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ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 11 (3): 339-343, 2012
ISSN 1812-5638 / DOI: 10.3923/itj.2012.339.343

A Mathematical Model of Fuzzy Multiple Objective Programming and Applications

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Abstract: For multiple objective programming problems with many constraint conditions, an improved objective programming approach is built up with fuzzy mathematics. First solve the deviation variables based on the priority level of the objectives and then determine the upper bound of the flexible indexes by considering the deviation variables. Hence, build up the mathematical model of fuzzy multiple objective programming and it can be applied to realistic transportation problems in which satisfactory results can be drawn.

Key words: Objective programming approach, priority level, fuzzy multiple objective programming, flexible indexes, satisfactory solution

INTRODUCTION

The goal programming, which can be used to deal with the problems in which containing a single main objective and multiple sub-objectives, or multiple main objectives and multiple sub-objectives. Previously, every object is assigned with an expected value and when they satisfy some constraint conditions of the system, the solutions which are most closed to the expected values can be found. By this method, the objects set by decision makers can be entirely reflected in the system model and subjective willing can be unified with objective possibility in one mathematical model.

When the calculation was processed, the deviation variable of goal programming was so large that the solution was not accurate enough. Since the objectives in multi-objective programming can be sorted as required goals and expected goals, where the required goals have veto and the expected goals are elastic in hierarchy of objectives. Because the goal constraint in multi-objective programming is a type of soft constraint which can be elastic, so we adopt a fuzzy mathematical method by introducing fuzzy programming with elastic constraint conditions into multi-objective programming, hence more satisfactory solutions can be gained (Kaicheng, 2004).

METHOD AND MODEL

Only multi-objective programming problems will be discussed.

Mathematical model for goal programming: In multi-objective optimization problems, since the objectives have connections and conflicts between each other, we need to distinguish main objectives and secondary objectives and sort multi-objective into different levels. The priority levels are represented by p_1, in which $P_1 >> P_2 >> ..... >> P_m$. Only when the objectives corresponding to higher priority factors are satisfied, the objectives with lower priority factors can de considered. For those objectives with the same priority level, the importance can be represented by their weight coefficient. (Changbing, 2007).

For every decision goal, we introduced deviation variables d^+ and d^- to represent the part that decision value exceeds or lack to the expectation value. And it is defined as $d^+ \geq 0$, $d^- \geq 0$.

The objective function consists of the deviation variables of goal constraint and their corresponding priority factors P_1, as well as the weight coefficients. Because goal programming pursues the solution that is closed to some given expectation value, which means that every corresponding deviation variable should be as small as possible, so the objective function should be minimized. The mathematical model is built up as:

$$\min\{P_l(\sum_{k=1}^{K}(W_{lk}^- d^- + W_{lk}^+ d^+)), l = 1, 2, ...L\}$$
$$\begin{cases} \sum_{j=1}^{n} c_{kj} x_j + d_k^- - d_k^+ = g_k, k = 1, 2, ...K \\ \sum_{j=1}^{n} a_{ij} x_j \leq (=, \geq) b_i, i = 1, 2, ...m \\ x_j \geq 0, j = 1, 2, ..., n \\ d_k^+, d_k^- \geq 0, k = 1, 2, ...K \end{cases} \quad (1)$$

where, g_k is the expectation value of the k-th goal constraint; W_{lk}^-, W_{lk}^+ are weights of the goals corresponding to P_1 priority level and $P_1 >> P_2 >> ..... P_m$.

