http://ansinet.com/itj

ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 11 (12): 1764-1769, 2012
ISSN 1812-5638 / DOI: 10.3923/itj.2012.1764.1769

A DS-AHP Approach for Multi-attribute Decision Making Problem with Intuitionistic Fuzzy Information

Xiangqian Feng, Xiaohua Qian and Qiang Wu
Department of Management Science and Engineering,
School of Computer Science and Technology,
Nanjing Normal University 210046, China

Abstract: A new method to solve multi-attribute decision making problem with Intuitionistic Fuzzy (IF) information is given here. This method is based on the Dempster Shafer-Analytical Hierarchy Process (DS-AHP) theory. The DS-AHP theory can solve a problem directly based on the decision matrix which is different from most of current methods. But in reality, the decision maker may provide several types of uncertain information such as fuzzy or interval values. So, the original DS-AHP method is combined with the IF information here. The expected utility is used to transform decision matrix with intuitionistic fuzzy information which is obtained from assessment. And then a non-linear optimization model is used to combine all the attributes. Using the combined results, the rank of the alternatives can be finally got. At the end of the paper a numerical example is studied to illustrate the details.

Key words: Dempster Shafer-analytical hierarchy process theory, intuitionistic fuzzy information, multi-attribute decision making problem, non-linear optimization model

INTRODUCTION

Multi-attribute Decision Making (MADM) is concerned with the elucidation of the levels of preference of decision alternatives, through judgments made over a number of criteria. Many complex MADM problems are characterized with both quantitative and qualitative-of preference of decision alternatives, through judgments made over a number of criteria. Many complex MADM problems are characterized with both quantitative and qualitative-attributes. For instance, the design evaluation of an engineering product may require the simultaneous consideration of several attributes such as cost, quality, safety, reliability, maintainability and environmental impact; in selection of its suppliers, an organization needs to take into account such attributes as quality, technical capability, supply chain management, financial soundness, environmental and so on. Most of such attributes are qualitative and could only be properly assessed using human judgments which are subjective in nature and are inevitably associated with uncertainties due to the human being's inability to provide complete judgment, or the lack of information, or the vagueness of the meanings about attributes and their assessments. For decades, many MADM methods have been developed, such as Analytical Hierarchy Process (AHP) (Saaty, 1980), TOPSIS (Lan, 2009), ELECTRE, PROMETHEE, LINMAP (Jiang and Fan, 2005) etc. for certain MADA and fuzzy multi-attribute decision making under uncertainty (Wang *et al.*, 2011). AHP has been widely used in many areas such as accounting (Webber *et al.*, 1997), assessment (Wu *et al.*, 2010), programming (Yang and Kuo, 2003), research and development management (Liu and Tsai, 2007) and information management (Liu and Shih, 2005). AHP can be applied under the precondition that the decision maker can make pairwise comparison between decision alternatives. This prerequisite, however, may not be satisfied in practice. For a practical MADM problem, information about decision alternatives may be incomplete because of time pressure, lack of data, intangible of some attributes (Kim and Ahn, 1997, 1999), limitation of attention, or limitations on information processing capabilities (Kahneman *et al.*, 1982), etc. The Dempster-Shafer (DS) theory of evidence (Dempster, 1967; Shafer, 1976) models have both quantitative and qualitative attributes with an appropriate framework. The power of the DS theory in handling uncertainties has found wide applications in many areas such as expert systems (Beynon *et al.*, 2001), diagnosis and reasoning (Jones *et al.*, 2002), pattern classification (Denoeux and Zouhal, 2001), information fusion (Telmoudi and Chakhar, 2004), sort (Xu, 2012). In recent years, there have been several attempts to use the DS theory of evidence for MADA (Yang *et al.*, 2006; Liu *et al.*, 2011; Zhang *et al.*,

Corresponding Author: Xiangqian Feng, Department of Management Science and Engineering, School of Computer Science and Technology, Nanjing Normal University 210046, China

2012). In many cases, the DS theory has been used as an alternative approach to Bayes decision theory (Beynon *et al*., 2000; Yager *et al*., 1994) incorporated the DS theory with the AHP process. The method can not only model both quantitative and qualitative attributes but also take advantage of the AHP to lower the number of alternatives that fit the limited number of opinions given so far, with only a few opinions stated.

