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Design of Observer and Observer-based Controller for a Class of Singularly Perturbed Discrete Bilinear Systems

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Abstract: This study has presented two types of observer and observer-based controller for stabilization of singularly perturbed discrete bilinear systems. The first one is an ϵ-dependent observer-based controller that stabilizes the closed-loop system for all $\varepsilon \in (0, \varepsilon^*_p)$ where, ε^*_p is the prespecified upper bound of the singular perturbation parameter. The other one is an ϵ-independent observer-based controller, which is able to stabilize the closed-loop system for all $\varepsilon \in (0, \varepsilon_{max})$ where, ε_{max} is the exact upper ϵ-bound. The designs of observer and observer-based controller are suitable for the quasi-separation property. An example is exploited to illustrate the proposed schemes.

Key words: Singularly perturbed systems, bilinear systems, observer, observer-based controller, Lyapunov function

INTRODUCTION

In the control process of practice systems may be appropriately described by the bilinear model, such as biology, ecology, socioeconomic, nuclear, thermal and chemical processes etc. (Mohler, 1991; Hsiao and Wang, 2000; Hsiao *et al*., 2010). Whereas the original systems are usually described by high-order difference equations, it requires large number of computer memory and considerable operation time to handle the systems. In general, singularly perturbed method can be solved this problem (Hsiao *et al*., 2003). Design of a state feedback controller for singularly perturbed discrete bilinear system was considered by Chiou *et al*. (2000) where design of all states for the controller must be measurable. But this assumption generally does not hold in practice (Xin *et al*., 2009; Asseu *et al*., 2008; Gholizade-Narm *et al*., 2008), i.e., the system's state may be unavailable to implement the state feedback. Hence, it is often desirable to design an observer and an observer-based controller leading to asymptotic stability (Li and Fang, 2009; Zhang and Li, 2010; Wang *et al*., 2009; Luo *et al*., 2009; Li *et al*., 2011; Hassanzadeh and Fallah, 2008; Asseu *et al*., 2011) for singularly perturbed discrete bilinear systems. In this study, the observer and observer-based controller designs of a system modeled as a singularly perturbed discrete bilinear system are examined. Two types of observer and observer-based controller are developed, one is that the controller gain depends on observer gain matrix, the other is that the controller gain and observer gain matrix can be designed separately once the so-called quasi-separation property holds.

The following notations will be used throughout the study: The identity matrix with dimension n is denoted by I_n, $\lambda_{min}(A)$ stands for the minimum eigenvalue of matrix A, the spectral radius of matrix A is denoted by r (A), $\|A\|$ presents the norm of matrix A, i.e., $\|A\| = \text{Max}\,(\lambda\,(A^T A))^{1/2}$ and the symbol "•" represents the bialternate product in (Fuller, 1968; Li and Chiou, 2002).

Observer and observer-based controller designs: Consider a singularly perturbed discrete bilinear system as described by the following difference equation:

$$X(k+1) = \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ \varepsilon A_{21} & \varepsilon A_{22} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} B_1 \\ \varepsilon B_2 \end{bmatrix} u(k) + \begin{bmatrix} N_{11} & N_{12} \\ \varepsilon N_{21} & \varepsilon N_{22} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} u(k)$$

$$= (A_0 + \in A_1)\, X(k) + B(\in)\, u(k) + N(\cup)\, X(k)\, u(k) \tag{1a}$$

$$y(k) = C_1 x_1(k) + C_2 x_2(k) = CX(k) \tag{1b}$$

where, $X \in R^n$, $x_1 \in R^{n1}$, $x_2 \in R^{n2}$, n $(=n_1+n_2)$ is the order of the whole system, u is a scalar control, $y \in R^m$ is the output, ε is the singular perturbation parameter and A_{ij}, B_i, N_{ij}, C_i are constant matrices with appropriate dimensions for i, j = 1, 2 Let:

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$$A_0 = \begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 0 \\ A_{21} & A_{22} \end{bmatrix}, B(\varepsilon) = \begin{bmatrix} B_1 \\ \varepsilon B_2 \end{bmatrix} \text{ and } N(\varepsilon) = \begin{bmatrix} N_{11} & N_{12} \\ \varepsilon N_{21} & \varepsilon N_{22} \end{bmatrix}$$

For the sake of saving space, we denote B = B (ϵ) and N = N (ϵ) in the following derivation.

The objective is to design a full state observer and an observer-based controller for the given system (1) to be stable. Now, it is required to construct an observer for the system (1) as below:

$$\hat{X}(k+1) = \begin{bmatrix} \hat{x}_1(k+1) \\ \hat{x}_2(k+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ \varepsilon A_{21} & \varepsilon A_{22} \end{bmatrix} \begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix} + \begin{bmatrix} B_1 \\ \varepsilon B_2 \end{bmatrix} u(k) + \begin{bmatrix} N_{11} & N_{12} \\ \varepsilon N_{21} & \varepsilon N_{22} \end{bmatrix} \begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix} u(k) + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \left(y(k) - \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix} \right)$$

$$= (A_0 + \varepsilon A_1)\hat{X}(k) + B(\varepsilon)u(k) + N(\varepsilon)\hat{X}(k)u(k) + L(y(k) - C\hat{X}(k)) \quad (2)$$

where, $\hat{x}_1 \in R^{n_1}$, $\hat{x}_2 \in R^{n_2}$ are the observer states, $L_1 \in R^{n1\times q}$ and $L_2 \in R^{n2\times q}$ are suitable observing gain matrices which will be designed so as to ensure the estimated state $\hat{x}_1 \to x_1$ and $\hat{x}_2 \to x_2$ as $k\to\infty$. It should also be added that the pair (A_{11}, C_1) is detectable.