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Mathematical model for fuzzy multi-objective programming: Here we take the idea of fuzzy linear programming into goal programming. First, we gain the optimal value f_{i0} (i = 1, 2,....., l) of each objective function:

$$f_i = \sum_{j=1}^{n} c_{ij} x_j$$

under the same constraint condition. If the expectation value of goal constraint is given, we can omit this step and then solve the deviation variable according to the priority level of goals. By considering the deviation variable the upper limit of the flexible index $d_i \geq 0$ ca be determined and the corresponding fuzzy goal set $\tilde{G}_i$ can be gained, whose subordinate function is defined as:

$$\tilde{G}_i(x) = \begin{cases} 0, & \sum_{j=1}^{n} c_{ij} x_j < f_{i0} - d_i; \\ 1 - \frac{1}{d_i}(f_{i0} - \sum_{j=1}^{n} c_{ij} x_{ij}), & f_{i0} - d_i < \sum_{j=1}^{n} c_{ij} x_j < f_{i0}; \\ 1, & \sum_{j=1}^{n} c_{ij} x_j \geq f_{i0}. \end{cases}$$

Take:

$$\tilde{G} = \bigcap_{i=1}^{L} \tilde{G}, A$$

as the feasible solution set satisfying the constraint conditions and solve X^* according to maximum subordination principle, such that:

$$\tilde{G}(x^*) = \max_{x \in A} \{\tilde{G}(x)\}$$

It equals to solving the normal linear programming problem:

$$\max \lambda; \quad \begin{cases} 1 - \frac{1}{d_i}(f_{i0} - \sum_{j=1}^{n} c_{ij} x_j) \geq \lambda, i = 1, 2, ...l; \\ \sum_{j=1}^{n} a_{kj} x_j \leq b_k, k = 1, 2, ...m; \\ \lambda \geq 0, x_1 \geq 0, x_2 \geq 0, ...x_n \geq 0. \end{cases} \quad (2)$$

The key of this problem is the determination of the flexible index $d_i \geq 0$. We combine the goal programming method in Eq. 1 and the fuzzy multi-objective programming in Eq. 2 and determine the appropriate $d_i \geq 0$, hence gain the optimal solution closed to the goal expectation within the feasible region.

A TRANSPORTATION PROBLEM CASE

A transportation enterprise provides the service that transport steels from steel producing enterprises in 3 different cities to 4 real estate enterprises in different cities. The steel producing enterprises A_i (i = 1, 2, 3, 4) have supply of a_i while the real estate enterprises B_j (j = 1, 2, 3, 4) have demand of b_j. The unit transportation fee c_{ij} is listed in Table 1.

When making the transportation schedule, these 6 goals should de considered by priority level:

P_1: The real estate enterprise B_4 is an important enterprise, its demand should be satisfied; P_2: The steel producing enterprise A_3 should supply no less than 100 to B1, P_3: Every real estate enterprise should get steels no less than 75~85% of its demand; P_4: The real total fee should be less than 110% of the minimum transportation fee without considering goals P_1 to P_5; P_5: Supply to B_1 and B_3 should be the same or similar; P_6: The total transportation fee should be minimized (Jiuping, 2009).

This is a transportation problem with unbalanced supply and demand. First we suppose transportation fee from A_i to B_j is x_{ij}, so the minimized fee is 2950 without considering goals P_1 to P_5. The minimum fee by goal programming is 3170 if P_1 to P_6 are considered (Yunquan, 2004; Anderson *et al.*, 2003).

We use the improved fuzzy goal programming. The deviation variables d_i of goal constraint can be treated as the upper limit of flexible index in fuzzy linear programming. The mathematical model is as follow:

$$\min f = 5x_{11}+2x_{12}+6x_{13}+7x_{14}+3x_{21} +5x_{22}+4x_{23}+6x_{24}+4x_{31}+5x_{32}+2x_{33}+3x_{34}$$

Constraint conditions:

$$\begin{cases} x_{11} + x_{12} + x_{13} + x_{14} \leq 300 \\ x_{21} + x_{22} + x_{23} + x_{24} \leq 200 \\ x_{31} + x_{32} + x_{33} + x_{34} \leq 400 \\ x_{11} + x_{21} + x_{31} \leq [200,10] \\ x_{12} + x_{22} + x_{32} \leq [100,5] \\ x_{13} + x_{23} + x_{33} \leq [450,25] \\ x_{14} + x_{24} + x_{34} \leq [250,15] \\ x_{31} \geq [100,5] \\ x_{11} + x_{21} + x_{31} \geq [160,10] \\ x_{12} + x_{22} + x_{32} \geq [80,5] \\ x_{13} + x_{23} + x_{33} \geq [360,25] \\ x_{14} + x_{24} + x_{34} \geq [200,15] \\ \frac{x_{11} + x_{21} + x_{31}}{200} + \frac{x_{13} + x_{23} + x_{33}}{450} = [0, 0.1] \end{cases}$$

