it is given by:

$$r_k(n) = \sum_{p=0}^{P-1} H_{k,p} X_{k,p} e^{j2\pi n(pQ+q+\xi_k)}$$

where, the subscript $(\cdot)_k$ denotes the kth user. $H_{k,p}$ denotes the channel frequency response on the pth subcarrier of the kth user's channel during one OFDMA block. $X_{k,p}$ denotes the data symbol on the pth subcarrier of the kth user. $\xi_k = \Delta f_k / \Delta f$ is the normalized CFO of the kth user. Δf denotes the subcarrier spacing and Δf_k represents the CFO between the kth user and the uplink receiver. For practical purpose, the absolute value of Δf_k is assumed to be less than half of OFDMA subcarrier spacing, that is, $\xi_k \in (-0.5, 0.5)$.

From (1), we construct a Q×P matrix Y by a data stacking technology. The matrix Y has the following form:

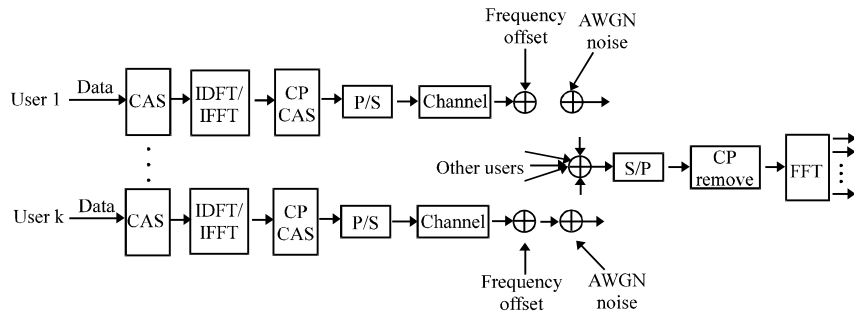


Fig. 1: The OFDMA uplink model

$$Y = \begin{bmatrix} r(0) & r(1) & \dots & r(P-1) \\ r(P) & r(P+1) & \dots & r(2P-1) \\ \vdots & \vdots & \ddots & \vdots \\ r(N-P) & r(N-P+1) & \dots & r(N-1) \end{bmatrix} = V[C \odot (BW)] + Z, = VS + Z \quad (2)$$

where, the Vandermonde matrix:

$$V_{Q \times K} = [v_1, v_2, \dots, v_K], v_k = [1, e^{j2\pi\theta_k}, \dots, e^{j2\pi(Q-1)\theta_k}]^T$$

(.)^T denotes the transpose operation and:

$$\theta_k = \frac{q + \xi_k}{Q}$$

being the effective CFO of the kth user. $S_{K \times P} = C_{K \times P} \odot (B_{K \times P} W_{P \times P})$, in which \odot denotes the Schur product, $C = [c_1^T, c_2^T, \dots, c_K^T]^T$ with $c_k = [1, e^{j2\pi\theta_k/P}, \dots, e^{j2\pi(P-1)\theta_k/P}]^T$, $B = [b_1^T, b_2^T, \dots, b_K^T]^T$ with $b_k = [H_{k,0} X_{k,0}, \dots, H_{k,P-1} X_{k,P-1}]$, $W = [w_1, w_2, \dots, w_P]$ is the IFFT matrix, with $w_p = [1, e^{j2\pi(p-1)/P}, \dots, e^{j2\pi(p-1)(P-1)/P}]^T$. The matrix $Z_{Q \times P}$ represents the noise term constructed from $z(n)$ in a similar form as Y .

Notice that the effective CFO has one important property, that is, different users have distinct effective CFOs. It can be shown that if one user occupies subchannel q , the range of its effective CFO is $((q-0.5)/Q, (q+0.5)/Q)$, since the range of its normalized CFO is $(-0.5, 0.5)$. Because different users occupy different subchannels, their effective CFOs fall in nonoverlapping ranges.

CFO ESTIMATION BASED ON CS THEORY

Assume that there exist a small number of users, the CFOs are sparse in the effective CFO space, i.e., $\eta = [\eta_1, \eta_2, \dots, \eta_J]^T$ is a sparse vector. A non-zero element with index j in η indicates that there is a user at the CFO θ_j .

Sparse representation and atom dictionary: In order to accomplish the CFO estimation based on CS theory, sparse representation and over-complete atom dictionary need to be constructed for CFO estimation problem.

Let y_l, s_l, z_l denote the l th column of $Y, S, Z, (l = 1, 2, \dots, P)$. Thus, we have the following equation:

$$y_l = Vs_l + z_l \quad (3)$$

where, V is defined in (2). Similarly, let v_k denote the k th column of the matrix V , namely, $V = [v_1, v_2, \dots, v_K]$ with $v_k = [1, e^{j2\pi\theta_k}, \dots, e^{j2\pi(Q-1)\theta_k}]^T, (k = 1, 2, \dots, K)$.

