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Adaptive Control Synchronization of Moving Agent Networks

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Abstract: This study investigates the synchronization of a moving agent network via adaptive control and propose a energy consumption moving agent network model. The new moving agent network model exhibits a time-varying topology in which each agent is abstracted as a random walker moving in a planar space and interactions between the agents are established by emitting and receiving waves. Under the condition of fast switching, the network synchronization can be achieved by acting adaptive feedback control on each node. Theoretical results show that the proposed controller can achieve quickly synchronization. And it can be extended to general nonlinear coupling in the future.

Key words: Complex network, synchronization, moving agent, adaptive

INTRODUCTION

Over the past decade years, the analysis of complex systems from the viewpoint of networks has become an important interdisciplinary issue (Albert and Barabasi, 2002). Complex networks have been intensively studied in many fields, such as social, biological, mathematical and engineering sciences. Generally, a complex network is made up of interconnected nodes in which a node is a basic unit with detailed contents. These interactions between nodes determine many basic properties of a network. To well understand the complex dynamical behaviors of many natural systems, we need to study their operating mechanism, dynamic behavior, synchronization, anti-jamming ability and so on.

Recently years, synchronization of complex dynamical networks has been a hot research topic. Synchronization of complex dynamical networks has received a great deal of attentions from various fields of science and engineering (Pandit and Amritkar, 1999; Strogatz, 2001; Albert and Barabasi, 2002). It is because network synchronization can not only explain many natural phenomena but also has many applications, such as secure communication, synchronous information exchange in the internet and the synchronous transfer of digital signals in communication network (Almaas *et al.*, 2002; Barahona and Pecora, 2002; Wang and Chen, 2003). Synchronization of complex networks, particularly, large-scale networks of coupled chaotic oscillators has been extensively investigated in the fields of science and engineering.

The properties of a complex network are mainly determined by its topological structures connections between nodes. In the current study of complex networks, most of the existing works on synchronization consider a static networks, that is, the topological structures of which do not change as time evolves (Barahona and Pecora, 2002; Hong *et al.*, 2002; Wang and Chen, 2002; Boccaletti *et al.*, 2006). The MSF approach allows us to determine the stability of a linearly coupled dynamical network with a constant coupling matrix (or Laplacian) (Pecora and Carroll, 1998; Barahona and Pecora, 2002; Nishikawa and Motter, 2006; Zhou and Kurths, 2006). However, numerous real-world networks such as biological, communication, social and epidemiological networks generally evolve with time-varying topological structures. Henceforth, researchers have devoted more and more efforts to complex networks with time varying topologies. Lu *et al.* (2009) investigated the local synchronization in networks with time-varying coupling strengths (Lu and Chen, 2005). Skufca and Bollt (2004) found that although there exists an instantaneously disconnected topology, a time-varying network could propagate sufficient information to make the complex networks achieve synchronization. Stilwell *et al.* (2006) found that if the network locally synchronizes for the fixed time-average of the topology and the time-average is achieved sufficiently fast, the network locally synchronizes for switching topologies. Furthermore, at the same conditions, Lu *et al.* (2009) found that the directed network with switching topology can reach global synchronization for

sufficiently large coupling strength if there exists a spanning directed tree in the network.

One interesting topic about time varying networks is synchronization in mobile agent network, where the network usually consists of a group of interactive moving agents (Frasca *et al.*, 2008). Due to its remarkable feature of switching topology, such an agent network model has been widely used to explore various practical problems, e.g., clock synchronization in mobile robots (Buscarino *et al.*, 2006), synchronized bulk oscillations (Buscarino *et al.*, 2006) and task coordination of swarming animals (Peng *et al.*, 2009).

In this study, we consider the case of mobile agents, each one associated with a oscillator coupled with those of the neighboring agents. we adopt the constraint of fast switching (Belykh *et al.*, 2004; Stilwell *et al.*, 2006) to derive synchronization conditions which relate synchronization to a scaled all-to-all Laplacian matrix. We study synchronization problems of moving agent dynamical networks with. By using Lyapunov stability theory, adaptive synchronization controllers are designed. Compared with some similar results, the adaptive controllers can ensure that the states of dynamical network is synchronization. The proposed scheme has faster convergence rate.

