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Control for Track of Swarm Systems

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Abstract: In this paper, we develop an algorithm for tracking the center of swarm systems to a desired trajectory by using the sliding-mode control method. The algorithm is robust with respect to system perturbations and external disturbance. Simulation further shows the effectiveness very well.

Key words: Swarm systems, center of swarm, sliding-mode control, dynamic trajectory

INTRODUCTION

Collective behavior in swarms of biological entities has long been observed in nature. Examples of swarms include flocks of birds, schools of fish, herds of animals and colonies of bacteria. Such a collective behavior has certain advantages such as threatening predators and increasing the chance of finding food.

Given the local rules of agents, how we control the collective behavior? Olfati-Saber (2006) and Su *et al.* (2009) presented the problem of multi-agent flocking. Hong *et al.* (2008) studied a multi-agent consensus problem with an active leader and variable interconnection topology.

Sliding-mode control is easy to come true. Sliding-mode manifold has full robustness for systemic perturbation and external disturbance. Gazi (2005) use sliding mode control theory to force vehicle dynamics motion to obey the dynamics of the swarm; Yao *et al.* (2007) presented a stable and decentralized control strategy for multiagent systems to capture a moving target in a specific formation; Mao *et al.* (2010) used sliding-mode control method to control the swarm agents to an expectant trajectory.

In this article we consider a swarm model with system parameter perturbations or external disturbance and develop an algorithm for tracking the center of swarm systems to a desired dynamic trajectory by controlling H agents using the method of sliding-mode control. Simulation further shows the effectiveness very well.

MODEL OF SWARM SYSTEMS

We consider a swarm of M+H individuals (members) in an n-dimensional Euclidean space and model the

individuals as points and ignore their dimensions. The position of member i of the swarm is described by $x^i \in \mathbb{R}^n$. We assume synchronous motion and no time delays, following the model of Gazi and Passino (2003) the equation of motion of individual i is given by:

$$\dot{x}^i = \sum_{j=1, j \neq i}^{M+H} g(x^i - x^j), i=1,2,\dots,M+H \quad (1)$$

We consider $g(\cdot)$ as:

$$g(y) = -y \left(a - b \exp\left(-\frac{\|y\|^2}{c}\right) \right) \quad (2)$$

where a, b and c are positive constants such that $b > a$ and $\|y\|$ is the Euclidean norm given by:

$$\|y\| = \sqrt{y^T y}$$

Suppose that $f^i(t)$ ($i = 1, 2, \dots, M+H$) represents the sum of system parameter perturbations and external disturbance of agent i and $\|f^i(t)\| \leq \bar{f}$, where $\bar{f} < \infty$ is positive constant. $x_d = [x_1(t), x_2(t), \dots, x_n(t)]^T$ represents the track of the center aim and there are positive constant Q such that for any time t, we have:

$$\|x_d(t)\| \leq Q$$

Suppose the agent M+1, M+2, ..., M+H as controlled agents, then the controlled systems can be expressed as:

$$\begin{cases} \dot{x}^i = \sum_{j=1, j \neq i}^{M+H} g(x^i - x^j) + f^i(t), & i=1,2,\dots,M \\ \dot{x}^{M+h} = \sum_{j=1, j \neq M+h}^{M+H} g(x^{M+h} - x^j) + f^{M+h}(t) + u_h, & h=1,2,\dots,H \end{cases} \quad (3)$$

DESIGN FOR CONTROL LAW

Now, we would like to design each of the control inputs u_k such that the center of swarm will be transferred effectively to x_d .

First, we define the n-dimensional sliding manifold for agent M+h as:

$$s^{M+h} = x^{M+h} - \frac{1}{H} \left[(M+H)x_d - \sum_{i=1}^M x^i \right] - \left(\frac{H+1}{2} - h \right) d, h=1,2,\dots,H \quad (4)$$

where, $d = [d_1, d_2, \dots, d_n]^T$, d_1, d_2, \dots, d_n is constant.

