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Intuitionistic Fuzzy Sets with Double Parameters and its Application to Pattern Recognition

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Abstract: A novel Intuitionistic Fuzzy Sets with Parameters (IFSP) is introduced in this paper. Based on membership function, non-membership function and hesitancy function, this paper concentrates on the construction of Intuitionistic Fuzzy Sets with Double Parameters (IFSDP). Then, we define the distance measures of IFSDP based on the distance measures of IFS. Finally, a pattern recognition example applied to medical diagnosis decision-making is given to demonstrate the application of IFSDP. The simulation results show that the method of IFSDP is more comprehensive and flexible than that of the traditional intuitionistic fuzzy sets.

Key words: Intuitionistic fuzzy sets, intuitionistic fuzzy sets with parameters, intuitionistic fuzzy sets with double parameters, pattern recognition; medical diagnosis decision making

INTRODUCTION

Atanassov (1986) introduced membership function, non-membership function and hesitancy function and presented Intuitionistic Fuzzy Sets (IFS). Hence, many scholars applied IFS to decision making analysis and pattern recognition widely. In the research field of IFS, Yager (2009) discussed the cut set characteristics of IFS in 2009, (Szmidt and Kacprzyk, 2004; Hung and Yang, 2007; Szmidt and Kacprzyk, 2008) applied it to medical diagnosis (2004-2008), Dengfeng and Chuntian (2002) and Li *et al.* (2007) applied it to pattern recognition (2002, 2007), Xu (2007) and Wei (2010) applied it to decision-making analysis (2007-2010) and Lei *et al.* (2010) studied intuitionistic fuzzy reasoning (2010). However, in the field of pattern recognition, conventional IFS method does not consider the detachment of hesitancy. Thus, the simulation results may be defective. Based on this defect, we present an IFSP method and mainly probe into the construction and application of IFSDP.

First, we propose a series of definitions and construction methods of the IFSP and focus on the model of IFSDP. Then, taking advantage of conventional distance measure, IFSDP is applied to medical diagnosis based on pattern recognition. The simulation results show that the result of the method introduced in this study is more comprehensive and flexible than that of the conventional IFS method. Hence, this paper can provide valuable conclusion for the field of application research of IFS and the model of IFSP is also useful to intuitionistic fuzzy reasoning. This method can also be generalized to interval-valued intuitionistic fuzzy sets as Zhao *et al.* (2010).

CONSTRUCTION OF IFSP

Definition 1: An IFS A in universe X is given by Atanassov (1986):

$$A = \{<x, \mu_A(x), \nu_A(x)>|x \in X\} \tag{1}$$

where, μ_A: X∈ [0, 1], ν_A: X∈[0, 1] with the condition $0 = \mu_A(x)+\nu_A(x) = 1$ for each $x \in X$. The numbers $\mu_A(x)$, $\nu_A(x) \in [0, 1]$ denote a degree of membership and a degree of non-membership of x to A, respectively. For each IFS in X, we will call $\pi_A(x) = 1-\mu_A(x)-\nu_A(x)$ the degree of hesitancy of x to A, $0 = \pi_A(x) = 1$ for each x∈X.

Definition 2: Let X be a universe of discourse. An IFSDP A in X is an object having the form:

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$$A=\{<x,\mu_A^*(x),\nu_A^*(x)>,x\in X\}$$

Let:

$$\mu_A^*(x)=\mu_A(x)+\alpha_A(x),\nu_A^*(x)=\nu_A(x)+\beta_A(x)$$

where, $\mu_A(x)$, $\nu_A(x)$ and $\pi_A(x)$ are the same as definition 1. And we have:

$$\pi_A(x)\geq\pi_A^*(x)\geq 0$$

and:

$$\alpha_A(x)+\beta_A(x)=\pi_A(x)-\pi_A^*(x),\alpha_A(x)\geq 0,\beta_A(x)\geq 0,$$
$$\mu_A^*(x)+\nu_A^*(x)+\pi_A^*(x)=\mu_A(x)+\nu_A(x)+\pi_A(x)=1$$

Thus, IFS is a special case of IFSDP.