A MOVING AGENT NETWORK MODEL

Consider a dynamical network consisting of N moving agents distributed in a two-dimensional space of size L with periodic boundary conditions. Each agent moves with velocity $v_i(t)$ and direction of motion $\theta_i(t)$. The velocity $v_i(t)$ is constant in module (denoted by v) and is updated in direction through the angle $v_i(t)$ for each time unit. The agents are considered as random walkers whose positions and orientations are updated according to:

$$\begin{aligned} y_i(t+\Delta t_M) &= y_i(t) + v_i(t)\Delta t_M \\ \theta_i(t+\Delta t_M) &= \eta_i(t+\Delta t_M) \end{aligned} \quad (1)$$

where, $y_i(t)$ is the position of agent i in the plane at time t , $\eta_i(t)$ with $i = 1, 2, \dots, N$ are N independent random variables chosen at each time unit with uniform probability in the interval $[0, 2\pi]$ and Δt_M is the time unit.

We consider each agent in network to be a wave source. Also, assume that each agent emits wave to all directions without energy absorption in the propagation medium, then we have:

$$E_{Tx}^i(k, d) = \begin{cases} E_{elec} \times k + E_{fs} \times k \times d^2 \times S^i(d) & d < d_0 \\ 0 & d \geq d_0 \end{cases} \quad (2)$$

where, E_{Tx}^i is the emission power of agent i , E_{elec} is RF energy consumption coefficient, E_{fs} is energy consumption coefficient of the amplifier circuit k is the sent number of bit, d is the distance from agent i and $S^i(d)$ is the wave intensity emitted by agent i at a distance d from agent i . It is obvious that the wave intensity $S^i(d)$ decreases with the increase of d . Taking into account that each agent has a limited sensing capability, we define a critical cut-off distance, denoted by d_0 . Then, for two agents i, j with distance d , agent j can perceive the wave from agent i if and only if $d < d_0$. The cut-off distance indicates that, for agent i with emission power, there exists an interaction radius of neighborhood $R_i = d_0$ and directed edges are then established from agent i to those located within R_i to agent i .

Furthermore, the states of these neighboring agents are changed to include diffusive couplings with the state of agent i . The coupling mechanism mentioned above can serve as a reasonable representation of interactions through sounds, rays, electromagnetic waves, etc., which are widely found both in natural world and in engineering fields.

Under these hypotheses, the state dynamics of agent i can be formulated as:

$$\dot{x}_i = f(x_i) - \sigma \sum_{j=1}^N g_{ij}(t)x_j + u_i \quad (3)$$

where, $i = 1, \dots, N$, $x_i = (x_{i1}, x_{i2}, \dots, x_{im})^T \in \mathbb{R}^n$ are the state variables of the node i ; $f: D \times \mathbb{R}^+ \rightarrow \mathbb{R}^n$ is a smooth nonlinear vector-valued function governs the local dynamics of oscillator, σ is the coupling strength, $g_{ij}(t)$ are the elements of a time-varying Laplacian matrix: $G(t) = [g_{ij}(t)] \in \mathbb{R}^{N \times N}$ which defines the coupling relationship of agents at a given time t , for two agents i, j with distance d , if $d < d_0$, then, $g_{ij}(t) = -1$, $g_{ji}(t) = -1$; if $d > d_0$, then $g_{ij}(t) = 0$, $g_{ji}(t) = 0$; $u_i \in \mathbb{R}^n$ are the control inputs, where s is a synchronous solution of the node system $\dot{x}_i(t) = f(x_i, t)$.

In this study, the control objective is to make the states of network (3) synchronize to a manifold defined in (4) by introducing a simple adaptive controller into each individual node:

$$x_1(t) = x_2(t) = \dots = x_N(t) = s(t) \quad (4)$$

where, $s(t)$ is a solution of an isolated node:

$$\dot{s}(t) = f(s(t), t) \quad (5)$$

We assume that $s(t)$ is an arbitrary desired state which can be an equilibrium point, a periodic orbit, an aperiodic orbit or even a chaotic orbit in the phase space.

Next, the rigorous mathematical definition of synchronization for dynamical network (3) is introduced.