Here, an observer-based control can be designed as follows:

$$u(k) = \frac{-\sigma D\hat{X}(k)}{\sqrt{1 + \hat{X}^T D^T D\hat{X}}} \quad (3)$$

where, $D \in R^{1\times n}$ can be arbitrarily designed, σ is constant. Hence, the state and error ($e_1 = x_1 - \hat{x}_1$, $e_2 = x_2 - \hat{x}_2$ and $E = [e_1 \quad e_2]^T$) equations which can be written as follows:

$$X(k+1) = \hat{A}_{11}X(k) + \hat{A}_{12}E(k) \quad (4a)$$

$$E(k+1) = \hat{A}_{22}E(k) \quad (4b)$$

where,

$$\hat{A}_{11} = A_0 + \varepsilon A_1 - \frac{\sigma}{\sqrt{1 + \hat{X}^T D^T D\hat{X}}} BD - \frac{\sigma}{\sqrt{1 + \hat{X}^T D^T D\hat{X}}} \hat{X}^T D^T N,$$

$$\hat{A}_{12} = \frac{\sigma}{\sqrt{1 + \hat{X}^T D^T D\hat{X}}} BD, \hat{A}_{22} = \hat{A}_0 + \varepsilon A_1 - \frac{\sigma\hat{X}^T D^T N}{\sqrt{1 + \hat{X}^T D^T D\hat{X}}}$$

in which $\hat{A}_0 = A_0 - LC$.

For the stability of the system (4), the eigen values of $\hat{A}_{11}$ and $\hat{A}_{22}$ are designed within the unit circle, i.e., $|\lambda(\hat{A}_{11})| < 1$ and $|\lambda(\hat{A}_{22})| < 1$. The objective is to find the suitable estimator's gain matrix L and the controller gain σ such that the system (4) is stable. In order to prove the stability of the system (4a) and the error equation (4b), the Lyapunov function candidates for the system (4a) and the error equation (4b) can be given by:

$$V[X(k)] = X^T(k) PX(k) \quad (5a)$$

$$V[E(k)] = E^T(k) P_e E(k) \quad (5b)$$

in which, P, $P_e \in R^{n\times n}$ may be obtained by solving the Lyapunov equations:

$$(1+\delta)^2 A_0^T P A_0 - P = -I_n \quad (6a)$$

$$(1+\delta)^2 \hat{A}_0^T P_e \hat{A}_0 - P_e = -I_n \quad (6b)$$

and where, δ is a positive constant satisfying $(1+\delta) < \text{Min}(r^{-1}(A_0), r^{-1}(\hat{A}_0))$.

For the latter development, the following lemma is helpful.

Lemma 1: (Furukawa and Shimemura, 1983): Consider the matrices F, G and H which have the same dimension and let F = G+H. For any real number $\delta>0$, the following relation holds:

$$FF^T \le (1+\delta) GG^T + (1+\delta^{-1}) HH^T \quad (7)$$

Theorem 1: Suppose A_{11} is stable and choose L such that $\hat{A}_0$ is stable in the discrete sense, then the system (1) and observer (2) are guaranteed to be stable under the observer-based controller (3) for all $\varepsilon \in (0, \varepsilon^*_p)$ and $\sigma \in (0, \sigma^*_p)$, if the upper bounds σ^*_p and ε^*_p are determined by the following algorithms:

$$\varepsilon^*_{oc-1} = \left(\frac{\delta}{(1+\delta)^2 \|A_1^T P A_1\|}\right)^{1/2} \quad (8)$$

$$q_{oc-1} = 1 - \frac{(1+\delta)^2 \varepsilon^2}{\delta} \|A_1^T P A_1\| \quad (9)$$

$$\sigma^*_{oc-1} = \text{Min}\left[\left(\frac{q_{oc-1}\delta}{(1+\delta)^2 \|D^T B^T P B D\|}\right)^{1/2}, \left(\frac{q_{oc-1}\delta^2}{(1+\delta)^2 \|N^T P N\|}\right)^{1/2}\right] \quad (10)$$

$$\varepsilon^*_{e-1} = \left(\frac{\delta}{(1+\delta)^2 \|A_1^T P_e A_1\|}\right)^{1/2} \quad (11)$$

$$q_{e-1} = 1 - \frac{(1+\delta)^2 \varepsilon^2}{\delta} \|A_1^T P_e A_1\| \quad (12)$$

$$\sigma^*_{e-1} = \left(\frac{\delta q_{e-1}}{(1+\delta)\|N^T P_e N\|}\right)^{1/2} \tag{13}$$

Find:

$$\varepsilon^*_p = \text{Min}\,(\varepsilon^*_{oc-1},\ \varepsilon^*_{e-1}) \tag{14}$$

$$\sigma^*_p = \text{Min}\,(\sigma^*_{oc-1},\ \sigma^*_{e-1}) \tag{15}$$

Proof: It will be useful to divide this proof into two parts, the stability of the system's state equation (4a) and the stability of the error equation (4b).