Table 1: Unit transportation fee (Transportation unit in ton, fee unit in ten thousand dollars)

c_{ij}	B_1	B_2	B_3	B_4	a_i
A_1	5	2	6	7	300
A_2	3	5	4	6	200
A_3	4	5	2	3	400
b_j	200	100	450	250	

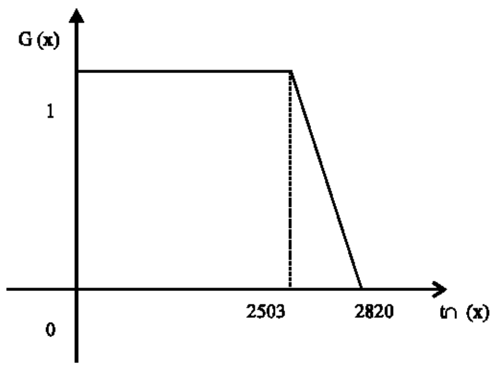

Fig. 1: Plot of the subordinate function

The equations can break up to two normal linear programming problems. By LINGO software we obtain two sets of optimal solutions:

$$f_0 = 2820,\ f_1 = 2503,\ d_0 = f_0 - f_1 = 317$$

The subordinate function of the objective function is plotted as Fig. 1.

Solve the normal linear programming: max λ with constraint conditions:

$$\begin{cases} 5x_{11} + 2x_{12} + 6x_{13} + 7x_{14} + 3x_{21} + 5x_{22} + 4x_{23} \\ +6x_{24} + 4x_{31} + 5x_{32} + 2x_{33} + 3x_{34} - 317\lambda \le 2820 \\ x_{11} + x_{12} + x_{13} + x_{14} \le 300 \\ x_{21} + x_{22} + x_{23} + x_{24} \le 200 \\ x_{31} + x_{32} + x_{33} + x_{34} \le 400 \\ x_{11} + x_{21} + x_{31} + 10\lambda \le 210 \\ x_{12} + x_{22} + x_{32} + 5\lambda \le 105 \\ x_{13} + x_{23} + x_{33} + 25\lambda \le 475 \\ x_{14} + x_{24} + x_{34} + 15\lambda \le 265 \\ x_{31} - 5\lambda \ge 95 \\ x_{11} + x_{21} + x_{31} - 10\lambda \ge 150 \\ x_{12} + x_{22} + x_{32} - 25\lambda \ge 75 \\ x_{13} + x_{23} + x_{33} - 25\lambda \ge 335 \\ x_{14} + x_{24} + x_{34} - 15\lambda \ge 185 \\ \frac{x_{11} + x_{21} + x_{31}}{200} + \frac{x_{13} + x_{23} + x_{33}}{450} - 0.1\lambda \ge -0.1 \\ \frac{x_{11} + x_{21} + x_{31}}{200} + \frac{x_{13} + x_{23} + x_{33}}{450} + 0.1\lambda \le 0.1 \end{cases}$$

Solve the linear programming by the software LINGO to get the optimal solution 2674, with the value of λ is 0.96 and the plan is:

$$x_{12} = 79,\ x_{13} = 118,\ x_{23} = 200,\ x_{31} = 159,\ x_{33} = 40,\ x_{34} = 199$$

Thus, we can gain the fuzzy optimal solution of the problem is 2674.

Calculation of estimate matrix is shown in appendix and program code are also presented.