Theorem 1: Let A be an IFSDP as mentioned above, then:

$$\mu_A^*(x)-\mu_A(x)+\nu_A^*(x)-\nu_A(x)=\pi_A(x)-\pi_A^*(x) \qquad (2)$$

Based on definition 2, we have Eq. 2. From definition 1, let all sample data be divided into three parts, $\mu_A(x)$ being the firm support party of event A, $\nu_A(x)$ representing the firm opposition party of event A and $\pi_A(x)$ showing all the absent party. In absent party, $\pi_A^*(x)$ is the firm absent party and $\pi_A(x)-\pi_A^*(x)$ is the convertible absent party, in which each sample may become one of support party and opposition party. If there is $\alpha_A(x)$ sample supporting event A and $\beta_A(x)$ sample opposing event A, then we have IFSDP as definition 2. If the proportion of the absent party being converted to support party is $\lambda_{A1}(x)$ and the proportion of the absent party being converted to opposition party is 1-$\lambda_{A1}(x)$, then the model will become intuitionistic fuzzy sets with single parameter, where:

$$\alpha_A(x)=\lambda_{A1}(x)(\pi_A(x)-\pi_A^*(x)),\beta_A(x)=(1-\lambda_{A1}(x))(\pi_A(x)-\pi_A^*(x))$$

If the firm absent party is $\pi_A^*(x)=(1-\lambda_{A0}(x))\pi_A(x)$, then the convertible absent party is $\pi_A(x)-\pi_A^*(x)=\lambda_{A0}(x)\pi_A(x)$, and then we get the other IFSDP definition:

Definition 3: An IFSDP A in universe X is given by:

$$A=\{<x,\mu_A^*(x),\nu_A^*(x)>,x\in X\}$$

Let:

$$\nu_A^*(x)=\nu_A(x)+\lambda_{A0}(x)(1-\lambda_{A1}(x))\pi_A(x),\pi_A^*(x)=(1-\lambda_{A0}(x))\pi_A(x),$$
$$\mu_A^*(x)=\mu_A(x)+\lambda_{A0}(x)\lambda_{A1}(x)\pi_A(x),\ 0\leq\lambda_{Ai}(x)\leq 1,i=0,1$$

and:

$$\mu_A^*(x),\nu_A^*(x),\pi_A^*(x)$$

represent the degree of membership, the degree of non-membership and the degree of hesitancy of x to A, respectively. Where $\mu_A(x)$, $\nu_A(x)$ and $\pi_A(x)$ are the same as definition 1.

It is clear that we will get the following conclusions: when $\lambda_{A0}(x)$ = 0, IFSDP is IFS; when $\lambda_{A0}(x)$ = 1 IFSDP is fuzzy sets.

DISTANCE MEASURES

In this study, we will use the following distance measures in simulation of pattern recognition.

$$d_{A,B}^{\mu,\nu,\pi}=\sqrt[p]{[\sum_{i=1}^{n}(|\mu_A^*(x_i)-\mu_B^*(x_i)|^p+|\nu_A^*(x_i)-\nu_B^*(x_i)|^p+|\pi_A^*(x_i)-\pi_B^*(x_i)|^p]/(2n)} \qquad (3)$$

where, $\mu_A^*(x),\nu_A^*(x),\pi_A^*(x)$ are defined as definition 3. If p = 1, the distance measure is Hamming distance, else if p = 2, it is Euclidean distance. Let:

$$d_{A,B}^{\mu,\nu,\pi}=\sqrt[p]{\sum_{i=1}^{n}d_{A,B}^{\mu,\nu,\pi}(x_i)/2n}$$

and then we will get the following Euclidean distance formulas when p = 2.

$$d_{A,B}^{\mu,\nu,\pi}(x_i)=(\mu_A^*(x)-\mu_B^*(x))^2+(\nu_A^*(x)-\nu_B^*(x))^2+(\pi_A^*(x)-\pi_B^*(x))^2$$
$$d_{A,B}^{\mu,\nu,\pi}(x_i)=(\mu_A(x_i)-\mu_B(x_i))^2+(\nu_A(x_i)-\nu_B(x_i))^2+(\pi_A(x_i)-\pi_B(x_i))^2$$
$$+2(\pi_A(x_i)-\pi_B(x_i))^2\lambda_{A0}^2(\lambda_{A1}{}^2-\lambda_{A1}+1)$$
$$+2(\pi_A(x_i)-\pi_B(x_i))[(\nu_A(x_i)-\nu_B(x_i))-(\pi_A(x_i)-\pi_B(x_i))]\lambda_{A0}$$
$$+2(\pi_A(x_i)-\pi_B(x_i))[(\mu_A(x_i)-\mu_B(x_i))-(\nu_A(x_i)-\nu_B(x_i))]\lambda_{A0}\lambda_{A1} \qquad (4)$$