- The stability of the system's state equation (4a)

In fact, this result is the same as Theorem 2 of (Chiou *et al*., 2000), i.e., $\varepsilon^*_{oc-1} = \varepsilon^*_0$, $q_{oc-1} = q$, $\sigma^*_{oc-1} = \sigma^*$ (ε^*_0, q and σ* can be obtained from equations (11)-(13) of (Chiou *et al*., 2000)). Hence, the system (1) can be guaranteed to stable for all $\varepsilon \in (0,\ \varepsilon^*_{oc-1})$ and $\sigma \in (0, \sigma^*_{oc-1})$.
(ii): The stability of the error equation (4b).

From (4b), (5b), (6b) and Lemma 1, the Lyapunov forward difference is given by:

$$\Delta V\,(E(k)) = E^T(\hat{A}_0 + \varepsilon A_1 - \frac{\sigma \hat{X}^T D^T N}{\sqrt{1+\hat{X}^T D^T D\hat{X}}})^T P_e (\hat{A}_0 + \varepsilon A_1 - \frac{\sigma \hat{X}^T D^T N}{\sqrt{1+\hat{X}^T D^T D\hat{X}}})E - E^T P_e E$$

$$\le E^T[(1+\delta)^2 \hat{A}_0^T P_e \hat{A}_0 + \frac{(1+\delta)^2 \varepsilon^2}{\delta} A_1^T P_e A_1 + \frac{(1+\delta)\sigma^2 \hat{X}^T D^T D\hat{X}}{\delta(1+\hat{X}^T D^T D\hat{X})} N^T P_e N - P_e)\,E$$

$$\le E^T(-I_n + \frac{(1+\delta)^2 \varepsilon^2}{\delta}\|A_1^T P_e A_1\| I_n + \frac{(1+\delta)\sigma^2}{\delta}\|N^T P_e N\| I_n)E$$

$$\le E^T(-q_{e-1} I_n + \frac{(1+\delta)\sigma^2}{\delta}\|N^T P_e N\| I_n)\,E$$

Inequality (16) is negative if:

$$\sigma < \left(\frac{\delta q_{e-1}}{(1+\delta)\|N^T P_e N\|}\right)^{1/2} \tag{16}$$

Hence, the error system (4b) can be stabilized for all $\varepsilon \in (0,\ \varepsilon^*_{e-1})$ and $\sigma \in (0,\ \sigma^*_{e-1})$.

- It is also obvious that the original system (1) and its observer (2) are both stable for all $\varepsilon \in (0, \varepsilon^*_p)$ and $\sigma \in (0, \sigma^*_p)$

Remark 1: The values ε^*_0 and σ* are always greater than the values ε^*_P and σ^*_P via Theorem 1, that is, the results are conservative by means of the observer-based controller.

Corollary 1: Suppose A_{11} is stable and choose L_1 such that A_{11}-L_1C_1 is more stable than A_{11} in the discrete sense, then the system (1) and observer (2) are guaranteed to be robustly stable under the observer-based controller (3), where the upper bounds ε^*_p and σ^*_p can be obtained as follows:

$$\varepsilon^*_p = \varepsilon^*_{oc-1} \tag{17}$$

$$\sigma^*_p = \sigma^*_{oc-1} \tag{18}$$

Proof: It follows from what has been said that $\varepsilon^*_{oc-1} < \varepsilon^*_{e-1}$, $q_{oc-1} < q_{e-1}$, $\sigma^*_{oc-1} < \sigma^*_{e-1}$ for A_{11}- L_1C_1 is more stable than A_{11} in the discrete sense. Therefore, reasonably conclude that the system (1) and observer (2) are guaranteed to be robustly stable for all $\varepsilon \in (0, \varepsilon^*_{oc-1})$ and $\sigma \in (0, \sigma^*_{oc-1})$. The proof is completed.

Remark 2: The controller constant gain σ depends on the observer gain matrix L in Theorem 1. For bilinear system, it would be possible to argue that the results do not possess the separation property. While, in Corollary 1, the value L cannot affect the value σ directly once A_{11}-L_1C_1 is more stable than A_{11}. Fundamentally, the property is called the quasi-separation property for Corollary 1.