Definition 1: Let $x_i(t, t_0, X_0)$ ($i = 1, 2, \dots, N$) be a solution of dynamical network (3), where $X_0 = (x_1^0, x_2^0, \dots, x_N^0)$ are initial conditions, $f: D \times \mathbb{R}^+ \rightarrow \mathbb{R}^n$ are continuously differentiable with $D \subseteq \mathbb{R}^n$. If there is a nonempty subset $\Lambda \subseteq D$ with $x_i^0 \in \Lambda$, $i = 1, 2, \dots, N$, for all $t \geq t_0$ and:

$$\lim_{t \rightarrow \infty} \|x_i(t, t_0, X_0) - x_j(t, t_0, X_0)\| = 0 \text{ for } i, j = 1, 2, \dots, N$$

(Hereafter, denote $\|\cdot\|$ as the Euclidean norm). Then the dynamical network (3) is said to realize synchronization. And $\Lambda \times \dots \times \Lambda$ is called the region of synchrony for dynamical network (3).

ADAPTIVE CONTROLLER OF MOVING AGENT NETWORKS FOR SYNCHRONIZATION

In this section, we study the synchronization of moving agent network (3) by designing adaptive controllers for each agent. In order to achieve the objective on the manifold (4), let us define the error vector:

$$e_i(t) = x_i(t) - s(t), \quad i = 1, 2, \dots, N \quad (6)$$

Subtracting (5) from (3) yields the error dynamical system:

$$\dot{e}_i(t) = f(x_i(t)) - f(s(t)) - \sigma \sum_{j=1}^N g_{ij}(t) e_j(t) + u_i \quad (7)$$

Then, synchronization problem of the dynamical network (3) is equivalent to the problem of stabilization of the error dynamical system (7).

Linearizing error system (7) about the synchronized states $s(t)$, we can get:

$$\dot{e}_i(t) = A(t) e_i(t) - \sigma \sum_{j=1}^N g_{ij}(t) e_j(t) + u_i \quad (8)$$

where, $i = 1, 2, \dots, N$ and $A(t) = J_f(s, t)$ is the Jacobian matrix of f evaluated at $x = s(t)$.

In the following, we give a useful hypotheses.

Assumption 1 (A1): Suppose there exists a nonnegative constant α , satisfying:

$$\|A(t)\| \leq \alpha$$

Based on (A1), a network synchronization criterion is deduced as follows:

Theorem 1: Suppose that (A1) hold. Then the dynamical moving agent network (3) is synchronized under the following adaptive controllers:

$$u_i = -d_i e_i, \quad i = 1, 2, \dots, N \quad (9a)$$

$$\dot{d}_i = k_i \|e_i\|_2^2 \quad (9b)$$

where, k_i ($1 \leq i \leq N$) are any positive constants.

Proof: Select a Lyapunov function as follows:

$$V = \frac{1}{2} \sum_{i=1}^N e_i^T e_i + \frac{1}{2} \sum_{i=1}^N \frac{(d_i - \hat{d}_i)^2}{k_i} \quad (10)$$

where, constant \hat{d}_i to be given below. Then the time derivative of $V(t)$ along the solution of the error system (8) is given as follows:

$$\begin{aligned} \dot{V} &= \frac{1}{2} \sum_{i=1}^N (e_i^T \dot{e}_i + \dot{e}_i^T e_i) + \sum_{i=1}^N \frac{(d_i - \hat{d}_i) \dot{d}_i}{k_i} \\ &= \frac{1}{2} \sum_{i=1}^N (J_f e_i - \sigma \sum_{j=1}^N g_{ij}(t) e_j - d_i e_i)^T e_i + e_i^T (J_f e_i - \sigma \sum_{j=1}^N g_{ij}(t) e_j - d_i e_i) + \sum_{i=1}^N (d_i - \hat{d}_i) \dot{e}_i^T e_i \\ &= \frac{1}{2} \sum_{i=1}^N (e_i^T J_f^T e_i - \sigma \sum_{j=1}^N g_{ij}(t) e_j^T e_i - d_i e_i^T e_i + e_i^T J_f e_i - \sigma \sum_{j=1}^N g_{ij}(t) e_i^T e_j - d_i e_i^T e_i) + \sum_{i=1}^N (d_i - \hat{d}_i) e_i^T e_i \\ &= \sum_{i=1}^N e_i^T \left(\frac{J_f^T + J_f}{2} \right) e_i - \frac{\sigma}{2} \sum_{i=1}^N \sum_{j=1}^N g_{ij}(t) e_j^T e_i - \frac{\sigma}{2} \sum_{i=1}^N \sum_{j=1}^N g_{ij}(t) e_i^T e_j - \sum_{i=1}^N d_i e_i^T e_i + \sum_{i=1}^N (d_i - \hat{d}_i) e_i^T e_i \\ &= \sum_{i=1}^N e_i^T \left(\frac{J_f^T + J_f}{2} \right) e_i - \sigma \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{ij}(t) e_i^T e_j - \sum_{i=1}^N \hat{d}_i e_i^T e_i \\ &= \sum_{i=1}^N e_i^T \left(\frac{J_f^T + J_f}{2} \right) e_i - \frac{\pi \sigma}{L^2} \sum_{i=1}^N e_i^T e_i (\sum_{j=1}^N R_j^2 - R_i^2) + \frac{\pi \sigma}{L^2} \sum_{i=1}^N \sum_{j=1, j \neq i}^N R_j^2 e_i^T e_j - \sum_{i=1}^N \hat{d}_i e_i^T e_i \\ &\leq \alpha \sum_{i=1}^N e_i^T e_i - \frac{\pi \sigma}{L^2} \sum_{i=1}^N e_i^T e_i (\sum_{j=1}^N R_j^2 - R_i^2) + \frac{\pi \sigma}{2L^2} \sum_{i=1}^N \sum_{j=1, j \neq i}^N R_j^2 (e_i^T e_i + e_j^T e_j) - \sum_{i=1}^N \hat{d}_i e_i^T e_i \\ &= \alpha \sum_{i=1}^N e_i^T e_i - \frac{\pi \sigma}{L^2} \sum_{i=1}^N e_i^T e_i (\sum_{j=1}^N R_j^2 - R_i^2) + \frac{\pi \sigma}{2L^2} \sum_{i=1}^N e_i^T e_i \sum_{j=1, j \neq i}^N R_j^2 + \frac{\pi \sigma}{2L^2} \sum_{i=1}^N \sum_{j=1, j \neq i}^N R_j^2 e_j^T e_i - \sum_{i=1}^N \hat{d}_i e_i^T e_i \end{aligned}$$

Since:

$$\begin{aligned} & - \frac{\pi \sigma}{L^2} \sum_{i=1}^N e_i^T e_i (\sum_{j=1}^N R_j^2 - R_i^2) + \frac{\pi \sigma}{2L^2} \sum_{i=1}^N e_i^T e_i \sum_{j=1, j \neq i}^N R_j^2 \\ &= - \frac{\pi \sigma}{L^2} [\|e_1\|_2^2 (R_2^2 + R_3^2 + \dots + R_N^2) + \dots + \|e_N\|_2^2 (R_1^2 + R_2^2 + \dots + R_{N-1}^2)] \\ & \quad + \frac{\pi \sigma}{2L^2} [\|e_1\|_2^2 (R_2^2 + R_3^2 + \dots + R_N^2) + \dots + \|e_N\|_2^2 (R_1^2 + R_2^2 + \dots + R_{N-1}^2)] \\ &= - \frac{\pi \sigma}{2L^2} [\|e_1\|_2^2 (R_2^2 + R_3^2 + \dots + R_N^2) + \dots + \|e_N\|_2^2 (R_1^2 + R_2^2 + \dots + R_{N-1}^2)] \end{aligned}$$

and:

$$\begin{aligned} & \frac{\pi \sigma}{2L^2} \sum_{i=1}^N \sum_{j=1, j \neq i}^N R_j^2 e_i^T e_j \\ &= \frac{\pi \sigma}{2L^2} [\|e_2\|_2^2 R_1^2 + \|e_3\|_2^2 R_1^2 + \dots + \|e_N\|_2^2 R_1^2 + \dots + \|e_1\|_2^2 R_2^2 + \|e_3\|_2^2 R_2^2 + \dots + \|e_{N-1}\|_2^2 R_2^2] \\ &= \frac{(N-1)\pi \sigma}{2L^2} [\|e_1\|_2^2 R_1^2 + \|e_2\|_2^2 R_2^2 + \dots + \|e_N\|_2^2 R_N^2] \end{aligned}$$