APPLICATION TO MEDICAL DIAGNOSIS DECISION MAKING BASED ON PATTERN RECOGNITION

To make a proper diagnosis D for a patient with given values of symptoms S, a medical knowledge base is necessary that involves elements described in terms of

Table 1: Symptoms characteristic for the diagnoses considered

Patient symptoms	Temperature	Headache	Stomach pain	Cough	Chest pain
AI	(0.8,0.1)	(0.6,0.1)	(0.2,0.8)	(0.6,0.1)	(0.1,0.6)
Bob	(0.0,0.8)	(0.4,0.4)	(0.6,0.1)	(0.1,0.7)	(0.1,0.8)
Joe	(0.8,0.1)	(0.8,0.1)	(0.0,0.6)	(0.2,0.7)	(0.0,0.5)
Ted	(0.6,0.1)	(0.5,0.4)	(0.3,0.4)	(0.7,0.2)	(0.3,0.4)

Table 2: Symptoms characteristic for the patients considered

Disease symptoms	Temperature	Headache	Stomach pain	Cough	Chest pain
Viral fever	(0.4,0.0)	(0.3,0.5)	(0.1,0.7)	(0.4,0.3)	(0.1,0.7)
Malaria	(0.7,0.0)	(0.2,0.6)	(0.0,0.9)	(0.7,0.0)	(0.1,0.8)
Typhoid	(0.3,0.3)	(0.6,0.1)	(0.2,0.7)	(0.2,0.6)	(0.1,0.9)
Stomach problem	(0.1,0.7)	(0.2,0.4)	(0.8,0.0)	(0.2,0.7)	(0.2,0.7)
Chest problem	(0.1,0.8)	(0.0,0.8)	(0.2,0.8)	(0.2,0.8)	(0.8,0.1)

IFS. We consider the same data that are introduced by Szmidt and Kacprzyk (2004, 2005): The set of disease diagnoses is D = {Viral fever, Malaria, Typhoid, Stomach problem, Chest problem} and the set of symptoms is a universe of discourse S = {temperature, headache, stomach pain, cough, chest pain}. The data are given in Table 1, where each symptom is described by: membership μ_A (x), non-membership ν_A (x), hesitancy margin π_A (x). For example, for malaria the temperature is high (μ_A (x) = 0.7, ν_A (x) = 0, π_A (x) = 0.3), for a chest problem the temperature is low (μ_A (x) = 0.1, ν_A (x) = 0.8, π_A (x) = 0.1), etc.

The set of patients is P = {AI, Bob, Joe, Ted}. The symptoms are given in Table 2, where we need all three parameters (μ_A (x), ν_A (x), π_A (x)) to describe each symptom (Szmidt and Kacprzyk, 2004, 2005) as before. We seek a diagnosis for each patient pi, i = 1, 2, 3, 4. They proposed to solve the problem in the following way: (1) to calculate a distance from a set of symptoms s_i (I = 1, 2, 3, 4, 5) (using normalized Hamming distance, Euclidean distance and ratio-based measure of similarity) between each patient p_j (j = 1, 2, 3, 4, Table 2) and each diagnosis d_k, (k = 1, 2, 3, 4, 5, Table 1) (2) to determine the lowest distance which points out to a proper diagnosis.