CONCLUSION

Compared with solutions of other multi-objective programming, the method of goal programming can solve the problems with different dimensions and the objectives can be modified more flexibly and instantaneously; at the meantime there would be very little additional calculation caused by the new objectives. Hence it is suitable for the changing situation of the current economics and has been a new type of tool to solving multi-objective programming.

APPENDIX

Appendix: 1: Calculation of estimate matrix\lambda_{max}: 7.0164

d	G1	G2	G3	G4	G5	G6	G7	Wi
G1	1.0000	1.4918	1.8221	2.2255	2.7183	3.3201	4.0552	0.2746
G2	0.6703	1.0000	1.4918	1.8221	2.2255	2.7183	3.3201	0.2123
G3	0.5488	0.6703	1.0000	1.2214	1.4918	1.8221	2.2255	0.1464
G4	0.4493	0.5488	0.8187	1.0000	1.4918	1.8221	1.8221	0.1269
G5	0.3679	0.4493	0.6703	0.6703	1.0000	1.2214	1.4918	0.0954
G6	0.3012	0.3679	0.5488	0.5488	0.8187	1.0000	1.4918	0.0804
G7	0.2466	0.3012	0.4493	0.5488	0.6703	0.6703	1.0000	0.0639

PROGRAM CODE

Model:

```
min = d 13;

x11+x12+x13+x14<=300;
x21+x22+x23+x24<=200;
x31+x32+x33+x34<=400;
x11+x21+x31+d1_-d1=200;
x12+x22+x32+d2_-d2=100;
x13+x23+x33+d3_-d3=450;
x14+x24+x34+d4_-d4=250;
x31+d5_-d5=100;
x11+x21+x31+d6_-d6=160;
x12+x22+x32+d7_-d7=80;
x13+x23+x33+d8_-d8=360;
x14+x24+x34+d9_-d9=200;
5*x11+2*x12+6*x13+7*x14+3*x21+5*x22+4*x23+6*x24+
4*x31+5*x32+2*x33+3*x34+d10_-d10=3245;
x24+d11_-d11=0;x11+x21+x31-0.4444444444444444*x13-
0.4444444444444444*x23-
0.4444444444444444*x33+d12_-d12=0;
```

```
5*x11+2*x12+6*x13+7*x14+3*x21+5*x22+4*x23+6*x24+
4*x31+5*x32+2*x33+3*x34+d13_-d13=2950;
d4_ = 0;
d5_ = 0;
d6_+d7_+d8_+d9_ = 0;
d10 = 0;
d11 = 0;
d12_+d12 = 0;
end
```

Global optimal solution found at iteration: 0
Objective value: 220.0000

Variable	Value	Reduced Cost
D13	220.0000	0.000000
X11	0.000000	0.000000
X12	80.00000	0.000000
X13	0.000000	0.000000
X14	170.0000	0.000000
X21	60.00000	0.000000
X22	0.000000	5.000000
X23	140.0000	0.000000
X24	0.000000	1.000000
X31	100.0000	0.000000
X32	0.000000	7.000000
X33	220.0000	0.000000
X34	80.00000	0.000000
D1_	40.00000	0.000000
D1	0.000000	0.000000
D2_	20.00000	0.000000
D2	0.000000	0.000000
D3_	90.00000	0.000000
D3	0.000000	0.000000
D4_	0.000000	0.000000
D4	0.000000	7.000000
D5_	0.000000	0.000000
D5	0.000000	3.000000
D6_	0.000000	1.000000
D6	0.000000	5.000000
D7_	0.000000	4.000000
D7	0.000000	2.000000
D8_	0.000000	0.000000
D8	0.000000	6.000000
D9_	0.000000	6.000000
D9	50.00000	0.000000
D10_	75.00000	0.000000
D10	0.000000	0.000000
D11_	0.000000	0.000000
D11	0.000000	0.000000
D12_	0.000000	0.000000
D12	0.000000	0.000000
D13_	0.000000	1.000000