In some uncertain decision problems with qualitative attributes, however, it may be difficult to define assessment grades as independent crisp sets. It would be more natural to define assessment grades using subjective and vague linguistic terms which may overlap in their meanings. While intuitionistic fuzzy set has been proven to be highly useful in dealing with uncertainty and vagueness, accordingly, intuitionistic fuzzy set is a very suitable tool to be used to describe the imprecise uncertain decision information (Zhanga *et al*., 2012; Amer *et al*., 2010). Nevertheless, the current DS-AHP approach does not take into account vagueness or fuzzy uncertainty. As such, there is a clear need to combine the DS-AHP theory for handling both types of uncertainties. The purpose of this paper is to investigate how to incorporate the intuitionistic fuzzy information with the DS-AHP method.

PRELIMINARY

The existing DS-AHP approach: On the base of Dempster-Shafer theory of evidence, the method first identifies all possible focal elements from the decision matrix and then it calculates the basic probability assignment of each focal element and the belief interval of each decision alternative. The AHP approach is used to describe the MADM problem. Using AHP approach, the MADM is decomposed into three levels. The first level is the MADM problem with incomplete information, the second level is the decision attributes of the MADM problem and the third level is focal elements identified from the decision matrix. Next, we will describe the main steps of the DS-AHP approach (Gong, 2007; Hua *et al*., 2008).

First, we identify focal elements from an incomplete decision matrix. Let $\Theta = \{a_1,\cdots, a_N\}$ be a collectively exhaustive and mutually exclusive set of decision alternatives, called the frame of discernment.

Given a decision matrix $V = |f(a_i, C_i)|$ where, $f(a_i, C_i)$ is the evaluation of the decision alternative a_i ($i = 1, 2,\cdots, N$) under the jth attribute C_j ($j = 1, 2,\cdots, M$). The decision matrix V implies the body of evidence of the MADM and a focal element can be defined from the decision matrix as follows:

For $\forall a_i, a_k \in \Theta$, and $a_i \neq a_k$, if $(a_i, C_j) = f(ak, C_j)$

then a_i and a_k belong to the same focal element.

According to above definition, we can obtain the focal elements of each attribute. Then, the hierarchical structure of DS-AHP can be constructed. And similar to AHP, the weights of importance of decision attributes ω_j, $j = 1, 2,\cdots, M$ in DS-AHP can be determined through pairwise comparison.

Second, construct belief interval of each decision alternative. Denote by A^j_k ($j = 1, 2,\cdots, M$; $k = 1, 2,\cdots, t$; $t<2^N$) the set that consists of all focal elements under decision attribute C_j and if $a_i \in A^j_k$, we can view $\omega_j f(a_i, C_j)$ as the decision maker's preference on the focal element A^j_k, where, ω_j is the importance weight of decision attribute C_j. Denote by $P(A^i_k)$ the decision maker's preference on the focal element A^j_k, then $p(A^j_k) = \omega_j f(a_i, C_j)$. Because Θ is the frame of discernment which consists of all decision alternatives, we let the $p(\Theta)$. Then we can define the basic probability assignment (BPA) of each focal element as follows:

$$m_j(A^j_k) = \frac{p(A^j_k)}{\sum_k p(A^j_k)}$$

Applying the operator of combination in the DS theory, the BPA of each focal element considering all decision attributes can be obtained. Suppose A^{j1}_k and A^{j2}_k are two focal elements under decision attribute C_{j1} and C_{j2}, respectively, $j_1, j_2 \in \{1, 2,\cdots, M\}$, $j_1 \neq j_2$. Denote the intersection of A^{j1}_k and A^{j2}_k as E, then according to DS rule of combination, the BPA of E is defines as follows:

$$\left[m_{j_1} \oplus m_{j_2}\right](E) = \begin{cases} 0, & E = \Phi, \\ \dfrac{\sum_E m_{j_1}(A^{j_1}_k) m_{j_2}(A^{j_2}_l)}{1 - \sum_{E=F} m_{j_1}(A^{j_1}_k) m_{j_2}(A^{j_2}_l)}, & E \neq \Phi. \end{cases}$$

After obtaining BPA of each focal element considering all decision attributes, we can define its belief measure (Bel) and plausibility measure (Pls).

