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Comparative Study of Target Function Definition in Linear Phase FIR Filter Design

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Abstract: The Weighted Least Squares (WLS) principle is popular in filter design due to its flexibility, where it can output equiripple filters with appropriate target function and weighted function. Therefore, this research studies the influence of the target function in linear phase Finite-Impulse-Response (FIR) filter design by comparing different target functions in WLS iterations. Compared with the traditional one, simulations show that the proposed target function can obtain more accurate stopband start frequencies which will be useful if the stopband start frequency must be precise in applications.

Key words: Target function, linear phase, FIR filter, weighted least square

INTRODUCTION

Filters are widely used in signal process, where they can filter the unwanted signal and maintain the useful signal. So far, the filter had been used in speech compression more than twenty-five years (Crochiere and Rabiner, 1983). Today they are used for the compression of image, video, audio signals as well as some applications in digital communication systems (Lyons, 2004), such as the perfect digital transmultiplexing (Vetterli, 1986) and the channel equalization (Scaglione *et al*., 1999). They have also been considered for matched filters in wireless channels. In general, filters are important in digital communication.

There are two kinds of filters which are named Finite Impulse Response (FIR) filter (McClellan and Parks, 2005; Lim, 1983; Johnson, 1990; Lai, 2009) and Infinite Impulse Response (IIR) filter (Konopacki and Moscinska, 2007; Lai and Lin, 2010; Rabiner and Gold, 1975). Taken into the consideration the environment of wireless communications, the linear phase property is important (Proakis, 2000), thus, both the IIR filter and the non-linear phase FIR filter are not suitable for wireless communications and our investigations focus on the linear phase FIR filter design.

In previous literature, there are many conventional methods to design FIR filter, such as the windowing method, the frequency sampling and the Parks-McClellan method (Lyons, 2004; McClellan and Parks, 2005). The windowing method is the simplest but it cannot control the approximation errors in different bands (Lim, 1983). On the other hand, the equiripple property in passband is necessary for communication applications (Proakis, 2000), which has been done through the minimax optimization and the Remez exchange technique (McClellan and Parks, 2005; Boyd and Vandenberghe, 2004; McClellan *et al*., 1973). Aside these methods, there are linear programming methods (Lim, 1983; Johnson, 1990) and WLS methods (Lim *et al*., 1992; Chi and Kou, 1991), the latter is also investigated in this study.

Generally, the WLS method uses a suitable frequency response weighting function to attach importance to different frequency band which produces an easier way to filter design. In the equiripple filter design, the determination of the frequency response weighting function is an essential and remarkable issue and an iterative process of the weighted function has been demonstrated by Lim *et al*. (1992) which can make the iteration converge rapidly. However, the iteration of Lim *et al*. (1992) cannot produce the precise stopband start frequency and passband cutoff frequency. Additionaly, due to its nature of error minimization, the WLS method should also account for the target frequency response function which is the main content in present study.

In order to tackle these problems above, this study attempts to define novel frequency response target functions instead of weighted function which yields more accurate stopband start frequency and passband cutoff frequency. This modification must be beneficial for wireless communications, since, the transition band location is also an important filter performance indicator at this time, i.e., at least with the same importance as the passband ripple and the stopband attenuation.

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THE LINEAR PHASE FIR FILTER AND LEAST SQUARE MODEL

The linear phase property is the essential issue for wireless applications (Proakis, 2000) and there are four types FIR linear-phase filters, where the type II is chosen as a example in our investigations. A type II linear phase filter is defined as:

$$h[n] = h[M-n], 0 \le n \le M \quad (1)$$

where, h(n) and M denote the filter coefficient and the filter length.

In study of Rabiner and Gold (1975) and Lyons (2004), we can find the frequency response expression of Eq. 1:

$$H(\omega) = \sum_{n=1}^{M} a_n \mathrm{trig}(\omega, n) \quad (2)$$

where, $\alpha[k] = 2h[(M/2)]-2$, k = 1, 2, ...M/2, $\alpha[0] = h[M/2]$, and trig(.,.) is an appropriate trigonometrically function defined in [10]. According to Eq. 2, the frequency response error function $E[\omega]$ is given by:

$$E(\omega) = \sum_{n=1}^{M} a_n \mathrm{trig}(\omega, n) - \hat{H}(\omega) \quad (3)$$

where, $\hat{H}(\omega)$ represents the frequency response target function. Formula 3 can be evaluated on a dense grid of frequencies linearly distributed from $\omega = 0$ to $\omega = \pi$ and therefore, form a set of linear equations to find the least-square solution. For a filter length N, L = 4N frequency grid points are adequate and those equations corresponding to ω's within the transition band are discarded (Lim *et al*., 1992).