The covariance matrix of vector y_l is given by:

$$R = E\{y_l y_l^H\} = VRV^H + \sigma^2 I \quad (4)$$

where $E\{\cdot\}$ and $(\cdot)^H$ denote the expectation and the conjugate transpose, respectively. The signal covariance matrix is $R_s = E\{s_l s_l^H\}$. σ^2 stands for the noise power. I presents the identity matrix.

Consider each column $r_m (m = 1, 2, \dots, Q)$, of $R = [r_1, r_2, \dots, r_Q]$. According to the linear algebra theory, it can be linearly represented by any complete basis in the Q -dimensional complex vector space. Given the overcomplete basis $\{v(\tilde{\theta}_k)\}_{k=1}^J (J \gg K)$ where, $\{\tilde{\theta}_k\}_{k=1}^J$ are the specified CFOs sampled in the CFO domain, e.g., from $(-0.5/Q, (Q-0.5)/Q)$ with δ spacing. The m th column vector r_m of R can be written as:

$$r_m = E\{y_l y_{1,m}^*\} = R e_m = V(\tilde{\theta}) b_m + \sigma^2 e_m, (m = 1, 2, \dots, Q) \quad (5)$$

where $(\cdot)^*$ denotes the complex conjugate operation. $Y_{1,m}$ is the m th element of the vector y_l . The atom dictionary $V(\tilde{\theta}) = [v(\tilde{\theta}_1), v(\tilde{\theta}_2), \dots, v(\tilde{\theta}_J)]$ in which the atom vector $v(\tilde{\theta}_k) = [1, e^{j2\pi\tilde{\theta}_k}, \dots, e^{j2\pi(Q-1)\tilde{\theta}_k}]^T, (k = 1, 2, \dots, J)$. e_m is a $Q \times 1$ column vector with 1 in the m th entry and 0 elsewhere. Assume that $\{\tilde{\theta}_k\}_{k=1}^J$ are dense enough, some K vectors in $\{v(\tilde{\theta}_k)\}_{k=1}^J$ can be expected to be very close to $\{v(\theta_k)\}_{k=1}^K$ so an ideal b_m should be the vector with all elements zero except for the K elements associated with these K basis vectors, namely, an ideal b_m has a sparse structure related the CFOs of the K users.

Observation matrix: In the normal circumstances, the Gaussian random matrix can act as the observation. The observation matrix Φ is the $L \times Q$ Gaussian random matrix, where, L presents the number of random sampling points. We can use the L points to accomplish the CFO estimation.

CFO estimation: In the noise environment, we can use the following formula to solve the problem according to the BP theory. The convex problem can be given as follows:

$$\min \|x_m - \Phi V(\tilde{\theta}) \eta\|_2^2 + \lambda \|\eta\|_1 \quad (6)$$

where, the down sampled data $x_m = \Phi r_m$. Φ presents the observation matrix. The matrix $V(\tilde{\theta})$ stands for the atom dictionary. The vector η is the sparse representation vector which contains the CFO information.

For the optimization problem (6), we can use the CVX toolbox (Grant and Boyd, 2009) to get the sparsest representation η , then figure out some biggest sparse coefficient components of η .

The positions of these K biggest sparse coefficient components located on the vector K should be recorded. Furthermore, according to the positions we should find

the corresponding atom in dictionary. Thus, we can obtain the CFO estimation on the basis of the atom.

SIMULATION RESULTS

In this section, some simulations are conducted to validate the performance of the proposed CFO estimation based on CS theory.

Case 1: The considered OFDMA system has $N = 1024$ subcarriers. QPSK modulation scheme is used to generate the transmitted symbols. The wireless channel is modeled as a frequency-selective fading channel consisting of six independent Rayleigh multipaths, with an exponentially decaying power-delay profile. In this simulation, the numbers of subchannels and users are set to $Q = 8$ and $K = 3$, respectively. The normalized CFOs are set to $-0.2, 0.1, 0.35$, respectively. The signal-to-noise-ratio (SNR) is set to 5 dB. Figure 2 gives the CFO curves estimated by the proposed method and MUSIC, respectively. As shown in this figure, all interesting CFOs are successfully estimated by the proposed method and MUSIC. Figure 2 shows that the performance of the proposed method is similar to that of MUSIC.