Denote by $Bel(\{a_i\})$ and $Pls(\{a_i\})$ the exact support to a decision alternative a_i ($i = 1, 2,\cdots, N$) and the possible support to a_i, respectively. The two values can be got as follows:

$$Bel(\{a_i\}) = \sum_{E=\{a_i\}} m(E) \text{ and } Pls(\{a_i\}) = \sum_{a_i \in E} m(E) \quad \forall i \in \{1, 2, \cdots, N\}$$

Using $Bel(\{a_i\})$ and $Pls(\{a_i\})$, we obtain the belief interval $[Bel(\{a_i\}), Pls(\{a_i\})]$ for all decision alternatives of the MADA problem with incomplete information. Then in order to obtain the preference relations among all decision alternatives, we need a mechanism to generate the rank of decision alternatives based in their belief intervals. There're many available methods to rank the alternatives. Here we cite the method described as follows.

We define the degree of preference of a_i over a_k, denoted by $P(a_i>a_k)$ as follows:

$$P(a_i > a_k) = \frac{\max[0, Pls(\{a_i\}) - Bel(\{a_k\})] - \max[0, Bel(\{a_i\}) - Pls(\{a_k\})]}{[Pls(\{a_i\}) - Bel(\{a_i\})] + [Pls(\{a_k\}) - Bel(\{a_k\})]}$$

with $P(a_i>a_k)\in[0, 1]$.

By applying the above definition, we define the preference relation between decision alternative as follows.

- Decision alternative a_i is said to be superior to a_k (denoted by $a_i \succ a_k$) if $P(a_i>a_k)>0.5$
- Decision alternative a_i is said to be inferior to a_k (denoted by $a_i \prec a_k$) if $P(a_i>a_k)<0.5$
- Decision alternative a_i is said to be indifferent to a_k (denoted by $a_i \sim a_k$) if $P(a_i>a_k) = 0.5$

The above formulas consist of the main process of the DS-AHP approach. After these steps, we can obtain the rank of all the decision alternatives.

The intuitionistic fuzzy set (IFS): An intuitionistic fuzzy set A on a universe U is defined as an object of the following form (Hua *et al*., 2008):

$$A = \{(u, \mu_A(u), v_A(u))\ u\in U\}$$

where, the functions μ_A: U→ [0, 1] and v_A: U→[0, 1] define the degree of membership and the degree of non-membership of the element $u\in U$ in A, respectively and for $\forall u\in U$, $0\le \mu_A(u)+v_A(u)\le 1$.

For convenience, we call u = (u, v) an intuitionistic fuzzy number.

THE COMBINATION OF THE DS-AHP METHOD WITH INTUITIONISTIC FUZZY INFORMATION

Just as the existing DS-AHP approach, we first need to obtain a decision matrix.. Here we assume there are n evaluation grades to which all alternatives can be assessed, denoted by $H = \{h_1, h_2,\cdots, h_n\}$ are mutually exclusive and collectively exhaustive. And we also need to define the utility of each grade as $u(h_r)$ with $u(h_{r+1})>u(h_r)$ if it is assumed that the grade h_{r+1} is preferred to h_r. And the assessment value is provided with intuitionistic fuzzy information. Then we have that: if an alternative a_i is assessed on an attribute C_j to a grade h_r with an intuitionistic fuzzy data, we denote this by:

$$S(C_j(a_i)) = \{(h_r, (\mu_{r,j}(a_i), v_{r,j}(a_i))), r = 1, 2,\cdots n\}),$$

where, $\mu_{r,j}(a_i)$ is the degree of membership of an alternative a_i on the attribute C_j associated with the grade h_r and $v_{r,j}(a_i)$ is the degree of non-membership. For convenience, we use $(\mu_{r,j}, v_{r,j})$ to represent the assessment $(\mu_{r,j}(a_i), v_{r,j}(a_i))$ in the following paragraphs. After N alternatives are all assessed on M attributes, we obtain the following decision matrix: $D = (S(C_j(a_i)))$.