Formula 3 can be written in matrix form:

$$E = Ua - \hat{H} \quad (4)$$

where:

$$\hat{H} = \left[\hat{H}(\omega_1), \hat{H}(\omega_2), \cdots, \hat{H}(\omega_M)\right]^T$$

$$U = \begin{bmatrix} \mathrm{trig}(\omega_1,1), & \mathrm{trig}(\omega_1,2), & \cdots & \mathrm{trig}(\omega_1,M), \\ \mathrm{trig}(\omega_2,1), & \mathrm{trig}(\omega_2,2), & \cdots & \mathrm{trig}(\omega_2,M), \\ \vdots & \vdots & & \vdots \\ \mathrm{trig}(\omega_L,1), & \mathrm{trig}(\omega_L,1), & \ldots & \mathrm{trig}(\omega_L,M), \end{bmatrix}$$

$$E = [E(\omega_1), E(\omega_2), \cdots, E(\omega_M)]^T$$

$$a = [a(1), a(2), \cdots, a(M)]^T$$

and $(.)^T$ denotes the transposing operation. From Eq. 4, the least squares design can be expressed as (Rabiner and Gold, 1975):

$$\hat{a} = (U^T U)^{-1} U^T \hat{H} \quad (5)$$

THE STUDY OF TARGET FUNCTION IN WLS DESIGN

The principle of weighted least square: In the weighted least square process, we have the following cost function:

$$\Lambda = \sum_{n=1}^{M} r(\omega_n) E(\omega_n)^2 \quad (6)$$

where, $r(\omega_n)$ is the least square weighting function. Minimizing Eq. 6 produces the optimum solution (Lim *et al*., 1992; Chi and Kou, 1991):

$$\hat{a} = (U^T R U)^{-1} U^T R \hat{H} \quad (7)$$

where, R is a diagonal matrix whose n-th diagonal element is $r(\omega_n)$, i.e., $R = \mathrm{diag}[r(\omega_n)]_{n=1..M}$.

Generally, the minimax design is superior to the least square design near the band edges whereas the least square design has a much smaller ripple magnitude elsewhere. Moreover, the performance of the least square design near the band edges can be improved by using a relatively larger $r(\omega_n)$ near the band edge. This is the essence of the weighted least squares technique (Lim *et al*., 1992). In order to accomplish the WLS process, Lim and Lee exploited an iterative process and proposed to use the following weighting function update:

$$r_{k+1}(\omega) = r_k(\omega)\beta_k(\omega), \beta_k(\omega) > 0 \quad (8)$$

where, $r_k(\omega)$ and $\beta_k(\omega)$ denote as the weighting function and the required update at the kth iteration. According to Lim *et al*. (1992), $\beta_k(\omega)$ can be determined to the extreme error frequency but we won't present its derivation in detail for the sake of simplicity. In present study, we employ the same iteration shown in Eq. 8.

Comparative study of target function definition: In Eq. 3, we have defined $\hat{H}(\omega)$ as the target function which together with the weighted function will finally affect the frequency response error. Thus, the target function definition will influence the performance of the WLS method and a good choice of target function may lead to a better filter. Hence, we try three definitions and analyze their performance.