Then, we design the control inputs u_k such as to enforce the occurrence of sliding mode. A sufficient condition for sliding mode to occur is given by (Decarlo *et al.*, 1988):

$$(s^{M+h})^T \dot{s}^{M+h} < 0 \quad (5)$$

which also guarantees that the sliding manifold is asymptotically reached. Later we will choose u_k which will actually guarantee finite time reaching of the sliding manifold. Differentiating the sliding manifold equation we obtain:

$$\begin{aligned} \dot{s}^{M+h} &= \dot{x}^{M+h} - \frac{1}{H} \left[(M+H)\dot{x}_d - \sum_{i=1}^M \dot{x}^i \right] \\ &= \sum_{j=1, j \neq M+h}^{M+H} g(x^{M+h} - x^j) + f^{M+h}(t) + u_h - \\ &\quad \frac{1}{H} \left[(M+H)\dot{x}_d - \sum_{i=1}^M \sum_{j=1, j \neq i}^{M+H} g(x^i - x^j) - \sum_{i=1}^M f^i(t) \right] \end{aligned} \quad (6)$$

Substituting it in Eq. 5, the condition for occurrence of sliding mode becomes:

$$\begin{aligned} &(s^{M+h})^T \left\{ \sum_{j=1, j \neq M+h}^{M+H} g(x^{M+h} - x^j) + f^{M+h}(t) + u_h - \right. \\ &\quad \left. \frac{1}{H} \left[(M+H)\dot{x}_d - \sum_{i=1}^M \sum_{j=1, j \neq i}^{M+H} g(x^i - x^j) - \sum_{i=1}^M f^i(t) \right] \right\} < 0 \end{aligned}$$

One can choose the control input u_k such that $(s^{M+h})^T \dot{s}^{M+h} < 0$ is satisfied. In particular, by choosing:

$$u_h = -u_0 \operatorname{sgn}(s^{M+h}) - \sum_{j=1, j \neq M+h}^{M+H} g(x^{M+h} - x^j) + \frac{1}{H} \left[(M+H)\dot{x}_d - \sum_{i=1}^M \sum_{j=1, j \neq i}^{M+H} g(x^i - x^j) \right] \quad (7)$$

where, $\operatorname{sgn}(s^{M+h}) = (\operatorname{sgn}(s_1^{M+h}), \operatorname{sgn}(s_2^{M+h}), \dots, \operatorname{sgn}(s_n^{M+h}))^T$ and $\operatorname{sgn}(y)$ is sign function:

$$\operatorname{sgn}(y) = \begin{cases} 1, & y > 0 \\ -1, & y < 0 \end{cases} (y \in \mathbb{R})$$

we obtain:

$$(s^{M+h})^T \dot{s}^{M+h} = (s^{M+h})^T [-u_0 \operatorname{sgn}(s^{M+h}) + f^{M+h}(t) + \frac{1}{H} \sum_{i=1}^M f^i(t)]$$

$$< -\|s^{M+h}\| \left[u_0 - \bar{f} - \frac{1}{H} \sum_{i=1}^M \bar{f}^i \right] < -\|s^{M+h}\| \left[u_0 - \frac{H+M}{H} \bar{f} \right]$$

Then, by choosing the gain u_0 of control input as:

$$u_0 > \frac{H+M}{H} \bar{f} + \varepsilon, \text{ for any } \varepsilon > 0$$

we can guarantee that $(s^{M+h})^T \dot{s}^{M+h} < -\varepsilon \|s^{M+h}\|$ is satisfied and that sliding mode occurs. Once the sliding manifold $s^{M+h} = 0$ is reached, the system remains on that manifold for all time. Choose the Lyapunov function as:

$$V^{M+h} = \frac{1}{2} (s^{M+h})^T s^{M+h}$$

and note also that the above inequality implies that:

$$\dot{V} \leq -\varepsilon \sqrt{V}$$

Solve it, we can obtain that the sliding manifold of agent M+h is reached at time:

$$T_{M+h} = \frac{2\sqrt{V^{M+h}(0)}}{\varepsilon} = \frac{2\|s^{M+h}(0)\|}{\varepsilon}$$