Furthermore, another type of full state observer and observer-based controller will be introduced for the given system (1) to be stable. Now, a new observer as follows:

$$\hat{x}_1(k+1) = A_{11}\hat{x}_1 + A_{12}\hat{x}_2 + N_{11}\hat{x}_1 u + N_{12}\hat{x}_2 u + B_1 u + L_1(y - C_1\hat{x}_1 - C_2\hat{x}_2) \tag{19a}$$

$$\hat{x}_2(k+1) = \varepsilon(A_{21}\hat{x}_1 + A_{22}\hat{x}_2 + N_{21}\hat{x}_1 u + N_{22}\hat{x}_2 u + B_2 u) \tag{19b}$$

where, $\hat{x}_1 \in R^{n_1}$, $\hat{x}_2 \in R^{n_2}$ are the observer states and $L_1 \in R^{n1\times q}$ is a suitable observing gain matrix that will be designed so as to ensure the estimated state $\hat{x}_1 \to x_1$ and $\hat{x}_2 \to x_2$ as . Here, an observer-based control of the form can be designed as follows:

$$u\,(k) = -\frac{\sigma D_1 \hat{x}_1}{\sqrt{1+\hat{x}_1^T D_1^T D_1 \hat{x}_1}} = -\sigma \sin\hat{\theta}$$
$$= -\sigma \cos\hat{\theta} D_1 \hat{x}_1 \tag{20}$$

to stabilize the system (1), where $D_1 \in R^{1\times n1}$ can be arbitrarily designed, σ is constant:

$$\sin\hat{\theta} = \frac{\hat{x}_1^T D_1^T}{\sqrt{1+\hat{x}_1^T D_1^T D_1 \hat{x}_1}},\quad \cos\hat{\theta} = \frac{1}{\sqrt{1+\hat{x}_1^T D_1^T D_1 \hat{x}_1}} \text{ and } -\tfrac{\pi}{2} \le \hat{\theta} \le \tfrac{\pi}{2}$$

Hence, the state and error equations which can be written as follows:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \tilde{A}_{11}(\hat{\theta}) \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \tilde{A}_{12}(\hat{\theta}) \begin{bmatrix} e_1(k) \\ e_2(k) \end{bmatrix} \quad (21a)$$

$$\begin{bmatrix} e_1(k+1) \\ e_2(k+1) \end{bmatrix} = \tilde{A}_{22}(\hat{\theta}) \begin{bmatrix} e_1(k) \\ e_2(k) \end{bmatrix} \quad (21b)$$

where,

$$\tilde{A}_{11}(\hat{\theta}) = \begin{bmatrix} A_{11} - \sigma B_1 D_1 \cos\hat{\theta} - \sigma N_{11} \sin\hat{\theta} & A_{12} - \sigma N_{12} \sin\hat{\theta} \\ \varepsilon(A_{21} - \sigma B_2 D_1 \cos\hat{\theta} - \sigma N_{21} \sin\hat{\theta}) & \varepsilon(A_{22} - \sigma N_{22} \sin\hat{\theta}) \end{bmatrix}$$

$$= \begin{bmatrix} \bar{A}_{11}(\hat{\theta}) & \bar{A}_{12}(\hat{\theta}) \\ \varepsilon \bar{A}_{21}(\hat{\theta}) & \varepsilon \bar{A}_{22}(\hat{\theta}) \end{bmatrix} = (M_0(\hat{\theta}) + \varepsilon M_1(\hat{\theta}))$$

$$\tilde{A}_{12}(\hat{\theta}) = \begin{bmatrix} \sigma B_1 D_1 \cos\hat{\theta} & 0 \\ \varepsilon\sigma B_2 D_1 \cos\hat{\theta} & 0 \end{bmatrix}, \tilde{A}_{22}(\hat{\theta}) = \begin{bmatrix} \hat{A}_{11} - \sigma N_{11} \sin\hat{\theta} & \hat{A}_{12} - \sigma N_{12} \sin\hat{\theta} \\ \varepsilon(A_{21} - \sigma N_{21} \sin\hat{\theta}) & \varepsilon(A_{22} - \sigma N_{22} \sin\hat{\theta}) \end{bmatrix}$$

$$= \begin{bmatrix} \bar{A}_{e11}(\hat{\theta}) & \bar{A}_{e12}(\hat{\theta}) \\ \varepsilon \bar{A}_{e21}(\hat{\theta}) & \varepsilon \bar{A}_{e22}(\hat{\theta}) \end{bmatrix} = M_{e0}(\hat{\theta}) + \varepsilon M_{e1}(\hat{\theta})$$

in which $\hat{A}_{11} = A_{11} - L_1C_1$ and $\hat{A}_{12} = A_{12} - L_1C_1$.

Before deriving the main result, a preliminary lemma is presented.

For the nominal discrete system:

$$X(k+1) = AX(k) \quad (22)$$

where, $A \in R^{n \times n}$, the critical stability criterion is described by the following lemma.