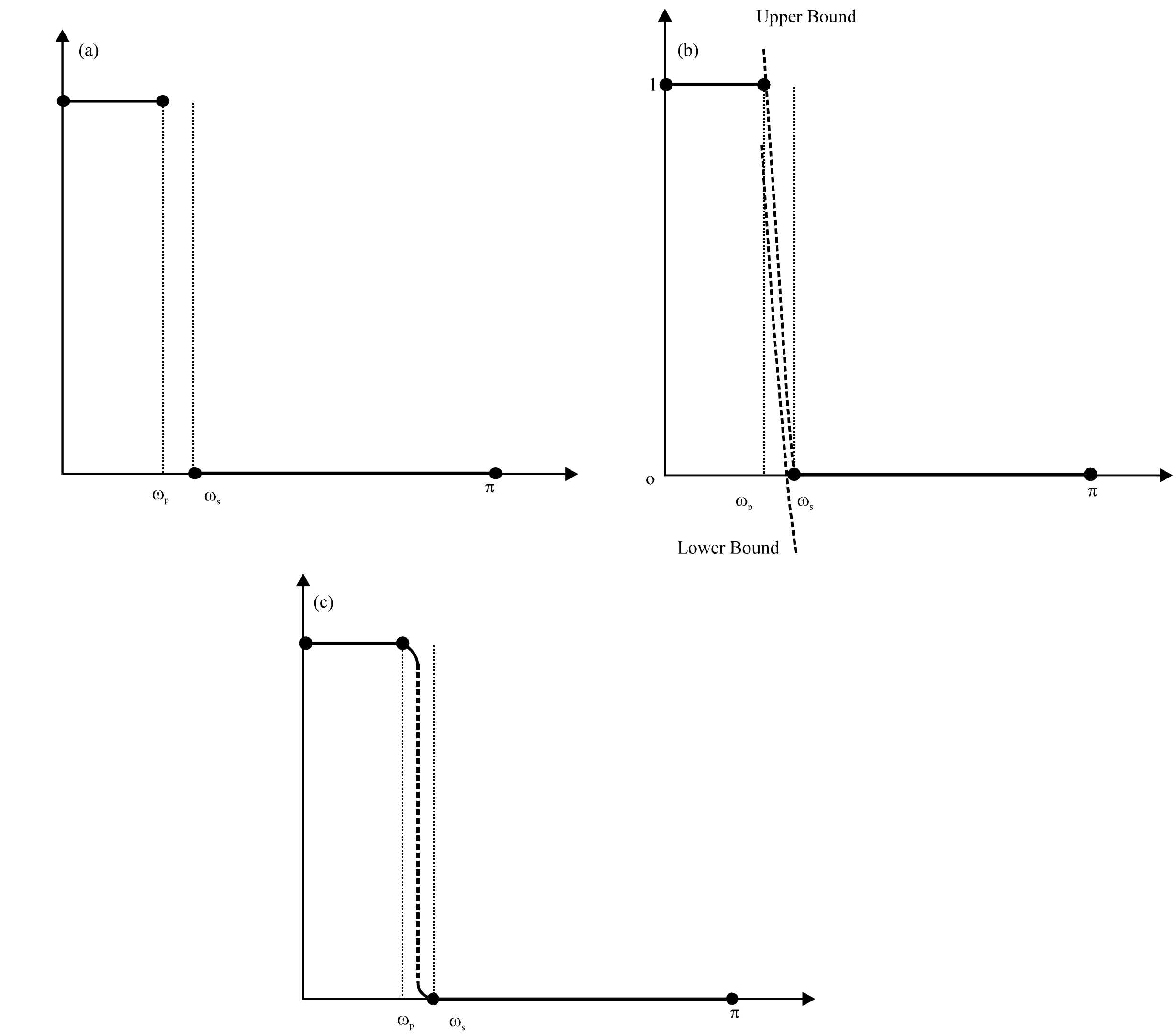

Fig. 1(a-c): Three target function definitions; (a) Definition in, (b) Area constraint definition and (c) Curve constraint definition

All three definitions are demonstrated in Fig. 1. In Fig. 1a, the first definition from Lim *et al.* (1992), named case A in present study, has the following form:

$$\begin{aligned} &\hat{H}(\omega_k) = 1, 0 \le k < ndwp \\ &\hat{H}(\omega_k) = 0, ndws \le k < 1024 \end{aligned} \quad (9)$$

where, L = 1024, ndwp means the passwand cut-off point and ndws represents the stopband start point. Note that ω_p and ω_s are the stopband start frequency and passband cutoff frequency which are known parameters for a given wireless system.

