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Research on Dynamic Pricing Between High Speed Rail and Air Transport Under the Influence of Induced Passenger Flow

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Abstract: This study first briefly describes the application of game theory and Changes arising from the combination of ticket price changes in passenger traffic to the dynamic pricing between high speed rail and air, selects Stackelberg model to describe the process of dynamic pricing process, then establishes multilayer planning dynamic pricing model between high speed rail and air transport with the thinking of bilevel programming model and calculated the constructed model ions with practical examples based on the heuristic algorithm of sensitivity analysis. With the analysis of the competition game pricing process between high speed rail and air transport, a conclusion under the influence of passenger has been drawn, that is high speed rail tends to balance ticket prices range in [38.8¥,39¥] and air transport tends to balance the ticket price range in [55.5¥,56¥]. Finally, this study systematically analyzes the passenger flow and the changes of profits in the game process of the pricing competition between high speed rail and air transport.

Key words: Induced passenger, game, multi-level programming, sensitivity analysis

INTRODUCTION

From current relevant researches perspective, highway transportation is the main competitor of railway transportation (Li *et al.*, 2005; Chen and Gao, 2005); however, the average speed of railway can reach 250 km h^{-1} which is highly beyond that of highway, therefore, the pricing of HSR tickets must be analyzed with air as the new competitor. The changing process of railway ticket prices is usually analyzed on the premise of a fixed highway ticket price. Such a premise goes against the regulation of ticket prices according to both sides' ticket price change and doesn't take the consideration of induced passenger flow generated by ticket price change of HSR and air, so the conclusion is incomprehensive (Chen and Gao, 2003).

Based on relevant researches, this paper analyses the ticket pricing under the condition that air is the competitor of HSR and puts full play of the influence of induced passenger flow resulted by the price change. And with the application of game theory in the ticket pricing of HSR and air, according to the description of both sides' ticket pricing in the dynamic games of complete information theory, a dynamic pricing model has been established in this study and the dynamic pricing region has been worked out by sensitivity analysis to provide reference for ticket pricing.

CALCULATION OF INDUCED PASSENGER FLOW BY PRICE CHANGE

The emerge of induced passenger flow is a complicated issue as it completely depends on a lot of social factors such as the condition of the vehicle, on whether the passenger can afford the ticket price and passengers' psychology (Bard, 1983). When passengers make their choice of transportation, they will consider the ticket price, spent time on the journey and convenience occurring before and after the whole journey. Thus, it is necessary for the transportation sectors to lower ticket price and improve the quality of transportation to meet passengers' need as possible as they can. However, to lower the ticket price and improve the quality of transportation will significantly reduce the profit of transportation sectors which will come into conflict with passengers' need.

The introduction of passenger traffic calculated the gravity model to seek to attract the rate of change of K (Ferrari, 1995) and then pushed to the next passenger ticket price changes induced rate k. gravity model for volume can be expressed as:

$$Q_{ij} \frac{KM_iM_j}{U_{ij}^{\beta}}$$

The induced rate k due to the change of ticket price is:

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$$k=(K_d-1)\times 100\%=\left[\left(\frac{U_{ij}^{(0)}}{U_{ij}^{(1)}}\right)^{\beta}-1\right]\times 100\%$$

DYNAMIC GAMES OF PRICING BETWEEN HSR AND AIR

Game refers to the combination of some interdependent decision-makings which influence each other and their results. The theory that studies the reasonable decision-making and interdependent decision-making bodies that influence each other and the equilibrium produced is called game theory. We assume that HSR and air know each other and the changing of ticket price is a dynamic gaming process, so we can use a kind of dynamic game of complete information model-Stackelberg's competition model to describe the competition of ticket pricing between HSR and air (Si and Gao, 2007). HSR is a new participant in the high-speed passenger transportation network and if its ticket price is $m_1^{(0)}$, the ticket price of air will be set at $m_1^{(0)}$; then the HSR sector will change the price at $m_1^{(0)}$ thereafter and air at $m_2^{(0)}$; after several rounds, the two sides will set their ticket price at $m_1^{(n)}, m_2^{(n)}$, respectively.

During the process of price game, based on HSR's ticket price m_1, air sector set its ticket price at m_2 according to m_1, therefore, the pricing strategy of air is a function: $s_2 = Q_1 \rightarrow Q_2$ (in this function, $Q_1 = (0, +\infty)$ means the ticket price region of HSR and $Q_2 = (0, +\infty)$ means that of air).