First, to every alternative assessed on each attribute, combine the assessment given to the different grades. With the utility of each grade, a comprehensive assessment value of alternative a_i on the attribute C_j can be got as follows:

$$u(a_i, C_j) = \sum_{r=1}^{n} u(h_r)(\mu_{r,j}, v_{r,j})$$

The operational rules of intuitionistic fuzzy set were defined by Atanassov (1986). Obviously, $u(a_i, C_j)$ is still an intuitionistic fuzzy data. As the process of assessment is often accompanied with incomplete information, so we need to revise the results obtained using the expected utilities.

We denote $\bar{\omega}_{ij}$ as the weight assigned to alternative a_i on the attribute C_j, where:

$$\bar{\omega}_{ij} = \frac{\sum_{r=1}^{n} d_r^{ij} u(h_r)}{\sum_{r=1}^{n} u(h_r)}$$

And δ^{ij}_r represents the counts of grades an alternative is assigned to δ^{ij}_r is about the following formula:

$$d_r^{ij} = \begin{cases} 1, & \text{alternative } a_i \text{ is assigned to } h_r \\ 0, & \text{others.} \end{cases}$$

Using the weight above, we can revise $u(a_i, C_j)$. We denote $(\mu_{ij}, v_{ij}) = \bar{\omega}_{ij}\, u(a_i, C_j)$. From now onwards, we can finally obtain the transformed decision matrix $V = (\mu_{i,j}, v_{i,j})\ N\times M$.

Second, identify the focal element of each attribute. On the base of the transformed decision matrix $V = (\mu_{i,j}, v_{i,j})\ N\times M$, we can define the following method. The decision alternatives which have the same assessment value or the same degree of influence on an attribute belong to the same focal element of the attribute. A focal element can be defined as follows:

Definition 1: For $\forall\ a_i, a_k \in \Theta$ and $a_i \neq a_k$, if $\mu_{ij} = \mu_{kj}$ then a_i and a_k belong to the same focal element.

The next step, we will define the interval probability masses of each focal element. Denote by A^j_k ($j = 1, 2, \cdots, M$; $k = 1, 2, \cdots, t$; $t<2^N$)the set that consists of all focal elements under decision attribute C_j. We can obtain the interval probability masses of A^j_k.

Definition 2: Considering the weight of C_j, for $\forall a_i \in \Theta$, $\forall A^j_k \in 2^\Theta$ if $a_i \in A^j_k$, then we define the interval probability masses as:

$$m_j(A^j_k) = [\omega_j\ \mu_{kj}, \omega_j\ (1\text{-min } v_{kj})]$$

We denote it as:

$$m_j(A^j_k) = \lfloor m_j(A^j_k)^-, m_j(A^j_k)^+ \rfloor$$

As to the whole set $\Theta = \{a_1, \cdots, a_N\}$, the followings are used to obtain its interval probability masses:

$$m_j(T) = \left[\max(0, 1-\sum_{k=1}^{t} m_j(A^j_k)^+), 1-\sum_{k=1}^{t} m_j(A^j_k)^-\right]$$

where, $t<2^N$.

Third, to obtain the interval probability masses of each focal element considering all decision attributes, we need to define the combination rule.

Definition 3: Let m_{j1} (A^{j2}_k), m_{j2} (A^{j1}_k) be two interval belief structure with interval probability masses m_{j1} $(A^{j1}_k)^- \leq m_{j1}$ $(A^{j1}_k) \leq m_{j1}$ $(A^{j1}_k)^+$ $k = 1, 2, \cdots,$ and m_{j2} $(A^{j2}_k)^- \leq m_{j2}$ $(A^{j2}_k) \leq m_{j2}$ $(A^{j2}_k)^+$, $l = 1, 2, \cdots, t$.