Therefore:

$$\begin{aligned} \dot{V} &\leq \alpha \sum_{i=1}^N e_i^T e_i - \frac{\pi\sigma}{2L^2} \sum_{i=1}^N e_i^T (\sum_{j=1}^N R_j^2 - R_i^2) + \frac{(N-1)\pi\sigma}{2L^2} \sum_{i=1}^N R_i^2 e_i^T e_i - \sum_{i=1}^N \hat{d}_i e_i^T e_i \\ &= \sum_{i=1}^N e_i^T e_i [\alpha + \frac{\pi\sigma}{2L^2} ((N-1)R_i^2 - \sum_{j=1, j \neq i}^N R_j^2) - \hat{d}_i] \\ &= e^T (\text{diag}(\alpha + \frac{\pi\sigma}{2L^2} ((N-1)R_1^2 - R_2^2 - \dots - R_N^2 - \hat{d}_1), \alpha + \frac{\pi\sigma}{2L^2} ((N-1)R_2^2 - R_1^2 - R_3^2 - \dots - R_N^2 - \hat{d}_2), \\ &\quad \dots, \alpha + \frac{\pi\sigma}{2L^2} ((N-1)R_N^2 - R_1^2 - \dots - R_{N-1}^2 - \hat{d}_N)) e \end{aligned}$$

Then, we select suitable constants \hat{d}_i ($1 \leq i \leq N$) to make:

$$\begin{aligned} &\text{diag}(\alpha + \frac{\pi\sigma}{2L^2} ((N-1)R_1^2 - R_2^2 - \dots - R_N^2 - \hat{d}_1), \alpha + \frac{\pi\sigma}{2L^2} ((N-1)R_2^2 - R_1^2 - R_3^2 - \dots - R_N^2 - \hat{d}_2), \\ &\quad \dots, \alpha + \frac{\pi\sigma}{2L^2} ((N-1)R_N^2 - R_1^2 - \dots - R_{N-1}^2 - \hat{d}_N)) \end{aligned}$$

be a negative definite matrix. Thus it follows that the error vector $E = [e_1^T, e_2^T, \dots, e_N^T]^T \rightarrow 0$ as $t \rightarrow \infty$. That is, the synchronous solution $S(t)$ of moving agent network (3) is synchronized under the adaptive controllers (9).

The proof is thus completed.

Remark 1: Because error dynamical system (7) is linearized, so the synchronous solution $S(t)$ of the moving agent system is local synchronization.

Remark 2: Linearizing error system (7) about the states $s(t)$, give $A(t) = Df(s(t), t)$ is the Jacobian of f evaluated at $x = s$. Suppose that there exists a nonnegative constant α satisfying $\|A(t)\| \leq \alpha$. In fact, the Assumption 1 is likely to be satisfied. For example, general chaotic system such as Lorenz system, Rossler and Chen system, there exists a constant α satisfying $\|A(t)\| \leq \alpha$.

Remark 3: In this study, we discussed the coupling scheme of dynamical network (3) is linear relationship. If the network coupling scheme is general nonlinear relationship, the network (3) is rewritten as follows:

$$\dot{x}_i = f(x_i) - \sigma \sum_{j=1}^N g_{ij}(t)h(x_j) + u_i \quad (11)$$

The controller (9) still be able to make the network (11) achieve synchronization by adding a assumption $\|h(x_i)\| < \beta$, where β is a nonnegative constant.

CONCLUSION

The problem of synchronization for moving agent dynamical network is investigated. The complex network with decentralized controllers is considered as a large-scale nonlinear system. An adequate Lyapunov function

is constructed to deal with the problem of controlled synchronization as to ensure the closed loop system stability. One novel network synchronization criteria have been proved by using Lyapunov stability theory. Decentralized adaptive controllers are designed to achieve synchronization for the moving agent networks. Compared with some similar results, the proposed controllers can achieve quickly synchronization.

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