Lemma 2: (Jury and Gutman, 1975): The critical stability criterion of discrete system (22) is given by the following three conditions:

$$\det(I_n - A) > 0 \quad (23a)$$

$$\det(I_n + A) > 0 \quad (23b)$$

$$(-1)^p \det(A \bullet A - I_p) > 0 \quad (23c)$$

where,

$$p = \frac{n(n-1)}{2}$$

For convenience, the following notations are introduced:

$$\bar{M}_{c\sigma 0} = A_{11} \bullet A_{11} - I_q, \bar{M}_{c0}(\hat{\theta}) = M_{c0}(\hat{\theta}) \bullet M_{c0}(\hat{\theta}) - I_p,$$

$$\bar{M}_{c\sigma 1}(\hat{\theta}) = A_{11} \bullet (-B_1 D_1 \cos\hat{\theta} - N_{11} \sin\hat{\theta}) + (-B_1 D_1 \cos\hat{\theta} - N_{11} \sin\hat{\theta}) \bullet A_{11},$$

$$\bar{M}_{c\sigma 2}(\hat{\theta}) = (-B_1 D_1 \cos\hat{\theta} - N_{11} \sin\hat{\theta}) \bullet (-B_1 D_1 \cos\hat{\theta} - N_{11} \sin\hat{\theta}),$$

$$\bar{M}_{c1}(\hat{\theta}) = M_{c0}(\hat{\theta}) \bullet M_{c1}(\hat{\theta}) + M_{c1}(\hat{\theta}) \bullet M_{c0}(\hat{\theta}), \bar{M}_{c2}(\hat{\theta}) = M_{c1}(\hat{\theta}) \bullet M_{c1}(\hat{\theta}),$$

$$\bar{A}_{c\sigma 3}(\hat{\theta}) = \begin{bmatrix} \bar{M}_{c\sigma 1}(\hat{\theta}) \bar{M}_{c\sigma 0}^{-1} & \bar{M}_{c\sigma 2}(\hat{\theta}) \bar{M}_{c\sigma 0}^{-1} \\ -I_q & 0 \end{bmatrix}, \bar{A}_{c3}(\hat{\theta}) = \begin{bmatrix} \bar{M}_{c1}(\hat{\theta}) \bar{M}_{c0}^{-1}(\hat{\theta}) & \bar{M}_{c2}(\hat{\theta}) \bar{M}_{c0}^{-1}(\hat{\theta}) \\ -I_p & 0 \end{bmatrix},$$

$$\bar{M}_{e\sigma 0} = \hat{A}_{11} \bullet \hat{A}_{11} - I_q, \bar{M}_{e\sigma 1}(\hat{\theta}) = \hat{A}_{11} \bullet (-N_{11} \sin\hat{\theta}) + (-N_{11} \sin\hat{\theta}) \bullet \hat{A}_{11},$$

$$\bar{M}_{e\sigma 2}(\hat{\theta}) = (-N_{11} \sin\hat{\theta}) \bullet (-N_{11} \sin\hat{\theta}), \bar{M}_{e0}(\hat{\theta}) = M_{e0}(\hat{\theta}) \bullet M_{e0}(\hat{\theta}) - I_p,$$

$$\bar{M}_{e1}(\hat{\theta}) = M_{e0}(\hat{\theta}) \bullet M_{e1}(\hat{\theta}) + M_{e1}(\hat{\theta}) \bullet M_{e0}(\hat{\theta}), \bar{M}_{e2}(\hat{\theta}) = M_{e1}(\hat{\theta}) \bullet M_{e1}(\hat{\theta}),$$

$$\bar{A}_{e\sigma 3}(\hat{\theta}) = \begin{bmatrix} \bar{M}_{e\sigma 1}(\hat{\theta}) \bar{M}_{e\sigma 0}^{-1} & \bar{M}_{e\sigma 2}(\hat{\theta}) \bar{M}_{e\sigma 0}^{-1} \\ -I_q & 0 \end{bmatrix}, \bar{A}_{e3}(\hat{\theta}) = \begin{bmatrix} \bar{M}_{e1}(\hat{\theta}) \bar{M}_{e0}^{-1}(\hat{\theta}) & \bar{M}_{e2}(\hat{\theta}) \bar{M}_{e0}^{-1}(\hat{\theta}) \\ -I_p & 0 \end{bmatrix}$$

$$\text{and } q = \frac{n_1(n_1 - 1)}{2}$$

By using the three conditions of stability criterion (23) in order to test the stability of the system (21), it is needed that $\bar{A}_{11}$ and $\bar{A}_{e11}$ are both stable, hence, the value σ must be designed such that $\bar{A}_{11}$ and $\bar{A}_{e11}$ are stable. Hence, the exact σ-bound can be also obtained by means of the three conditions of stability criterion (23) for the stability of $\bar{A}_{11}$ and $\bar{A}_{e11}$, the result is written as the following corollary.