We define E as the intersection of two focal elements A^{j1}_k and A^{j2}_k. Their combination, denoted by $m_{j1} \oplus m_{j2}$ is also an interval belief structure defined by:

$$[m_{j_1} \oplus m_{j_2}](E) = \begin{cases} 0, & E = \Phi, \\ \left[(m_{j_1} \oplus m_{j_2})^-(E), (m_{j_1} \oplus m_{j_2})^+(E)\right], & E \neq \Phi, \end{cases}$$

where, $(m_{ji} \oplus m_{j2})^-$ (E) and $(m_{ji} \oplus m_{j2})^+$ (E) are, respectively the minimum and the maximum of the following optimization model:

$$\text{ax/Min } [m_{j_1} \oplus m_{j_2}](E) = \frac{\sum_{E} m_{j_1}(A^{j_1}_k) m_{j_2}(A^{j_2}_l)}{1 - \sum_{E=F} m_{j_1}(A^{j_1}_k) m_{j_2}(A^{j_2}_l)}$$

$$\begin{aligned} \text{s.t. } & \sum_{k}^{t} m_{j_1}(A^{j_1}_k) = 1, \\ & \sum_{l}^{t} m_{j_2}(A^{j_2}_l) = 1, \\ & m_{j_1}(A^{j_1}_k)^- \leq m_{j_1}(A^{j_1}_k) \leq m_{j_1}(A^{j_1}_k)^+, k = 1, 2, \cdots, t, \\ & m_{j_2}(A^{j_2}_l)^- \leq m_{j_2}(A^{j_2}_l) \leq m_{j_2}(A^{j_2}_l)^+, l = 1, 2, \cdots, t. \end{aligned}$$

When there are three or more attributes to be combined, we can use the combination rule recursively to obtain the final combined results. It should be pointed out that when using the combination rule recursively, we should do follow the next processes. We can first do the combination between two attributes and then combine the intersections of the two with the third attribute. This process will be stopped when all decision attributes are considered. These non-linear programming models can be solved by some mathematical software such as LINGO. Similar models can also be constructed for $m_j(\Theta)$.

The last step is to construct belief interval of each decision alternative.

Definition 4: The belief measure (Bel) and the plausibility measure (Pls) of alternatives a_j are defined, respectively by:

$$\text{Bel}(\{a_i\}) = \left[\text{Bel}(\{a_i\})^-, \text{Bel}(\{a_i\})^+\right],$$
$$\text{Pls}(\{a_i\}) = \left[\text{Pls}(\{a_i\})^-, \text{Pls}(\{a_i\})^+\right],$$

Where:

$$\text{Bel}(\{a_i\})^- = \min \sum_{E=\{a_i\}} m(E) = \max\left[\sum_{E=\{a_i\}} m(E)^-, (1 - \sum_{E\neq\{a_i\}} m(E)^+)\right],$$
$$\text{Bel}(\{a_i\})^+ = \max \sum_{E=\{a_i\}} m(E) = \min\left[\sum_{E=\{a_i\}} m(E)^+, (1 - \sum_{E\neq\{a_i\}} m(E)^-)\right],$$
$$\text{Pls}(\{a_i\})^- = \min \sum_{\{a_i\}\in E} m(E) = \max\left[\sum_{\{a_i\}\in E} m(E)^-, (1 - \sum_{\{a_i\}\notin E} m(E)^+)\right],$$
$$\text{Pls}(\{a_i\})^+ = \max \sum_{\{a_i\}\in E} m(E) = \min\left[\sum_{\{a_i\}\in E} m(E)^+, (1 - \sum_{\{a_i\}\notin E} m(E)^-)\right].$$

Finally, we obtain the belief interval of all the decision alternatives which can be used to rank the alternatives. As for the ranking problem, there are a lot of methods provided by many scholars. Here we use the following formula:

$$P(a_i > a_k) = \max\left\{1 - \max\left[\frac{\text{Pls}^+(\{a_k\}) - \text{Bel}^-(\{a_i\})}{[\text{Pls}^+(\{a_k\}) - \text{Bel}^-(\{a_i\})] + [\text{Pls}^+(\{a_i\}) - \text{Bel}^-(\{a_k\})]}, 0\right], 0\right\} \tag{1}$$

If $P(a_i > a_k) > 0.5$ then $a_i \succ a_k$; else if $P(a_i > a_k) < 0.5$; else $a_i \prec a_k$.