The second definition in Fig. 1b, named case B in present study, must obey:

$$\begin{aligned} &\bullet \hat{H}(\omega_k) = 1, \text{ for } 0 \le k < ndwp \\ &\bullet \hat{H}(\omega_k) = 0, \text{ for } ndws \le k < 1024 \\ &\bullet lb(\omega_k) \le \hat{H}(\omega_k) \le ub(\omega_k), \text{ for } ndwp \le \omega_k \le ndws \end{aligned} \quad (10)$$

where, lb(w) and ub(w) mean the lower and the upper amplitude limit in the transition band. This target function different from the first one on the constraint in transition bands. It offers more information to the design process and we can control the WLS algorithm more flexible. The most important in this method is that we need to choose appropriate lower and upper limits which is a nuisance work and can be done by simulations. Without loss of generality, both lb(w) and ub(w) are chosen equi-spaced around the line through (ω_p, 1) and (ω_s, 0).

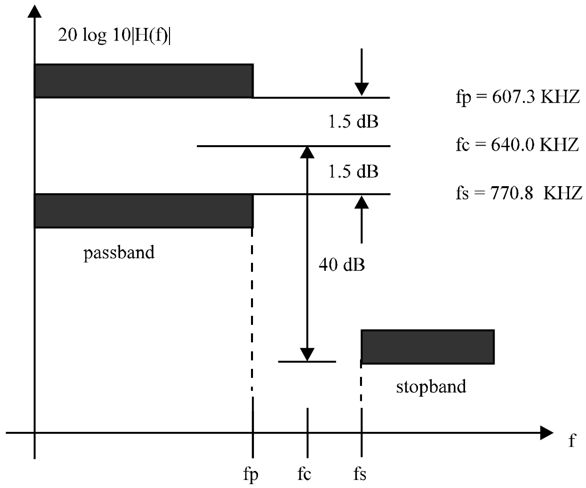

Fig. 2: The requirement of TD-SCDMA matched filter

The final definition named as case C, as seen in Fig. 1c, is a curve through (ω_p, 1) and (ω_s, 0), i.e.,

$$\bullet\hat{H}(\omega) = 1, 0 \le k < ndwp$$

$$\bullet\hat{H}(\omega_k) = \left[\left(\frac{ndws - 1 - ndwp}{2 * P}\right)^2 - \left(\frac{k - ndwp}{P}\right)^2 \right]^{\frac{1}{2}} + 1 - \frac{ndws - 1 - ndwp}{2 * P}, \text{ for } ndwp \le k \le \frac{ndws - 1 + ndwp}{2} \quad (11)$$

$$\bullet\hat{H}(\omega_k) = -\left[\left(\frac{ndws - 1 - ndwp}{2 * P}\right)^2 - \left(\frac{k - ndwp}{P}\right)^2 \right]^{\frac{1}{2}} - \frac{ndws - 1 - ndwp}{2 * P}, \text{ for } \frac{ndws - 1 + ndwp}{2} \le k \le ndws$$

$$\bullet\hat{H}(\omega_k) = 0, ndws \le k < 1024$$

where, P = 1024.

APPLICATION IN DESIGNING TD-SCDMA SIGNAL SOURCE

Here, the simulation result are presented to prove the effectivity of transition band constraints in WLS filter design. Without loss of generality, we assume that the filter must meet the requirement of the Time Division Synchronous Code Division Multiple Access (TD-SCDMA) matched filter (3GPP, 2008) which is shown in Fig. 2. Moreover, extensions to other standards are straightforward.

In our simulations, the four times oversampling is assumed thus the digital frequencies of ω_p, ω_c and ω_s in Fig. 2 are 0.237227π, 0.25π and 0.301094π.

From Fig. 3-5, the subfigure shows the zoomed passband ripple. We can explicitly see that the passband ripple is approximately equiripple and less than 2 dB, thus, satisfying the requirements of TD-SCDMA. Moreover, once the filter length is larger than 48, the stopband attenuation is also adequate. However, the transition performance is difficult to identify, accordingly, Table 1 is presented to solve this problem, where Rp and As represent the passband ripple and the stopband attenuation.