Corollary 2: $\bar{A}_{11}(\hat{\theta})$ and $\bar{A}_{e11}(\hat{\theta})$ are both asymptotically stable matrices for all $\sigma \in (0, \sigma_{max})$, if σ_{max} is determined by the following algorithm:

- Calculate to satisfy stability criteria by:

$$\sigma_{oc-max} = \underset{i=1,2,3}{Min}(\sigma_{oci}) \quad (24)$$

Where:

$$\begin{cases} \sigma_{oci} = -\dfrac{1}{\eta_{oci}} & \text{for } \eta_{oci} < 0 \\ \sigma_{oci} = \infty & \text{otherwise} \end{cases} \quad (25)$$

in which

$$\eta_{oci} = \underset{\hat{\theta} \in [-\frac{\pi}{2}, \frac{\pi}{2}]}{Min} \{\lambda_{min}(\bar{A}_{c\sigma i}(\hat{\theta}))\}, \bar{A}_{c\sigma 1}(\hat{\theta}) = (I_{n_1} - A_{11})^{-1}(B_1 D_1 \cos\hat{\theta} + N_{11} \sin\hat{\theta}) \text{ and } \hat{\theta} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\bar{A}_{c\sigma 2}(\hat{\theta}) = (I_{n_1} - A_{11})^{-1}(B_1 D_1 \cos\hat{\theta} + N_{11} \sin\hat{\theta})$$

- Calculate to satisfy stability criteria by:

$$\sigma_{e-max} = \underset{i=1,2,3}{Min}(\sigma_{ei}) \quad (26)$$

where,

$$\begin{cases} \sigma_{ei} = -\dfrac{1}{\eta_{ei}} & \text{for } \eta_{ei} < 0 \\ \sigma_{ei} = \infty & \text{otherwise} \end{cases} \quad (27)$$

in which:

$$\eta_{ei} = \underset{\hat\theta \in[-\frac{\pi}{2},\frac{\pi}{2}]}{\mathrm{Min}}\{\lambda_{\min}(\bar A_{e\sigma i}(\hat\theta))\},\ \bar A_{e\sigma 1}(\hat\theta) = (I_{n_1} - \hat A_{11})^{-1}N_{11}\sin\hat\theta \text{ and}$$

$$\bar A_{e\sigma 2}(\hat\theta) = -(I_{n_1} + \hat A_{11})^{-1}N_{11}\sin\hat\theta$$

- Find:

$$\sigma_{max} = \mathrm{Min}\ (\sigma_{oc-max},\ \sigma_{e-max})$$

Theorem 2: Consider the singularly perturbed discrete bilinear system (1) and the observer (19), the observer-based controller (20) stabilizes the systems (1) and (19) for all $\varepsilon \in (0, \varepsilon_{max})$, if ε_{max} is determined by the following algorithms:

- Calculate ε^*_{oc-2} to satisfy stability criteria by:

$$\varepsilon^*_{oc-2} = \underset{i=1,2,3}{\mathrm{Min}}(\varepsilon_{oci}) \quad (29)$$

where,

$$\begin{cases}\varepsilon_{oci} = -\dfrac{1}{\gamma_{oci}} & \text{for } \gamma_{oci} < 0\\ \varepsilon_{oci} = \infty & \text{otherwise}\end{cases} \quad (30)$$

in which:

$$\gamma_{oci} = \underset{\hat\theta \in[-\frac{\pi}{2},\frac{\pi}{2}]}{\mathrm{Min}}\{\lambda_{\min}(\bar A_{ci}(\hat\theta))\},\ \bar A_{ci}(\hat\theta) = -[\bar A_{22}(\hat\theta) + \bar A_{21}(\hat\theta)(I_{n_1} - \bar A_{11}(\hat\theta))^{-1}\bar A_{12}(\hat\theta)]\ \hat\theta \in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$

$$\text{and } \bar A_{c2}(\hat\theta) = \bar A_{22}(\hat\theta) - \bar A_{21}(\hat\theta)(I_{n_1} + \bar A_{11}(\hat\theta))^{-1}\bar A_{12}(\hat\theta)$$

- Calculate ε^*_{e-2} to satisfy stability criteria by:

$$\varepsilon^*_{e-2} = \underset{i=1,2,3}{\mathrm{Min}}(\varepsilon_{ei}) \quad (31)$$

Where:

$$\begin{cases}\varepsilon_{ei} = -\dfrac{1}{\gamma_{oci}} & \text{for } \gamma_{ei} < 0\\ \varepsilon_{ei} = \infty & \text{otherwise}\end{cases} \quad (32)$$

in which:

$$\gamma_{ei} = \underset{\hat\theta \in[-\frac{\pi}{2},\frac{\pi}{2}]}{\mathrm{Min}}\{\lambda_{\min}(\bar A_{ei}(\hat\theta))\},\ \bar A_{ei}(\hat\theta) = -[\bar A_{e22}(\hat\theta) + \bar A_{e21}(\hat\theta)(I_{n_1} - \bar A_{e11}(\hat\theta))^{-1}\bar A_{e12}(\hat\theta)\ \hat\theta \in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$

$$\text{and } \bar A_{e2}(\hat\theta) = \bar A_{e22}(\hat\theta) - \bar A_{e21}(\hat\theta)(I_{n_1} + \bar A_{e11}(\hat\theta))^{-1}\bar A_{e12}(\hat\theta)$$

- Find:

$$\varepsilon^*_{oc-e-2} = \mathrm{Min}\ (\varepsilon^*_{oc-2},\ \varepsilon^*_{e-2}) \quad (33)$$

- Find:

$$\varepsilon_{max} = \underset{\sigma\in(0,\ \sigma_{max})}{\mathrm{Max}}\ (\varepsilon^*_{oc-e-2}) \quad (34)$$

Proof: The proof can be also divided into two parts, the stability of the system's state equation (21a) for all $\varepsilon \in (0,\ \varepsilon^*_{oc-2})$ and the stability of the error equation (21b) for all $\varepsilon \in (0,\ \varepsilon^*_{e-2})$.