NUMERICAL EXAMPLE

Here, we give an example on how to select a good project when the investment is carried on. The problem is described as follows: there are four available projects which consist of the decision alternative set $\Theta = \{a_1, a_2, a_3, a_4\}$ and there are four attributes related to the investment problem: marketing opportunity (C_1), support degree of policy (C_2), economy (C_3), technical capability (C_4) and the

Table 1: Decision matrix of the project selection problem

	C_1	C_2	C_3	C_4
a_1	h_1 (0.6, 0.2)	h_2 (0.3, 0.4)	h_1 (0.8, 0.1)	h_3 (0.5, 0.4)
	h_2 (0.5, 0.3)	h_4 (0.8, 0.1)	h_2 (0.5, 0.3)	h_4 (0.3, 0.5)
	h_3 (0.2, 0.5)	h_5 (0.1, 0.7)	h_4 (0.1, 0.5)	h_2 (0.1, 0.8)
a_2	h_1 (0.6, 0.3)	h_2 (0.8, 0.1)	h_4 (0.1, 0.8)	h_1 (0.6, 0.3)
	h_3 (0.5, 0.4)	h_3 (0.3, 0.6)	h_5 (0.3, 0.4)	h_2 (0.5, 0.3)
	h_5 (0.4, 0.5)	h_4 (0.1, 0.7)		h_3 (0.4, 0.4)
				h_4 (0.2, 0.6)
a_3	h_2 (0.8, 0.1)	h_3 (0.5, 0.4)	h_1 (0.6, 0.2)	h_1 (0.4, 0.5)
	h_3 (0.6, 0.2)	h_4 (0.2, 0.6)	h_2 (0.5, 0.4)	h_2 (0.8, 0.1)
	h_4 (0.1, 0.6)	h_5 (0.1, 0.8)	h_5 (0.1, 0.6)	h_3 (0.2, 0.6)
a_4	h_3 (0.5, 0.4)	h_2 (0.6, 0.2)	h_1 (0.1, 0.7)	h_2 (0.8, 0.1)
	h_4 (0.6, 0.3)	h_3 (0.8, 0.1)	h_2 (0.6, 0.2)	h_3 (0.5, 0.4)
	h_5 (0.2, 0.7)		h_3 (0.1, 0.7)	h_4 (0.2, 0.5)
				h_5 (0.3, 0.5)

Table 2: Transformed decision matrix

	C_1	C_2	C_3	C_4
a_1	0.212, 0.614	0.676, 0.151	0.272, 0.497	0.416, 0.346
a_2	0.486, 0.411	0.432, 0.403	0.233, 0.519	0.359, 0.45
a_3	0.535, 0.253	0.428, 0.389	0.26, 0.527	0.297, 0.579
a_4	0.666, 0.224	0.382, 0.51	0.236, 0.59	0.774, 0.079

weight of importance of decision attributes are $\omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$. All alternatives can be assessed to five grades $H = \{h_1, h_2, h_3, h_4, h_5\}$, here we assume the expected utility of each grades are:

$$u(h_1) = 0.2,\ u(h_2) = 0.4,\ u(h_3) = 0.6,\ u(h_4) = 0.8,\ u(h_5) = 1$$

First of all, we should calculate the weight of importance. By comparing with each other and using AHP algorithm, we can obtain ω = (0.517, 0.168, 0.077 and 0.238). After we assessed every alternative on each attribute, we obtain the decision matrix as shown in Table 1.

With the original information, we can obtain the transformed matrix $V = ((\mu_{i,j}, v_{i,j}))$ as shown in Table 2.