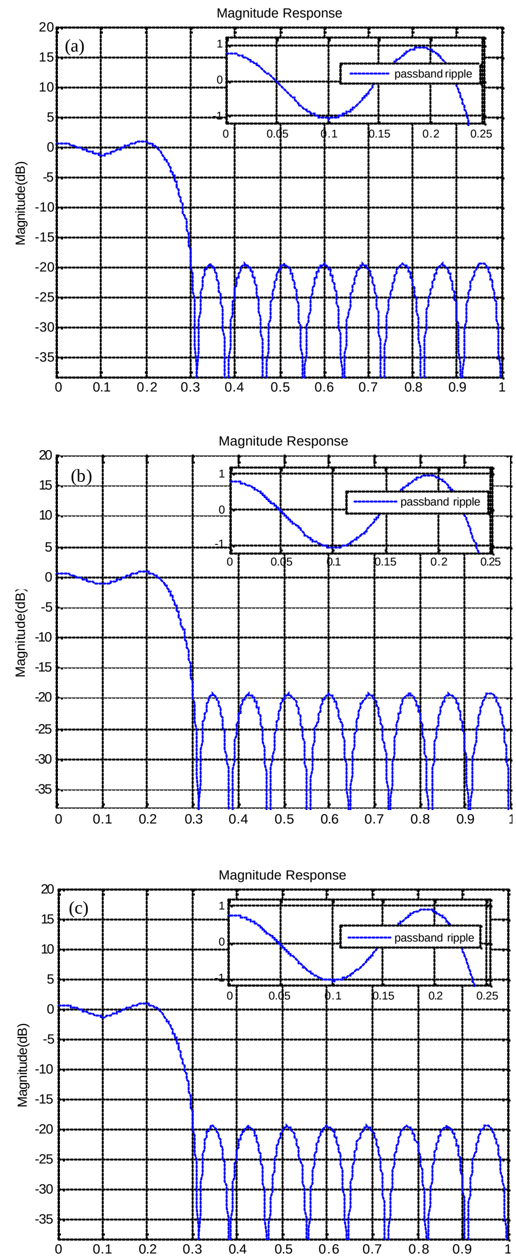

Fig. 3(a-c): The magnitude response of FIR filter with length 24, (a) Case A, (b) Case B and (c) Case C

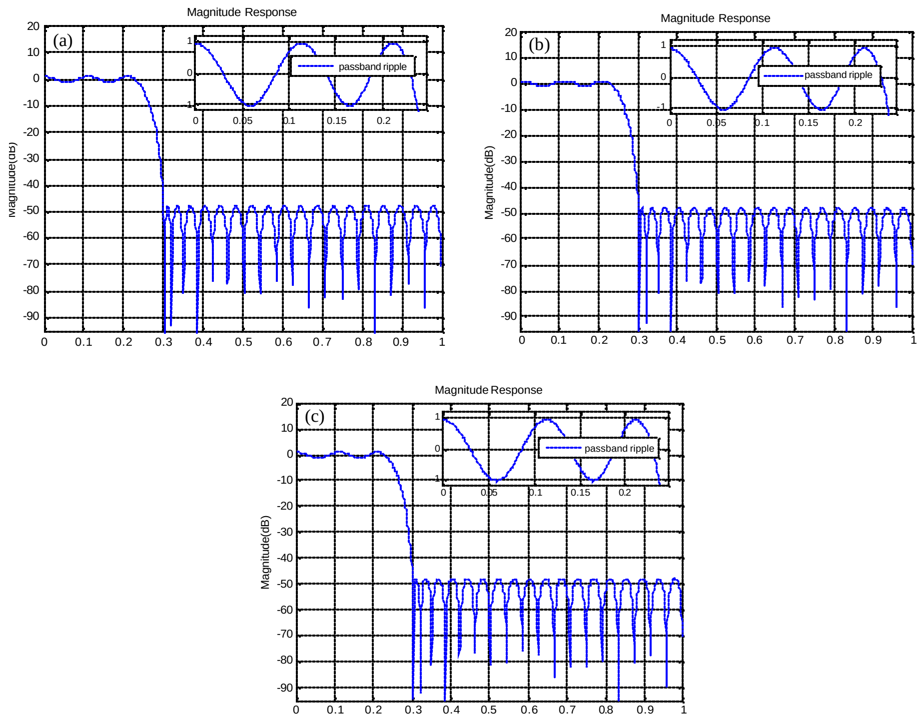

Fig. 4(a-c): The magnitude response of FIR filter with length 48, (a) Case A, (b) Case B and (c) Case C