- The stability of the system's state equation (21a) for all $\varepsilon \in (0,\ \varepsilon^*_{oc-2})$.

In fact, this result is the same as Theorem 3 of Chiou *et al*. (2000). The proof of this part is benign neglect.

- The stability of the error equation (21b) for all $\varepsilon \in (0,\ \varepsilon^*_{e-2})$.

From condition (a) of Lemma 2 and (21.a):

$$\det[I_n - \tilde A_{22}(\hat\theta)] = \begin{bmatrix} I_{n_1} - \bar A_{e11}(\hat\theta) & -\bar A_{e12}(\hat\theta)\\ -\varepsilon\bar A_{e21}(\hat\theta) & I_{n_2} - \varepsilon\bar A_{e22}(\hat\theta)\end{bmatrix}, = \det(I_{n_1} - \bar A_{e11})\det(I_{n_2} + \varepsilon\bar A_{e2}) > 0$$

According to Corollary 2, $\bar A_{e11}$ is an asymptotically stable matrix for σ satisfies (28), hence the above inequality can be satisfied if $\epsilon < \epsilon_{e1}$.

From condition (b) of Lemma 2 and (21.a):

$$\det[I_n + \tilde A_{22}(\hat\theta)] = \begin{bmatrix} I_{n_1} + \bar A_{e11}(\hat\theta) & \bar A_{e12}(\hat\theta)\\ \varepsilon\bar A_{e21}(\hat\theta) & I_{n_2} + \varepsilon\bar A_{e22}(\hat\theta)\end{bmatrix}, = \det(I_{n_1} + \bar A_{e11})\det(I_{n_2} + \varepsilon\bar A_{e2}) > 0$$

Since , then the above inequality holds if $\epsilon < \epsilon_{e2}$.

The third condition of Lemma 2 can be operated as follows:

$$\begin{aligned}(-1)^p\det(\tilde A_{22}\bullet\tilde A_{22} - I_p) &= (-1)^p\det[(M_{e0} + \varepsilon M_{e1})\bullet(M_{e0} + \varepsilon M_{e1}) - I_p]\\ &= (-1)^p\det[(M_{e0}\bullet M_{e0} - I_p) + \varepsilon(M_{e0}\bullet M_{e1} + M_{e1}\bullet M_{e0}) + \varepsilon^2 M_{e1}\bullet M_{e1}]\\ &= (-1)^p\det(\bar M_{e0})\det(I_p + \varepsilon\bar M_{e1}\bar M_{e0}^{-1} + \varepsilon^2\bar M_{e2}\bar M_{e0}^{-1}) > 0\end{aligned}$$

Since, $\bar A_{e11}$ is assumed to be stable, i.e., $(-1)^p\det(\bar M_{e0}) > 0$ then the critical stability for the above inequality is:

$$\det\ (I_p + \bar M_{e1}\bar M_{e0}^{-1} + \bar M_{e2}\bar M_{e0}^{-1}) = 0$$

$$\text{for } \epsilon < \epsilon_{e3} \quad (35)$$

or, equivalently:

$$\det\left(I_{2p} + \varepsilon_{e3}\bar{A}_{e3}\right) = 0$$

Hence, the upper bound ϵ_{e3}, to guarantee the condition (c) of Lemma 2, can be given by $\epsilon < \epsilon_{e3}$.

It is also obvious that the original system (1) and its observer (19) are both stable for all $\epsilon \in (0,\ \varepsilon^*_{oc-e-2})$ by each σ which $\sigma \in (0, \sigma_{max})$. It follows from what has been proved that the upper bound can be represented by (34) for all $\sigma \in (0, \sigma_{max})$. The proof is completed.

Remark 3: From the observer-based controller (20), one can see that different ε^*_{oc-e-2} can be determined by using different gain σ ($< \sigma_{max}$) in Theorem 2 for a fixed matrix D_1. The ε_{max} is the largest ε-bound for the system (1) and observer (19) under the observer-based controller (20) for a fixed matrix D_1.