According to definition 2, we calculate the interval probability masses of each focal element as follows:
Under the decision attribute C_1:

$$m_1(\{a_1\}) = [0.11, 0.2],\ m_1(\{a_2\}) = [0.251, 0.305],\ m_1(\{a_3\}) = [0.277, 0.386],$$
$$m_1(\{a_4\}) = [0.344, 0.401],\ m_1(T) = [0, 0.018].$$

Under the decision attribute C_2:

$$m_2(\{a_1\}) = [0.114, 0.143],\ m_2(\{a_2\}) = [0.073, 0.1],\ m_2(\{a_3\}) = [0.072, 0.103],$$
$$m_2(\{a_4\}) = [0.064, 0.082],\ m_2(T) = [0.572, 0.677]$$

Under the decision attribute C_3:

$$m_3(\{a_1\}) = [0.021, 0.039],\ m_3(\{a_2\}) = [0.018, 0.037],\ m_3(\{a_3\}) = [0.02, 0.036],$$
$$m_3(\{a_4\}) = [0.018, 0.032],\ m_3(T) = [0.856, 0.923]$$

Under the decision attribute C_4:

$$m_4(\{a_1\}) = [0.113, 0.156],\ m_4(\{a_2\}) = [0.085, 0.131],\ m_4(\{a_3\}) = [0.071, 0.1],$$
$$m_4(\{a_4\}) = [0.184, 0.219],\ m_4(T) = [0.394, 0.547]$$

According to definition 3 and 4, we can obtain the belief measure and the plausible measure of alternative a_1 as follows:

$$\begin{array}{ll}
Bel(\{a_1\}) = [0.1029, 0.151] & Pls(\{a_1\}) = [0.1029, 0.1627] \\
Bel(\{a_2\}) = [0.1964, 0.2726] & Pls(\{a_2\}) = [0.1964, 0.2843] \\
Bel(\{a_3\}) = [0.2007, 0.2742] & Pls(\{a_3\}) = [0.2007, 0.2859] \\
Bel(\{a_4\}) = [0.3866, 0.4647] & Pls(\{a_4\}) = [0.3866, 0.4764] \\
Bel(\{\Theta\}) = [0, 0.0117] & Pls(\{\Theta\}) = 1
\end{array}$$

Then according to Eq. 1, we have the final rank of the four alternatives: $a_1 \prec a_1 \prec a_3 \prec a_4$, that is to say a_4 is the best project to invest.

CONCLUSION

The DS-AHP approach is a novel, flexible and systematic method. It can solve the problems directly based on its decision matrix. In this paper, we introduced something about intuitionistic fuzzy data and gave the basic steps of the DS-AHP method. We used the expected utilities to transform the original decision matrix. And then we defined interval probability masses which is different from the original basic probability assignment. In fact, it is an interval BPA. With the interval probability masses, we used a non-linear model to combine the focal element. In the end, we obtain the belief interval, where belief measure and plausible measure are both intervals. As we know the intuitionistic fuzzy information is used commonly in real assessment, so this combination is meaningful.

Further extension about DS-AHP approach includes developing methods with information expressed in interval intuitionistic fuzzy values or other uncertain values.

ACKNOWLEDGMENTS

The authors acknowledge support of Chinese Humanities and Social Sciences Project of Ministry of Education (10YJC630269) and University Science Research Project of Jiangsu Province (11KJD630001).

REFERENCES

Amer, A.A., R. Rakesh and B. Ranjit, 2010. Applying a new method of soft-computing for system reliability. J. Applied Sci., 10: 1951-1956.

Atanassov, K.T., 1986. Intuitionistic fuzzy sets. Fuzzy Sets Syst., 20: 87-96.

Beynon, M., B. Curry and P. Morgan, 2000. The Dempster-Shafer theory of evidence: An alternative approach to multicriteria decision modeling. Omega, 28: 37-50.

Beynon, M., D. Cosker and D. Marshall, 2001. An expert system for multi-criteria decision making using Dempster Shafer theory. Expert Syst. Appl., 20: 357-367.

Dempster, A.P., 1967. Upper and lower probabilities induced by a multivalued mapping. Ann. Math. Stat., 38: 325-339.

Denoeux, T. and L.M. Zouhal, 2001. Handling possibilistic labels in pattern classification using evidential reasoning. Fuzzy Sets Sys., 122: 409-424.

Gong, B.G., 2007. Methods based on Dempster-Shafer theory for multi-attribute decision making with incomplete information. Ph.D. Thesis, University of Science and Technology of China, China.