DYNAMIC PRICING MODEL OF HSR AND AIR UNDER THE INFLUENCE OF INDUCED PASSENGERS FLOW

Similar to as mentioned earlier, the price competition between HSR and air can be considered as a process of dynamic game consisting of several static games. Therefore, it is crucial premise that ticket price of air must remain the same in terms of how to set the ticket price of HSR should be deeply taken into account. The decision-making department of HSR can be as the leader while the choice of passengers or the traffic flow of HSR and air could be the follower. According to the cost of transport and expectation of profit, the department set the ticket price which will influence passengers' choice on one hand. On the other hand, passengers will choose their own way of transportation according to their own financial status, personal need and preference as well as others. The relationship can be illustrated in the following model--the linear bi-level programming:

$$\begin{aligned}&\max F(x, y(x))\\ &\text{s.t.}\quad G(x, y(x))\leq 0\end{aligned} \tag{1}$$

where, y (x) is obtained by lower-level programming as follows:

$$\begin{aligned}&\min f(x, y)\\ &\text{s.t.}\quad g(x, y)\leq 0\end{aligned} \tag{2}$$

Function F is the objective function of the first level programming which refers to the profit of HSR sector and x is the decision variable of the first level programming which refers to the ticket price of HSR; f is the objective function of the second level programming which stands for the general expenses of passengers and y is the decision variable of the second level, y (x) is the decision variable function of the first level programming (Fan and Zhu, 2004). The function of HSR's profit F can be expressed by the cost and earning of transporting, so the first level programming can be illustrated as follows:

$$\begin{aligned}&\max F = x^{HSR}(p^{HSR})\times(p^{HSR}-c^{HSR})\\ &\text{s.t.}\quad p^{min}\leq p^{HSR}\leq p^{max}\end{aligned} \tag{3}$$

where, c^{HSR} that means the transportation costs is assumed to be constant. P^{min}, p^{max} mean the lower bound and the upper bound of ticket price that operation can allow separately. v^{HSR} (p^{HSR}) is given by the lower plan. Suppose the change of passenger traffic is a continuous process, the lower-level can be expressed by:

$$\begin{aligned}&\min Z(q)=\sum_n\int_0^{v_{HSR}} f(x)dx\\ &\text{s.t.}\quad \sum_n (v_{HSR}+v_{air}) = Q + v_{HSR}^{induced}, v_{HSR}\geq 0\end{aligned} \tag{4}$$

where, v_{HSR} that we use the function of logit to express means the traffic of high speed railway, f(x) is the generalized cost function. The generalized cost function always takes the forms of power function or logarithmic function to express. This text takes the form of power function as follows:

$$f(v^{HSR}) = a(v^{HSR})^b\text{-}V^{HSR}$$

where, a, b in the formula are parameters, V^{HSR} means that the high speed railway, a way of passenger transportation, can observe the utility value or the degree of attraction to the passengers. The higher the utility value is, the bigger profit that passengers can achieve in the mode of transport.

Form the formula of 1, 2, 3 and 4, we can formulate the price optimization models of high speed railway when the ticket price of air is fixed, that is:

$$\begin{cases} \max F = v^{HSR}(p^{HSR}) \times (p^{HSR} - c^{HSR}) \\ \text{s.t. } p^{min} \le p^{HSR} \le p^{min} \\ \text{where } v^{HSR}(p^{HSR}) \text{ is obtained by solving lower level programming} \\ \min Z(v) = \sum_n \int_0^{v_{HSR}} a(v^{HSR})^b - (a_1 p^{HSR} + a_2 t^{HSR} + a_3 c^{HSR} + \varepsilon_i) dx \\ \text{s.t. } \sum_n (V_{HSR} + v_{air}) = Q + v_{HSR}^{induced}, v_{HSR} \ge 0 \end{cases} \tag{5}$$

Also we can formulate the price optimization models of air when the ticket price of HSR is fixed, that is:

$$\begin{cases} \max F = v^{air}(p^{air}) \times (p^{air} - c^{air}) \\ \text{s.t. } p^{min} \le p^{air} \le p^{min} \\ \text{where } v^{air}(p^{air}) \text{ is obtained by solving lower level programming} \\ \min Z(v) = \sum_n \int_0^{v_{air}} a(v^{air})^b - (a_1 p^{air} + a_2 t^{air} + a_3 c^{air} + \varepsilon_i) dx \\ \text{s.t. } \sum_n (V_{HSR} + v_{air}) = Q + v_{air}^{induced}, v_{air} \ge 0 \end{cases} \tag{6}$$