The normalized Hamming distance for all symptoms of patient j-th from diagnosis kth is:

$$d_H(p_j, d_k) = \sum_{i=1}^{5} \frac{|\mu_{p_j}(s_i) - \mu_{d_k}(s_i)| + |\nu_{p_j}(s_i) - \nu_{d_k}(s_i)| + |\pi_{p_j}(s_i) - \pi_{d_k}(s_i)|}{10} \tag{5}$$

The normalized Euclidean distance for all symptoms of patient j-th from diagnosis kth is:

$$d_E(p_j, d_k) = \sqrt{\frac{\sum_{i=1}^{5}(|\mu_{p_j}(s_i) - \mu_{d_k}(s_i)|^2 + |\nu_{p_j}(s_i) - \nu_{d_k}(s_i)|^2 + |\pi_{p_j}(s_i) - \pi_{d_k}(s_i)|^2)}{10}} \tag{6}$$

The ratio-based measure of similarity for all symptoms of patient j-th from diagnosis k-th is:

$$dRB\,(pj, dk) = d_H(p_j, d_k) / d_H(p_j, d_k^C) \tag{7}$$

where, they describe d_k^C to be a set (ν_A (x), μ_A (x), π_A (x)). For example, for malaria the temperature d_k is high (μ_A (x) = 0.7, ν_A (x) = 0, π_A (x) = 0.3), d_k^C is low (μ_A (x) = 0, ν_A (μ_A (x) = 0.7, ν_A (x) = 0, π_A (x) = 0.3), d_k^C is low (μ_A (x) = 0, ν_A (x) = 0.7, π_A (x) = 0.3), etc.

From the obtained distance measures and similarity measures, they get the results as Table 3. Three types of distances given in Table 3 are described as follows: the first data is Hamming distance as Eq. 5, the second data is Euclidean distance as Eq. 6 and the third data is ratio-based measure as Eq. 7. For example, d_H (AI, viral fever), d_E (AI, viral fever) and d_{RB} (AI, viral fever) are 0.28, 0.29, 0.75, respectively. Similarly, we can obtain the other results. Thus, they considered that Bob suffers from stomach problem and Joe from typhoid based on Eq. 5-7 and that AI suffers from malaria and Ted from viral fever calculated by Hamming distance and Euclidean distance, while AI suffers from viral fever and ted from malaria calculated by the ratio-based measure.

From definition 3, using the distance Eq. 4, we will get Table 4. And the distances given in Table 4 are described as follows: for example, from the first data of viral fever and malaria, we can describe the distance measure between AI and viral fever and the distance measure between AI and malaria by Eq. 8 and 9, respectively. Similarly, we can get all the results in Table 4.

$$\sum_{i=1}^{n} d_{AI,Viralfever}^{\mu,\nu,\pi}(s_i) = 0.84 - 0.86\lambda_{A0} - 0.14\lambda_{A0}\lambda_{AI} + 0.62\lambda_{A0}^2(\lambda_{A1}^2 - \lambda_{A1} + 1) \tag{8}$$

$$\sum_{i=1}^{n} d_{AI,malaria}^{\mu,\nu,\pi}(s_i) = 0.64 - 0.4\lambda_{A0} + 0.2\lambda_{A0}\lambda_{AI} + 0.2\lambda_{A0}^2(\lambda_{A1}^2 - \lambda_{AI} + 1) \tag{9}$$

Obviously, if:

$$\sum_{i=1}^{n} d_{AI,disease}^{\mu,\nu,\pi}(s_i)$$

is the smallest, then:

$$d_{AI,disease}^{\mu,\nu,\pi} = \sqrt{\sum_{i=1}^{n} d_{AI,disease}^{\mu,\nu,\pi}(s_i) / 2n}$$

is also the smallest. Therefore, the minimum among all these:

Table 3: Distance description between each patient and each disease based on IFS

Patient/disease	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
AI	(0.28,0.29,0.75)	(0.24,0.25,1.19)	(0.28,0.32,1.31)	(0.54,0.53, 3.27)	(0.56,0.58,8)
Bob	(0.40,0.43,2.1)	(0.50,0.56,3.73)	(0.31,0.33,1.10)	(0.14,0.14,0.35)	(0.42,0.46, 8)
Joe	(0.38,0.36,0.87)	(0.44,0.41,1.52)	(0.32,0.32,0.46)	(0.50,0.52,2.61)	(0.55,0.57, 8)
Ted	(0.28,0.25,0.95)	(0.30,0.29,0.77)	(0.38,0.35,1.67)	(0.44,0.43, 8)	(0.54,0.50,2.56)