Case 2: Except for the normalized CFOs of all active users ($\xi_{p,k}, k = 1, 2, \dots, K$) are generated as random variables that are uniformly distributed in $(-0.5, 0.5)$ with 500 random trial runs, other simulation conditions are similar to Case 1. Figure 3 gives the normalized Root Mean Square Error (RMSE) curves of the CFOs estimated by the aforementioned methods. The RMSE curves with SNRs ranging from 0 to 20 dB. The RMSE is defined as:

$$\sqrt{\frac{1}{K\Pi} \sum_{p=1}^{\Pi} \sum_{k=1}^K (\hat{\xi}_{p,k} - \xi_{p,k})^2} \quad (7)$$

where, K is the number of all users, Π is the total number of Monte Carlo trials, $\hat{\xi}_{p,k}$ is the estimate of $\xi_{p,k}$. ρ is the index of the Monte Carlo trials. Figure 3 gives the RMSE curves of the proposed method, MUSIC and ESPRIT, respectively. As shown in Fig. 2, the proposed method can work well for different SNR and outperform ESPRIT.

Case 3: Except for the number of users, other simulation conditions are similar to Case 2. Figure 3 gives the normalized root mean square error (RMSE) curves of the CFOs estimated via the proposed algorithm when the number of users varies. As can be seen in Fig. 4, the proposed estimator can work well in different users. Meanwhile, as expected, the less the user number, the better performance we will obtain.

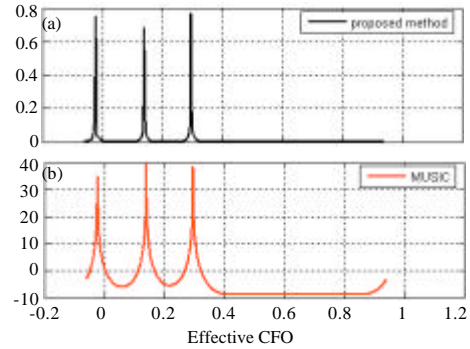


Fig. 2(a-b): Comparison of the CFO estimation between MUSIC and the proposed method

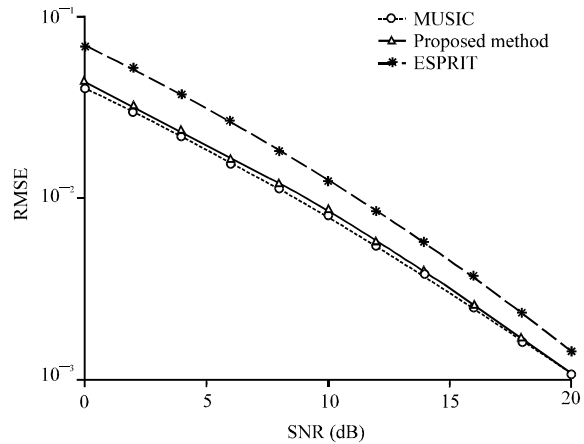


Fig. 3: Comparison of the RMSE curves versus SNR for MUSIC, ESPRIT and the proposed method

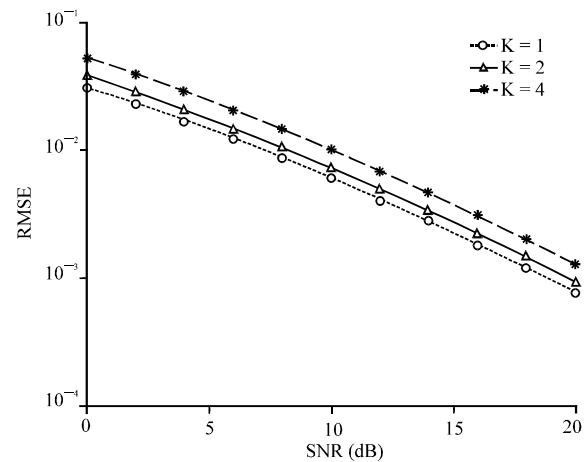


Fig. 4: Comparison of the RMSE curves versus SNR for various K

CONCLUSIONS

In this study, a new CFO estimation method is proposed for the interleaved OFDMA uplink systems. According to the compressive sensing theory, the presented method can simultaneously estimate the CFOs of all active users by finding the sparsest coefficients which can be obtained by solving the convex optimization problem. Simulation results verify that the proposed CFO estimation method is effective, practical and easily achieved.

ACKNOWLEDGMENTS

This study has been supported by the National Natural Science Foundation of China under Grant No. 61104005, by Directive Plan of Science Research from the Bureau of Education of Hebei Province, China, under Grant Nos.Z2011129 and by Science and Technology Support planning project of Hebei Province, China, under Grand No. 11213502D.