SOLUTION TO THE MULTI-LEVEL PROGRAMMING WITH SENSITIVITY ANALYSIS

The solution to Eq. 1 and 2 that stand for the multi-level programming can help set the reasonable ticket price of HSR and air respectively to realize the decision-making departments' goal. Since the multi-level programming model is made up by two bilevel programming Eq. 1 and 2, the first step is to solve these two equations. It is very difficult to get the best global solution for the problem of multilevel programming is nonconvex and there is not an accurate solution to polynomial to solve the problem of bilevel programming which is a nonfinite polynomial. Many heuristic algorithms such as the Simulated Annealing Algorithm, Artificial Neural Network, Tabu Search and Ant Algorithm can be used to simplify the process.

Obviously, it is the key to solve the problem of bilevel programming that figures out the concrete form of reaction function v (p) (Shuai and Sun, 2009); the sensitivity analysis can be used to find out the derivative relationship between the volume of passenger and ticket price of some transportation in the model of multiple transportations; then Taylor's expansion can be used to simplify v (p) so as to improve the efficiency to solve bi-level programming. This is the SAB with sensitivity analysis (Yang and Lam, 1996). SAB is applied in this paper and the steps are as follows:

Step 1: Assume the ticket price of HSR and air is:

$$\{p_{HSR}^{k(j)}, p_{air}^{k(i)}\}$$

and i = 0, j = 0, k = 0.

Step 2: If the initial ticket price is:

$$\{p_{HSR}^{k(j)}, p_{air}^{k(i)}\}$$

with the Eq. 5, the equilibrium value of passengers' choice of transportation can be worked out: v_{air}.

Step 3: With sensitivity analysis, the derivative of ticket price by the passenger flow of air will be got; and with:

$$v^{air}(p_{air}) = v^{air}(p_{air}^{k(0)}) + \frac{\partial v^{air}}{p_{air}}\left(p_{air} - p_{air}^{k(0)}\right)$$

the similar form of response function $v(p_{air}^{k(0)})$ can be worked out.

Step 4: Put response function $v(p_{air}^{k(0)})$ into Eq. 1, first level of programming, the new ticket price of air will be $\{p_{air}^{k(0)}\}$ and its passenger flow will be $Q + v_{air}^{induced(1)}$.

Step 5: To evaluate the solution: if:

$$\left|p_{air}^{k(i+1)} - p_{air}^{k(i)}\right| \le \delta_1$$

(δ_1 is the iteration accuracy of air pricing competition), the optimum ticket price of air will be $p_{air}^{k(i^*)}$ and the passenger flow will be:

$$Q + \sum_{n=1}^{i} v_{air}^{induced(n)}$$

Otherwise, make I = i+1 and return to step 2 to get the solution.

Step 6: If the ticket price of two sides is:

$$\left\{p_{HSR}^{k(0)}, p_{air}^{k(i^*)}\right\}$$

and the volume of passenger is:

$$Q+\sum_{n=1}^{i} v_{air}^{induced(n)}$$

with the Eq. 5, the equilibrium value of passengers' choice of transportation can be worked out v_{HSR}.

CASE STUDY

As regards to HSR, the ticket prices vary with different lines. This is the case in air and at the same time, the price may differ because the discount is different when passengers book at different time. So to make the pricing game universally applicable, the average price at every hundred kilometers will be used in this study. To simplify the arithmetic process, the transporting time is the time running every hundred kilometers and the total number of passengers is 10,000. The transportation cost remains the same regardless of the number of passengers and in meanwhile, some of parameters are from other relevant studies and some data observed have been corrected. Model calibration coefficients in Table 1.

Table 1: Parameters table

Parameter	p/¥	t/hour	c	a	b	α_1	α_2	α_3
HSR	50	0.4	3	1.2	0.4	5.0	0.5	3
air	75	0.13	5	1.2	0.4	2.4	0.5	5

From Table 2, after 10 times' iteration, the dynamic ticket pricing of HSR and air tends to Nash equilibrium. However, it is difficult to get a best global solution with the sensitivity analysis used in multi-level programming since the bi-level programming is nonconvex. The gaming between two sides is actually a competition of ticket price. They both lower their ticket price down to attract more passengers. Similarly, after 10 times' iteration, the ticket price region of HSR tends to [38.8¥, 39¥] and that of air [55.5¥, 56¥].

