

<http://ansinet.com/itj>

ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSI*net*

Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Losses of Small Decision-making Problems

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Abstract: In this paper, we extend the SAM and explore SDM problems, in the situation that the DMs should estimate their losses not payoffs when they make decisions under risk. The methodologies are motivated and illustrated by a data example. We also extend the consideration of losses into other representative research of SDM problems.

Key words: SDM problem, search-assessment model, binary choice, loss, extension of SDM

INTRODUCTION

Experimentally we can define a problem as a SDM problem when it satisfies the following conditions, first, in the situation in which the information available to DMs is limited to feedback concerning the outcomes of their previous decisions made in similar situations in the past, that is, the DMs do not have objective prior information as to payoff or loss distribution; second, the Expected Value (EV) that may be fairly small of the alternatives tend to be similar; third, in the similar situations the DMs face the same choice repeatedly and need to make decisions for many times.

Kahneman and Tversky (1979) experimental research on these problems presented a critique of Expected Utility Theory (EUT) (Von Neumann and Morgenstern, 1944) as a descriptive model of decision making under risk and they proposed two particular hypotheses of decision weights which underweight all probabilities less than unity. In this paper, we believe that the DMs' risk attitude can not explain the exploration tendency in SDM problems. The DMs' behaviors can be easily tested by the Search-Assessment model by Fujikawa and Oda (2007) the model insisted that in SDM problems the tendency to select best reply to the past and misestimation of payoff distribution can lead to robust deviations from EV maximization, we consider that this model is rational. Barron and Erev (2003) have also reported their results about these problems and their results were rather different from Kahneman and Tversky (1979) but they also proved that the EUT could not explain the problems of decision making under risk well.

We are interest in the situations in which the DMs should estimate their loss when they make decisions under risk. In this paper, we start with a review about

Fujikawa and Oda (2007) research on the SDM problem and the Search-Assessment model. Then we extend the problem to the situation in which we are interest. We reveal a motivated and illustrated analysis of this problem; it is also the key of this paper.

Kahneman and Tversky (1979) experimental research on "Big Decision-Making" (BDM) problems and Barron and Erev, 2003 experimental research on SDM problems are reviewed as well. BDM problems and SDM problems are both about the decision making under risk, the reason why they have different names because of the probability of the risk that if the DMs make the choice which has a higher EV in a situation, the probability of the BDM problems is rather higher than in the SDM problems. The experiment included the treatment in which the choice situation was repeated many times, the alternatives had similar and small EV and the distribution of lotteries was unknown to the subjects. We extend the experiments to which can be used in the situations in which the losses should be estimated.

FUJIKAWA AND ODA (2007) SEARCH-ASSESSMENT MODEL

Decision making under risk can be viewed as a choice between prospects or gambles. A prospects is a contract that yields outcome x_i with probability p_i , namely, (x_i, p_i) , $(x_2, p_2), \dots, (x_n, p_n)$ where $p_1 + p_2 + \dots + p_n = 1$. We can describe the problems generally as, Problem: Choose between:

- H: $x(\theta p); 0(1-\theta p)$
- L: $y(\theta); 0(1-\theta)$,

where, $p, \theta \in (0, 1)$.

Fujikawa and Oda (2007) claimed that it was not safe to make direct comparison between the results by Barron and Erev (2003), Fujikawa and Oda (2007) as by Barron and Erev (2003), there were only hundreds of trials; it was ambiguous whether the subjects could estimate each alternative correctly. They pointed out that the DMs were likely to misestimate the payoff structure of binary choice problems by Fujikawa and Oda (2007) and they proposed the search-assessment model to demonstrate that the probability of such misestimation is rather high. They proposed that:

Problem 1: Choose between:

- H: $x(p)$; $0(1-p)$
- L: $1(1)$

where, $p \in (0, 1)$ and $px > 1$.

The posterior average points of H (posteriorH) was defined as the points the DM has earned from H after it was chosen m times and $P(Hm)$ was denoted as the probability of the event that the DM's posteriorH became greater than or equal to one that was EV of L (EVL) after m selections of H:

$$P(Hm) = \sum_{\text{all } k: \frac{kx}{m} \geq 1} C_m^k p^k (1-p)^{m-k} = \sum_{k=\lceil \frac{m}{x} \rceil}^m C_m^k p^k (1-p)^{m-k}$$

Problem 2: Choose between:

- H: $x(\theta p)$; $0(1-\theta p)$
- L: $1(\theta)$; $0(1-\theta)$

where, $p, \theta \in (0, 1)$ and x such that $\theta px > \theta$.