REFERENCES

- Abdellaoui, M., N. Nasri, B. Gassara, N. Kachouri, M. Samet and T. Val, 2006. Determination of the underwater channel characteristics to improve a multiband OFDM communication. *Trends Applied Sci. Res.*, 1: 431-443.
- Berger, C.R., S.L. Zhou, J.C. Preisig and P. Willett, 2010. Sparse channel estimation for multicarrier underwater acoustic communication: from subspace methods to compressed sensing. *IEEE Trans. Signal Process.*, 58: 1708-1721.
- Bolton W., Y. Xiao and M. Guizani, 2007. IEEE 802.20: Mobile broadband wireless access. *IEEE Wireless Commun.*, 14: 84-95.
- Candes, E., J. Romberg and T. Tao, 2006. Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information. *IEEE Trans. Inform. Theory*, 52: 489-509.
- Cao, Z., U. Tureli and Y.D. Yao, 2004. Deterministic multiuser carrier frequency offset estimation for interleaved OFDMA uplink. *IEEE Trans. Commun.*, 52: 1585-1594.
- Cevher, V., A.C. Gurbuz, J.H. McClellan and R. Chellappa, 2008. Compressive wireless arrays for bearing estimation. *Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing*, 31 March-4 April, 2008, ICASSP, Las Vegas, pp: 2497-2500.
- Cordeiro, C., K. Challapali and D. Birru, 2006. IEEE 802.22: An introduction to the first wireless standard based on cognitive radios. *J. Commun.*, 1: 38-47.
- Deng, J., G. Ren, Y. Jin and W. Ning, 2011. Iterative weighted gradient projection for sparse reconstruction. *Inform. Technol. J.*, 10: 1409-1414.
- Deng, Z., G. Hu, Z. Pan and Y. Zhang, 2012. Kernel sparse feature selection based on semantics in text classification. *Inform. Technol. J.* 11: 319-323.
- Donoho, D.L., 2006. Compressed sensing. *IEEE Trans. Inform. Theory*, 52: 1289-1306.
- Grant, M. and S. Boyd, 2009. CVX: Matlab software for disciplined convex programming. <http://cvxr.com/cvx/>
- Hsieh, H.T. and W.R. Wu, 2011. Blind maximum-likelihood carrier-frequency-offset estimation for interleaved OFDMA uplink systems. *IEEE Transaction on Vehicular Technology*, 60: 160-173.
- Jiang, W., Z. Zhang, Y. Xu and L. Sun, 2010. Dynamic inter-cell interference cancellation for uplink in multi-cell OFDMA systems. *Inform. Technol. J.*, 9: 1490-1494.
- Kumar, R., S. Malarvizhi and S. Jayashri, 2008. Time-domain equalization technique for intercarrier interference suppression in OFDM systems. *Inform. Technol. J.*, 7: 149-154.
- Lee, J., S. Lee and K.J. Bang, 2007. Carrier frequency offset estimation using ESPRIT for interleaved OFDMA uplink systems. *IEEE Trans. Vehicular Technol.*, 56: 3227-3231.
- Liu, Z.M., Z.T. Huang and Y.Y. Zhou, 2011. Direction of arrival estimation of wideband signals via covariance matrix sparse representation. *IEEE Trans. Signal Process.*, 59: 4256-4270.
- Pun, M.O., M. Morelli and C.C.J. Kuo, 2006. Maximum-likelihood synchronization and channel estimation for OFDMA uplink transmissions. *IEEE Trans. Commun.*, 54: 726-736.
- Ren, G., Z. Wu, N. Zhao and M. Lin, 2008. A mutated ant colony optimization algorithm for multiuser detection. *Inform. Technol. J.*, 7: 948-952.
- Schmidl, T.M. and D.C. Cox, 1997. Robust frequency and timing synchronization for OFDM. *IEEE Trans. Commun.*, 45: 1613-1621.
- Sezginer, S. and P. Bianchi, 2008. Asymptotically efficient reduced complexity frequency offset and channel estimators for uplink MIMO-OFDMA systems. *IEEE Trans. Signal Process.*, 56: 964-979.
- Sun, P. and L. Zhang, 2011. A simple iterative carrier frequency synchronization technique for OFDMA uplink transmissions. *Wireless Commun. Mobile Comput.*, 11: 121-128.

- Venkatesan, G.K.D.P. and V.C. Ravichandran, 2007. Performance analysis of MC-CDMA for wide band channels. *Inform. Technol. J.*, 6: 267-270.
- Wu, Z., N. Zhao, G. Ren and T. Quan, 2010. Anti-interference strategies review of unified spread spectrum telemetry tracking and control system. *Inform. Technol. J.*, 9: 979-983.
- Wu, Y., W.M. Bergmans and S. Attallah, 2011. Carrier frequency offset estimation for multiuser MIMO OFDM uplink using CAZAC sequences: performance and sequence optimization. *EURASIP J. Wireless Commun. Networking*, 10.1155/2011/570680 .
- Zhao, H., Y. Liu, J. Zhang and J. Zhou, 2011. A novel joint synchronization algorithm for OFDM systems based on single training symbol. *Int. J. Digital Content Technol. Appl.*, 5: 217-225.