Table 4: Distance description between each patient and each disease on IFSDP

(AI, Bob, Joe, Ted)	Constant	λ_{A0}	λ_{A0}, λ_{A1}	$\lambda_{A0}^2\ (\lambda_{A1}^2\text{-}\lambda_{A1}+1)$
Viral fever	(0.84,1.88., 1.28,0.62)	(-0.86,-1.24, -1.20,-0.44)	(-0.14,1.34, -0.18,-0.08)	(0.62,0.38, 0.86,0.32)
Malaria	(0.64,3.12, 1.90,0.86)	(-0.40,-0.74, -1.32,-0.66)	(0.20,1.12, 0.60,0.54)	(0.20,0.12, 0.68,0.26)
Typhoid	(1.00,1.12, 1.04,1.22)	(-0.58,-0.72, -1.32,-0.80)	(-0.04,0.84, -0.24,0.46)	(0.40,0.20, 0.96,0.38)
Stomach problem	(2.78,0.20, 2.68,1.86)	(-0.70,-0.10, -0.22,-0.46)	(0.56,-0.16, -1.36,0.02)	(0.28,0.12, 0.60,0.30)
Chest problem	(3.36,2.12, 3.34,2.46)	(-0.64,-0.74, -0.40,-0.82)	(0.44,0.64, -1.24,0.50)	0.28,0.28, 0.68,0.38)

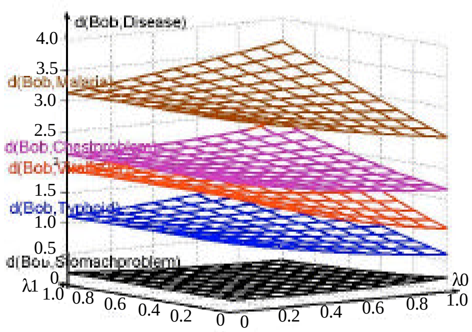

Fig. 1: Distance (Bob, disease)

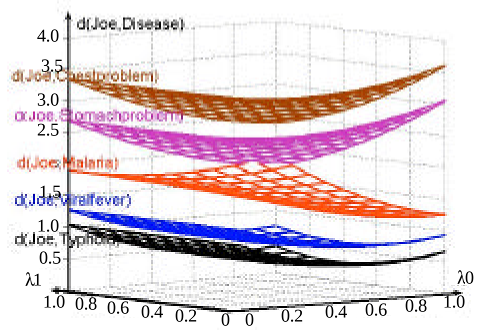

Fig. 2: Distance (Joe, disease)

$$\sum_{i=1}^{n} d^{\mu,\nu,\pi}_{AI,disease}(s_i)$$

is the pattern we need. When:

$$\sum_{i=1}^{n} d^{\mu,\nu,\pi}_{AI,viralfever}(x_i) > \sum_{i=1}^{n} d^{\mu,\nu,\pi}_{AI,malaria}(x_i)$$

AI is diagnosed as malaria, otherwise, AI is viral fever. Then, we have the results: If:

$$0.2 - 0.46\lambda_{A0} - 0.34\lambda_{A0}\lambda_{A1} + 0.42\lambda_{A0}^2(\lambda_{A1}^2 - \lambda_{A1} + 1) > 0$$

AI is diagnosed as malaria and on the contrary, AI is diagnosed as viral fever. For example, let λ_{A0}=1, then we will have the results as follows: if λ_{A1}<0.243, AI is diagnosed as malaria, otherwise, AI is viral fever. Similarly, we can calculate the distance between each patient and each disease and then draw a conclusion.

It is obvious that Bob suffers from stomach problem and Joe from typhoid for each $\lambda_{A0}\in[0,1]$ and for each $\lambda_{A1}\in[0,1]$. Then, we calculate the distances between Ted and each disease and we have the results as follows:

If $-0.16 + 0.22\lambda_{A0} - 0.62\lambda_{A0}\lambda_{A1} + 0.06\lambda_{A0}^2(\lambda_{A1}^2 - \lambda_{A1} + 1) > 0$ Ted is diagnosed as malaria; otherwise, Ted is diagnosed as viral fever. For example, let $\lambda_{A0} = 1$, if λ_{A1}<0.18, then Ted is diagnosed as malaria, otherwise, Ted is viral fever.