$P(Ln)$ was defined as the probability of the event that posteriorH becomes greater than or equal to posteriorL after the DM has chosen H and L m and n times in Problem E respectively:

$$P(Ln) = \sum_{k=0}^m \left[C_m^k (\theta p)^k (1-\theta p)^{m-k} \times \sum_{j=0}^{\lceil \frac{nkx}{m} \rceil} C_n^j (\theta)^j (1-\theta)^{n-j} \right]$$

The search-assessment model offered better explanation of Barron and Erev (2003) results, rather than the assertion of under weighting of rare events they insisted and it asserted that the subjects were very likely to misestimate the payoff structure with only 400 trials.

EXTENSION OF SDM PROBLEM

The representative research on SDM problems above focused on the DMs' payoffs when they should make decisions of binary choice. In this paper, we extend the situations in which the DMs should estimate their loss distribution of binary choice such that we have the Extension of SDM problem. This is also the key of our paper.

Fujikawa and Oda, 2007 have proposed that the exploration tendency in SDM problems seemed to be explained by the search-assessment model, rather than each DM's risk attitude. The model has insisted that in SDM problems the tendency to select best reply to the past and misestimation of payoff distribution can lead to robust deviations from EV maximization. We conduct Problem to examine our extension:

Problem 3: Choose between:

- H: a with certainty;
- L: b with probability P ; 0 with probability $(1-p)$

where, $p \in (0, 1)$ and $a > bp$. The DM loses a points for certain by choosing H; DM loses b points with probability p and zero point with probability $(1-p)$. Note that EV of H is greater than EV of L.

We define $P(Lm)$ as the probability of the event that the average points the DM has lost from L after it was chosen m times becomes greater than or equal to one that is Expect Value of H (EVH). Here is the extension of SAM:

$$P(Lm) = \sum_{\text{all } k: \frac{2(m-k)}{m} \leq 0}^m \left[C_m^k p^k (1-p)^{m-k} \right] = \sum_{k=\lceil \frac{m}{2} \rceil}^m \left[C_m^k p^k (1-p)^{m-k} \right],$$

with which we have the extension of SAM. Now let us examine the following Problem 1 to analyze Problem.

Problem 1: Choose between:

- H: 1 with certainty.
- L: 0 with probability 0.4 ; 1.5 with probability 0.6

where, the DM will lose 1 for certain if she/he chooses H otherwise the DM will not lose with a probability of 0.4 meanwhile with a probability of 0.6 to lose 1.5 if chooses L. We illustrate the extension of SAM for $m = 1000$ times in one round.

By Fig. 1, we know the probability of judging that EV (L) = EV (H) after m selections of L in Problem 1 Calibration of $P(Lm)$ tells us that if the DM chooses H

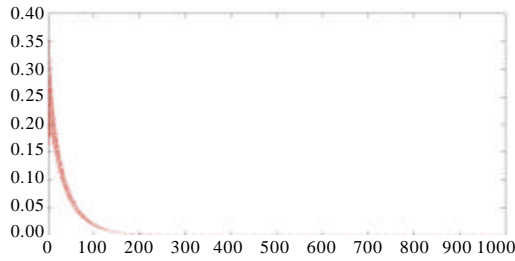


Fig. 1: P (Lm) for m = 1000 times

and L each 100 times in Problem E1, 0.0168 is the probability for an event that she/he can judge that $EV(L) = EV(H)$ as shown in Fig. 1.

OTHER REPRESENTATIVE RESEARCH ON SDM PROBLEM AND EXTENSION OF THE EXPERIMENTS

In other representative researches about the problems of making decision under risk we choose (Kahneman and Tversky 1979; Barron and Erev, 2003). We present the representative experimental research problems in them and extend the experiments as follows.