Theorem: Adopt the control inputs Eq. 7 to agents M+1, M+2, ..., M+H, the center of members will transfer to x_d in finite time:

$$t_0 = \max_{1 \leq k \leq H} \{T_{M+k}\}$$

Proof: Once all the agents M+1, M+2, ..., M+H reach their sliding manifolds $s^{M+h} = 0$ we have:

$$x^{M+h} = \frac{1}{H} \left[(M+H)x_d - \sum_{i=1}^M x^i \right] + \left(\frac{H+1}{2} - h \right) d, h=1,2,\dots,H$$

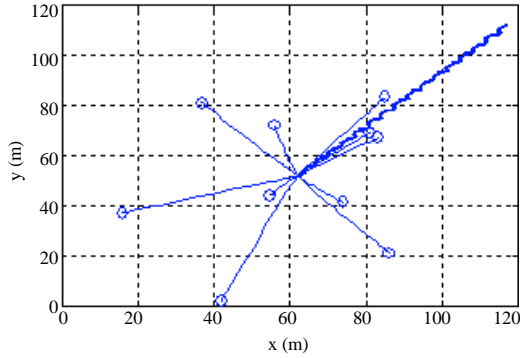


Fig. 1: The trajectories with disturbance

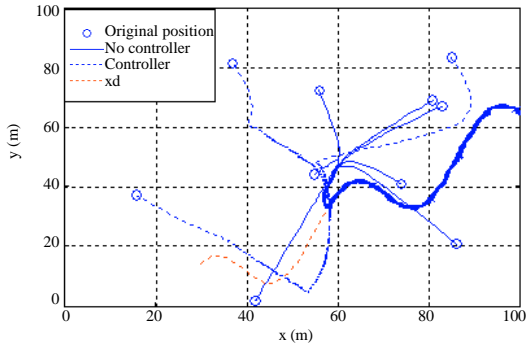


Fig. 2: The trajectories with controlled agents

then:

$$\sum_{h=1}^H x^{M+h} = \sum_{h=1}^H \frac{1}{H} \left[(M+H)x_d - \sum_{i=1}^M x^i \right] + \sum_{h=1}^H \left(\frac{H+1}{2} \right) d - \sum_{h=1}^H hd$$

$$= \left[(M+H)x_d - \sum_{i=1}^M x^i \right] + \frac{H(H+1)}{2} d - \frac{H(H+1)}{2} d = (M+H)x_d - \sum_{i=1}^M x^i$$

The center is:

$$\bar{x} = \frac{1}{M+H} \left(\sum_{i=1}^M x^i + \sum_{h=1}^H x^{M+h} \right) = x_d.$$

So the center of members will transfer to x_d in the finite time:

$$t_0 = \max_{1 \leq h \leq H} \{T_{M+h}\}$$

SIMULATION

In these simulations, Let $a = 3, b = 8$ and $c = 0.2$ in (2). Choose two-dimensional space as the practical space and the original positions of 10 agents are obtained randomly. The sign o represents the original position.

Figure 1 shows the trajectories of the agents with external disturbance and without control. Figure 2 shows the trajectories of the agents with three controlled agents. The red broken line represents x_d , the other broken lines represent the trajectories of controlled agents and full lines represent the trajectories of no control members. It shows that the center of swarm systems will transfer effectively to x_d by controlling three agents using the control inputs (Eq. 7).

CONCLUSION

In this paper, we present a procedure based on sliding mode control theory, which can control the center of swarm systems to a desired dynamic trajectory by controlling a few agents. The algorithm is robust with respect to system perturbations and external disturbance. It solves well the stabilization problem of motion tracking for complex systems. Simulation further shows the effectiveness very well.

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