From Table 4, we have Fig. 1 to 4. From Fig. 1 and 2, it is clear that Bob is diagnosed as stomach problem and Joe is diagnosed as typhoid for each $\lambda_{A0}\in[0,1]$ and for each $\lambda_{A1}\in[0,1]$. However, from Fig. 3 and 4, AI is diagnosed as one of viral fever and malaria when $\lambda_{A0}\in[0,1]$ and $\lambda_{A1}\in[0,1]$ and the diagnosis of Ted is similar to that of AI. Figure 3 shows the difference of AI to be diagnosed as viral fever and malaria. Figure 4 shows the difference of Ted to be diagnosed as viral fever and malaria. From Fig. 3, we draw a conclusion: If λ_{A0}>0.3 and λ_{A1}>0.3, which means that the proportion of the absent party being converted is more than 0.3 and the proportion of the absent party being converted into support party is more than 0.3, then AI is diagnosed as viral fever, otherwise, AI is diagnosed as malaria. From Fig. 4, we also have: If $\lambda_{A0}\in 1$ and $\lambda_{A1}\in 0$ which means that the majority of the absent party can be converted and most of them are converted into opposition party, then Ted is diagnosed as malaria, otherwise, Ted is diagnosed as viral fever.

If λ_{A0}= 0, then IFSDP is equivalent to IFS and then $\mu^*_A(x) = \mu_A(x), \nu^*_A(x) = \nu_A(x), \pi^*_A(x) = \pi_A(x)$, which means that the results of the left coordinate axis and the left-lower plane

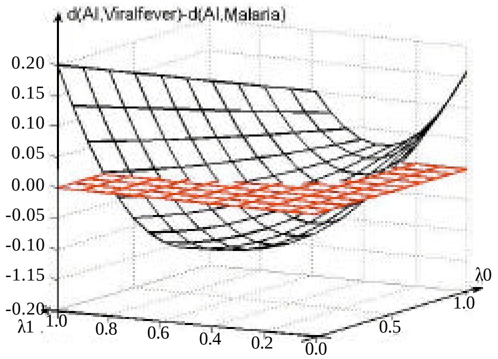

Fig. 3: d(AI, Viral fever)-d(AI, Malaria)

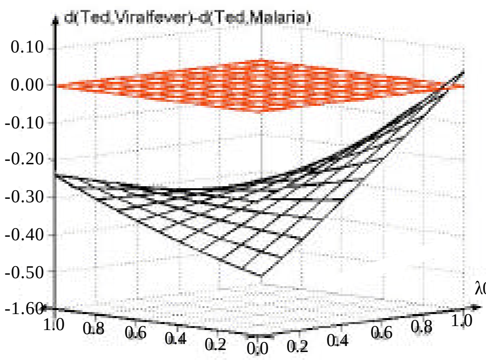

Fig. 4: d(Ted, Viral fever) -d(Ted, Malaria)

in Fig. 3, 4 are the results calculated by IFS method. And then we have: If λ_{A0}= 0, AI, Bob, Joe and Ted are identified as malaria, stomach problem, typhoid and viral fever respectively. And if λ_{A0}>0, Bob and Joe are also identified as stomach problem and typhoid respectively which are the same as the results calculated by Eq. 5-7. However, AI and Ted are identified as viral fever and malaria under different known conditions about λ_{A0} and λ_{A1}.

The experimental results above show that there is much difference between the recognition results of IFSDP and that of IFS. Conventional IFS method is simple but its recognition results are fixed when using conventional distance formulas. Therefore, it is difficult to reveal the potential law from all available information when using the IFS method. And the results of the IFSDP method are variable which can be adjusted to sample data with parameters. Furthermore, if the fixed pattern recognition results are different from sample data, the IFS method will fall into fail. However, when using the IFSDP method, we can meet the needs of sample data by adjusting parameters to appropriate values. All the results above show that the IFSDP method is more effective than the IFS method.

CONCLUSION

We propose a method for the evaluation of a degree of agreement in a group of individuals by calculating distances between intuitionistic fuzzy preference relations with parameters. The IFSP method not only considers membership function and non-membership function but also considers the detachment of hesitancy function. Therefore, it is more comprehensive and flexible than the IFS method.

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