Row	slack or surplus	dual price
1	220.0000	-1.000000
2	50.00000	0.000000
3	0.000000	2.000000
4	0.000000	4.000000
5	0.000000	0.000000
6	0.000000	0.000000
7	0.000000	0.000000
8	0.000000	-7.000000
9	0.000000	-3.000000
10	0.000000	-5.000000
11	0.000000	-2.000000
12	0.000000	-6.000000
13	0.000000	0.000000
14	0.000000	0.000000
15	0.000000	0.000000
16	0.000000	0.000000
17	0.000000	1.000000
18	0.000000	7.000000
19	0.000000	3.000000
20	0.000000	6.000000
21	0.000000	0.000000
22	0.000000	0.000000
23	0.000000	0.000000

Model:

```
min=d12_+d12;

x11+x12+x13+x14<=300;
x21+x22+x23+x24<=200;
x31+x32+x33+x34<=400;
x11+x21+x31+d1_-d1=200;
x12+x22+x32+d2_-d2=100;
x13+x23+x33+d3_-d3=450;
x14+x24+x34+d4_-d4=250;
x31+d5_-d5=100;
x11+x21+x31+d6_-d6=160;
x12+x22+x32+d7_-d7=80;
x13+x23+x33+d8_-d8=360;
x14+x24+x34+d9_-d9=200;
5*x11+2*x12+6*x13+7*x14+3*x21+5*x22+4*x23+6*x24+
4*x31+5*x32+2*x33+3*x34+d10_-d10=3245;
x24+d11_-d11=0;
x11+x21+x31-0.4444444444444444*x13-
0.4444444444444444*x23-
0.4444444444444444*x33+d12_-d12=0;
5*x11+2*x12+6*x13+7*x14+3*x21+5*x22+4*x23+6*x24+
4*x31+5*x32+2*x33+3*x34+d13_-d13=2950;
d4_ = 0;
d5_ = 0;
d6_+d7_+d8_+d9_ = 0;
d10 = 0;
```

d11 = 0;
end

Global optimal solution found at iteration: 34
Objective value: 0.000000
Variable Value Reduced Cost
D12_ 0.000000 1.000000
D12 0.000000 1.000000
X11 0.000000 0.000000
X12 80.00000 0.000000
X13 0.000000 0.000000
X14 207.5000 0.000000
X21 60.00000 0.000000
X22 0.000000 0.000000
X23 102.5000 0.000000
X24 0.000000 0.000000
X31 100.0000 0.000000
X32 0.000000 0.000000
X33 257.5000 0.000000
X34 42.50000 0.000000
D1_ 40.00000 0.000000
D1 0.000000 0.000000
D2_ 20.00000 0.000000
D2 0.000000 0.000000
D3_ 90.00000 0.000000
D3 0.000000 0.000000
D4_ 0.000000 0.000000
D4 0.000000 0.000000
D5_ 0.000000 0.000000
D5 0.000000 0.000000
D6_ 0.000000 0.000000
D6 0.000000 0.000000
D7_ 0.000000 0.000000
D7 0.000000 0.000000
D8_ 0.000000 0.000000
D8 0.000000 0.000000
D9_ 0.000000 0.000000
D9 50.00000 0.000000
D10_ 0.000000 0.000000
D10 0.000000 0.000000
D11_ 0.000000 0.000000
D11 0.000000 0.000000
D13_ 0.000000 0.000000
D13 295.0000 0.000000

Row slack or surplus dual price
1 0.000000 -1.000000
2 12.50000 0.000000
3 37.50000 0.000000
4 0.000000 0.000000
5 0.000000 0.000000
6 0.000000 0.000000
7 0.000000 0.000000
8 0.000000 0.000000
9 0.000000 0.000000
10 0.000000 0.000000
11 0.000000 0.000000
12 0.000000 0.000000
13 0.000000 0.000000
14 0.000000 0.000000
15 0.000000 0.000000
16 0.000000 0.000000
17 0.000000 0.000000
18 0.000000 0.000000
19 0.000000 0.000000
20 0.000000 0.000000
21 0.000000 0.000000
22 0.000000 0.000000

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