AN EXAMPLE

Consider the system (1) as described below:

$$X(k+1) = \begin{bmatrix} x_1^{(1)}(k+1) \\ x_1^{(2)}(k+1) \\ x_1^{(3)}(k+1) \\ x_2^{(1)}(k+1) \\ x_2^{(2)}(k+1) \\ x_2^{(3)}(k+1) \end{bmatrix} = \begin{bmatrix} 0.6 & 0.4 & 0 & 0.4 & 0.3 & 0.1 \\ 0 & 0.3 & 0.1 & 0.2 & 0.5 & 0.3 \\ 0 & 0 & -0.5 & 0 & 0.1 & 0.3 \\ 0.1\varepsilon & 0 & -0.3\varepsilon & 0.1\varepsilon & 0 & 0.3\varepsilon \\ 0 & -0.5\varepsilon & 0.2\varepsilon & -0.2\varepsilon & 0.4\varepsilon & 0.3\varepsilon \\ 0.1\varepsilon & 0.2\varepsilon & -0.2\varepsilon & 0.2\varepsilon & 0.5\varepsilon & -0.2\varepsilon \end{bmatrix} \begin{bmatrix} x_1^{(1)}(k) \\ x_1^{(2)}(k) \\ x_1^{(3)}(k) \\ x_2^{(1)}(k) \\ x_2^{(2)}(k) \\ x_2^{(3)}(k) \end{bmatrix} + \begin{bmatrix} 0.1 \\ -1 \\ 0.1 \\ -0.2\varepsilon \\ 0 \\ 1\varepsilon \end{bmatrix} u(k)$$

$$+ \begin{bmatrix} 0.3 & 0 & -0.1 & 0.2 & -0.8 & 0 \\ 0.5 & 0.4 & 0 & 0 & 0.6 & 0.3 \\ 0.2 & 0.3 & 0 & 0.2 & -0.5 & -0.1 \\ 0 & 0.7\varepsilon & 0.1\varepsilon & 0.6\varepsilon & 0.2\varepsilon & 0 \\ -0.8\varepsilon & 0.1\varepsilon & 0 & 0 & 0.3\varepsilon & -0.2\varepsilon \\ 0.2\varepsilon & 0.3\varepsilon & 0.3\varepsilon & 0.3\varepsilon & 0.5\varepsilon & 0.6\varepsilon \end{bmatrix} \begin{bmatrix} x_1^{(1)}(k) \\ x_1^{(2)}(k) \\ x_1^{(3)}(k) \\ x_2^{(1)}(k) \\ x_2^{(2)}(k) \\ x_2^{(3)}(k) \end{bmatrix} u(k) \quad (36)$$

$$y(k) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} X(k)$$

where:

$$x_1 = \left[x_1^{(1)}\ x_1^{(2)}\ x_1^{(3)}\right]^T \text{ and } x_2 = \left[x_2^{(1)}\ x_2^{(2)}\ x_2^{(3)}\right]^T.$$

From Corollary 1, choosing L = [0.163 0.0519 -0.4148 0 0 0]^T implies to place all the eigenvalues of $\hat{A}_0$ to be 0.1, 0.2, 0.3, 0, 0 and 0. It is easy to choose δ = 0.5 for solving Lyapunov equation (6) and ε^*_{oc-1} =0.1758 can be obtained from (17). The observer-based controller to stabilize the state system and the observer is then given by:

$$u(k) = \frac{-\sigma[\hat{x}_1^{(1)}(k) + \hat{x}_1^{(2)}(k) + \hat{x}_1^{(3)}(k)]}{\sqrt{1 + [\hat{x}_1^{(1)}(k) + \hat{x}_1^{(2)}(k) + \hat{x}_1^{(3)}(k)]^2}},\ \sigma < \sigma^*_{oc-1} = 0.0963 \quad (37)$$

with ϵ = 0.1, D = [1 1 1 0 0 0] and q_{nc-1} = 0.6764.

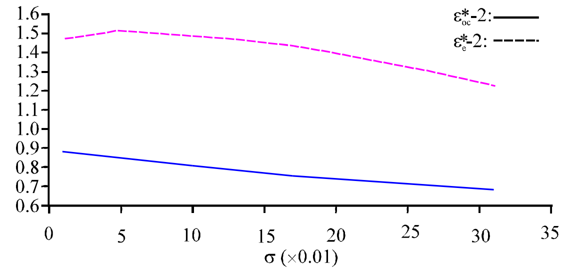

Fig. 1: The relation between ε^*_{oc-2}, ε^*_{e-2} (Y axis) and σ (X axis)

From Corollary 2, let D_1 = [1 1 1] and choose L_1 = [0.163 0.0514-0.4148]^T, then σ_{max} = 0.3199 and σ_{max} = 1.7278 can be calculated from Eq. 24 and 26, respectively. Hence, σ_{max} = 0.3199. Furthermore, from Theorem 2, ε_{max} = 0.875 can be obtained. Finally, the closed-loop system can be stabilized for all $\varepsilon \in (0,\ 0.875)$ if the observer-based controller is given by:

$$u(k) = \frac{-0.01[\hat{x}_1^{(1)}(k) + \hat{x}_1^{(2)}(k) + \hat{x}_1^{(3)}(k)]}{\sqrt{1 + [\hat{x}_1^{(1)}(k) + \hat{x}_1^{(2)}(k) + \hat{x}_1^{(3)}(k)]^2}} \quad (38)$$

and the relation between ε^*_{oc-2} , ε^*_{e-2} and σ are shown in Fig. 1.

CONCLUSION

The ε-dependent and ε-independent observer and observer-based controllers for singularly perturbed discrete systems have been presented. It is illustrated in the simulation results that the ϵ-bounds of the observer-based controller are more conservative than those of the direct state feedback controller.

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