Kahneman and Tversky (1979) experimental research: Allais^[5] maintained that the more risky of two prospects became relatively the more attractive when the probability of winning in both prospects were multiplied by a common ratio whose effect was of great importance since it represented a robust violation of the tenets of Expected Utility Theory (EUT) (Von Neumann and Morgenstern, 1944). To ascertain the validity of the common ratio effect, (Kahneman and Tversky, 1979) did experimental research, we simplify the problem they studied as follows:

Problem A: Choose between:

- H: 4 with probability 0.8; 0 with probability 0.2
- L: 3 with certainty

Problem B: Choose between:

- H: 4 with probability 0.2; 0 with probability 0.8
- L: 3 with probability 0.25; 0 with probability 0.75

In both of the problems, if the DMs make choices, they will get the corresponding payoffs with the probability in the choice. Kahneman and Tversky 1979 results represented that in problem A 20% of their subjects chose H while 65% of their subjects preferred H

Table 1: KT Experimental Results

Choice	Problem	Probability of subjects
H	A	20%
	B	65%
L	A	80%
	B	35%

in problem B. We can also regard the probability as a DM's preference to H when she/he should make a decision of binary choice. From these results we have Table 1.

The results presented a violation of the independence axiom of EUT that asserts that the DMs should have the same preferences in the two problems. The comparison of Problem A and B described a series of choice problems in which the DMs' preferences systematically violated such axiom. The results showed that people under weighted outcomes that were merely probable in comparison with outcomes that were obtained with certainty; this phenomenon was called the certainty effect.

Considering the extension of the SDM problem, we could extend the experiments as the following problems:

Problem A1: Choose between:

- H: 2 with probability 0.8; 4 with probability 0.2
- L: 3 with certainty

Problem B1: Choose between:

- H: 2 with probability 0.2; 4 with probability 0.8
- L: 3 with probability 0.25; 4 with probability 0.75

In both of the problems, the DMs should consider their losses of each choice with the corresponding probability.

Barron and Erev (2003) experiment on SDM problem:

Barron and Erev (2003) replicated Problem A and B and reported the results revealing differences in risk attitude in SDM problems, they also conducted Problem C as well as Problem A and Problem B.

Problem C: Choose between:

- H: 32 with probability 0.1; 0 with probability 0.9
- L: 3 with certainty.

The subjects were asked to choose either H or L 400 times in each of Problem A, B and C. From Barron and Erev, 2003 results we have in Table 2.

Table 2: BE Experimental Results

Problem	Choice					
	H			L		
	A	B	C	A	B	C
Probability of the subjects	0.63	0.51	0.28	0.27	0.49	0.72

Barron and Erev (2003) results represented the observation of the reversed certain effect and their results were rather different from Kahneman and Tversky (1979).

We also could extend the experiments into the situation in which the losses should be taken account as problem C1: Choose between:

- H: 0 with probability 0.1; 40 with probability 0.9
- L: 32 with certainty.

If H is chosen, the DM is likely to lose 0 with probability 0.1 and 40 with probability 0.9. Otherwise, when L is chosen, the DM must lose 32 with certainty.

It is apparent to see that the Problem 1 can be applied to Problem A by setting $p = 0.8$ and $x = 4/3$ and to Problem C by setting $p = 0.1$ and $x = 32/3$. Problem 2 applied to Problem B by setting $p = 0.8$, $\theta = 0.25$ and $x = 4/3$.

CONCLUSION

In this paper, for the investigation of SDM problems, we viewed Fujikawa and Oda (2007). search-assessment model which explained the exploration tendency in SDM problems rather than each DM’s risk attitude. The model had insisted that in SDM problems the tendency to select best reply to the past and misestimation of payoff distribution could lead to robust deviations from EV maximization. As the researches on SDM problems were focused on the DM’s estimations of payoffs distribution. A key contribution to this paper was that we extended the

problem to the situations in which the DMs have to estimate the loss distribution of binary choice and we presented a motivated and illustrated analysis of this problem.

Kahneman and Tversky 1979 and Barron and Erev, 2003 experiments on these problems were viewed as well and were extended to the situations in which the DMs needed to take their losses into account.

ACKNOWLEDGMENTS

The authors were granted by National NSF of China (111061008/A0112), Guizhou Department of Science and Technology (QianKeHeGYZi (2011) 3055; Qian Ke HeWaiGZi (2010) 7011; Qian Ke HeJZ (2010) 2136 Guiyang Bureau of Science and Technology (ZhuKeHeTong (2011101), Bijie Government: Cooperation of Bijie District and Guizhou University (BiXunZhuanHeZi (2010) SK003), State Ethnic Affairs Commission Funding 2010 for Guizhou University for Nationalities.

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