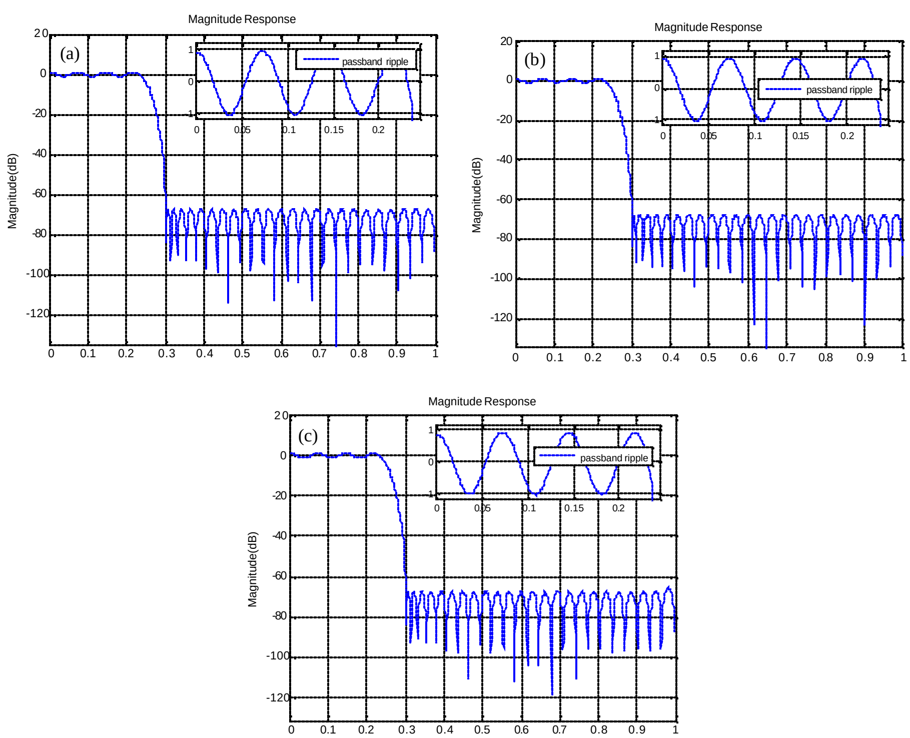

Fig. 5(a-c): The magnitude response of FIR filter with length 64, (a) Case A, (b) Case B and (c) Case C

Table 1: The transition band performance of the FIR filter

Target function	N = 24	N = 48	N = 64
Case A	fp = 0.237227 π	fp = 0.237227 π	fp = 0.237227 π
	fc = 0.252988 π	fc = 0.246714 π	fc = 0.253294 π
	fs = 0.300738 π	fs = 0.301001 π	fs = 0.301011 π
	Rp = ±1.0000 dB	Rp = ±1.0000 dB	Rp = ±1.0000 dB
	As = 19.012700 dB	As = 47.736370 dB	As = 67.535509 dB
Case B	fp = 0.237227 π	fp = 0.237227 π	fp = 0.237227 π
	fc = 0.251978π	fc = 0.248714 π	fc = 0.249927 π
	fs = 0.300922 π	fs = 0.301099 π	fs = 0.301089 π
	Rp = ±1.0000 dB	Rp = ±1.0000 dB	Rp = ±1.0000 dB
	As = 19.0137 dB	As = 47.728577 dB	As = 67.4658 dB
Case C	fp = 0.237227 π	fp = 0.237227 π	fp = 0.237227 π
	fc = 0.252986π	fc = 0.246713 π	fc = 0.247910 π
	fs = 0.300784 π	fs = 0.301067 π	fs = 0.300993 π
	Rp = ±1.0000 dB	Rp = ±1.0000 dB	Rp = ±1.0000 dB
	As = 19.01844 dB	As = 47.687389 dB	As = 65.3478125 dB

From Table 1, it's obviously that case B and case C produce small transition bands. Moreover, the stopband start frequency of case B is closer to its expected value when compared with that of case A. This advantage is important in communication applications, since the accurate stopband start frequency absolutely reduce the adjacent channel interference. Hence, we conclude that appropriate constraints on transition bands will be beneficial for the filter design.

CONCLUSIONS

In this study, we presented comparisons between different target functions in WLS filter design. Three target functions are defined and simulated, results show that reasonable constraints on the transition band will yield the better stopband start frequency which is important in some wireless applications.

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