The data in Fig. 1 shows how induced passenger flow, the volume of passenger of two sides, profit, ticket price change in every iteration. The detailed analysis to Fig. 1 can show that with the ticket price changing, the

Table 2: Calculations of HSR and air transport dynamic pricing

Iteration time	Passenger flow of HSR	Passenger flow of air	Profit of HSR	Profit of air	Dynamic pricing of HSR	Dynamic pricing of air	Influence of passenger
1	6806	5482	1392955	229412	39.535	61.8483	2288
2	6746	6101	135289	235361	40.0548	56.9384	2847
3	6779	6393	126914	229362	38.7217	55.8772	3172
4	6768	6407	128666	228066	38.9815	55.5964	3175
5	6773	6431	127509	228613	38.8262	55.5487	3204
6	6771	6433	127789	228582	38.8730	55.5328	3204
7	6772	6436	127669	228680	38.8525	55.5314	3208
8	6770	6437	127680	228708	38.8597	55.5303	3207
9	6771	6437	127689	228709	38.8568	55.5304	3208
10	6771	6437	127686	228708	38.8579	55.5303	3208

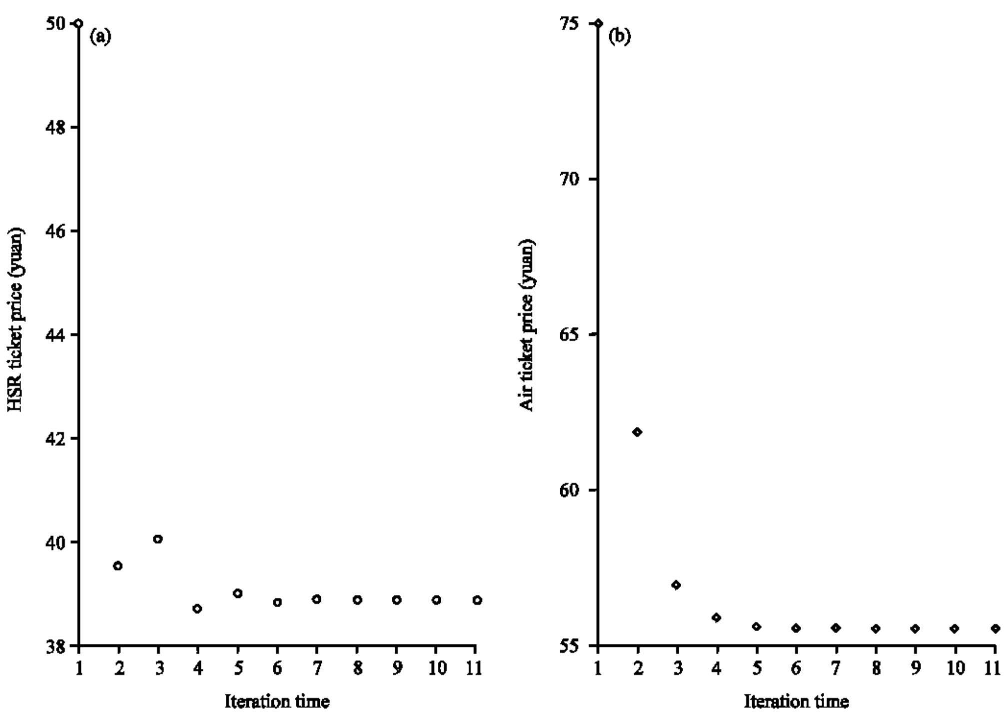

Fig. 1(a-b): HSR and air transport dynamic pricing chart; (a) Dynamic pricing process of HSR and (b) Dynamic pricing process of HSR number of passengers and the profit of two sides fluctuate slightly instead of changing linearly. In the first iteration, the number of passengers of HSR is 6,806 and its profit reaches 1,392,955¥ which reaches the peak; whereas the number of passengers of air is 5,482 and its profit is 229,412¥ which is the minimum. Afterwards, the price of air began to lower down to enlarge its market share. So in the second round of game, the number of passengers of air rises to 6,101 and its profit increases to peak 235, 361¥.

CONCLUSION

The study does demonstration which is on the basis of Game Theory, in relation to the ticket pricing competition between HSR and air by drawing on Stackelberg's competition model. A dynamic ticket pricing model of competition between HSR and air under the influence of induced passenger flow is worked out as integrated with the analysis on the variation of induced passenger flow with the change of ticket price. According to the analysis on the model of multi-level programming and the solution to the model by sensitivity analysis with SAB, it is apparent to get the conclusion that the balanced ticket price of HSR under the influence of induced passenger flow is [38.8¥, 39¥] and that of air is [55.5¥, 56¥].

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