Hua, Z.S., B.G. Gong and X.Y. Xu, 2008. A DS-AHP approach for multi-attribute decision making problem with incomplete information. Expert. Syst. Appl., 34: 2221-2227.

Jiang, Y.P. and Z.P. Fan, 2005. Method for multiple attribute decision making with attribute interval numbers and preference information on alternatives. Sys. Eng. Electron., 27: 250-252.

Jones, R.W., A. Lowe and M.J. Harrison, 2002. A framework for intelligent medical diagnosis using the theory of evidence. Knowledge-Based Syst., 15: 77-84.

Kahneman, D., P. Slovic and A. Tversky, 1982. Judgement Under Uncertainty: Heuristics and Biases. Cambridge University Press, Cambridge, UK.

Kim, S.H. and B.S. Ahn, 1997. Group decision making procedure considering preference strength under incomplete information. Comput. Oper. Res., 24: 1101-1112.

Kim, S.H. and B.S. Ahn, 1999. Interactive group decision making procedure under incomplete information. Eur. J. Oper. Res., 116: 498-507.

Lan, T.S., 2009. Taguchi optimization of multi-objective CNC machining using TOPSIS. Inform. Technol. J., 8: 917-922.

Liu, D.R. and Y.Y. Shih, 2005. Integrating AHP and data mining for product recommendation based on customer lifetime value. Inform. Manage., 42: 387-400.

Liu, H.C., L. Liu, Q.H. Bian, Q.L. Lin, N. Dong and P.C. Xu, 2011. Failure mode and effects analysis using fuzzy evidential reasoning approach and grey theory. Expert Syst. Appl., 38: 4403-4415.

Liu, P.L. and C.H. Tsai, 2007. A study on the application of fuzzy analytic hierarchy process to construct an R and D management effectiveness evaluation index for taiwan`s high-tech industry. J. Applied Sci., 7: 1908-1915.

Saaty, T.L., 1980. The Analytic Hierarchy Process. McGraw Hill, New York.

Shafer, G., 1976. A Mathematical Theory of Evidence. 1st Edn., Princeton University Press, Princeton, New Jersey, USA., ISBN: 9780691081755, Pages: 297.

Telmoudi, A. and S. Chakhar, 2004. Data fusion application from evidential databases as a support for decision making. nform. Software Technol., 46: 547-555.

Wang, H., G. Qian and X. Feng, 2011. An intuitionistic fuzzy AHP based on synthesis of eigenvectors and its application. Inform. Technol. J., 10: 1850-1866.

Webber, S.A., B. Apostolou and J.M. Hassell, 1997. The sensitivity of the analytic hierarchy process to alternative scale and cue presentations. Eur. J. Oper. Res., 96: 351-362.

Wu, H.H., J.I. Shieh, Y. Li and H.K. Chen, 2010. A combination of AHP and DEMATEL in evaluating the criteria of employment service outreach program personnel. Inform. Technol. J., 9: 569-575.

Xu, D., 2012. The research on sort method of consumer's brand preference under uncertain environment. Inform. Technol. J., 11: 637-641.

Yager, R.R., M. Fedrizzi and J. Kacprzyk, 1994. Advances in the Dempster-Shafer Theory of Evidence. John Wiley, New York, USA.

Yang, J.B., Y.M. Wang, D.L. Xu and K.S. Chin, 2006. The evidential reasoning approach for MADA under both probabilistic and fuzzy uncertainties. Eur. J. Oper. Res., 171: 309-343.

Yang, T. and C. Kuo, 2003. A hierarchical DEA-AHP methodology for the facilities layout design problem. Eur. J. Opert. Res., 147: 128-136.

Zhang, Z., J. Yang, Y. Ye, Y. Hu and Q. Zhang, 2012. Intuitionistic fuzzy sets with double parameters and its application to pattern recognition. Inform. Technol. J., 11: 313-318.

Zhanga, Y., X. Denga, D. Weia and Y. Denga, 2012. Assessment of E-Commerce security using AHP and evidential reasoning. Expert Syst. Appl., 39: 3611-3623.