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Invariant Decomposition Conditions for Petri Nets Based on the Index of Transitions

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Abstract: With the decomposition method of Petri net by assigning an index of the transition set, a structure-complex net system can be decomposed to a set of structure-simple subnets, named T-net. There is a projection relation of the reachable marking set and languages between the original net system and the subnet systems decomposed. However, some unnecessary states and languages are also added in the subnet systems. This study presents the deep research results on the decomposition method of Petri nets based on the index of transitions. A set of necessary and sufficient conditions for keeping the states and languages invariant between the original system and the subnet systems is obtained. Based on the simplified reachable marking graph, an algorithm is given to decide the states and languages invariant.

Key words: Petri net, index of transitions, invariant decomposition, reachable marking graph, petri net language, reachable states

INTRODUCTION

As tools of modeling and analyzing physical systems, Petri nets have been widely applied in a large number of areas to model, analyze and manipulate real systems, which provides users with an integrated modeling, analyzing and manipulating environment, so as to give a reliable basis for the analyze of real system (Sun *et al*., 2011; Zeng *et al*., 2008a; Zeng and Duan, 2007; Wang and Zeng, 2008; Meng *et al*., 2011; Du and Guo, 2009; Liu and Du, 2009; Liu *et al*., 2009). The greatest obstacle to the application of Petri nets in real systems is the state space explosion, which arises while analyzing the structural properties of Petri nets. Because of the complexity of the real system, this problem has not been solved properly. To deal with the state space explosion of Petri nets, large numbers of researchers have done much work. They have presented the idea of Petri net reduction, Petri net operation, Petri net composition, stepwise refinement and so on (Jiang, 1997, 2000; Wang, 2001a, b; Jiang and Wu, 1992; Zeng, 2004; Lee-Kwang *et al*., 1987; Suzuki and Murata, 1983; Wang, 1999; Zeng and Wu, 2002, 2004). These works can be divided into two methods. One method is to compose structure-complex net system using a set of structure-simple nets so as to obtain the property of the complex system by analyzing the property of simple nets (Jiang, 1997, 2000; Wang, 2001a; Jiang and Wu, 1992; Zeng, 2004). The concept of synchronous composition of Petri nets and analyzes some conditions to keep states and behaviors invariant during the process of composition was given (Jiang, 1997, 2000). Wang (2001b) introduced a modeling method for FMS (Flexible Manufacture System) based on the synchronous composition of Petri net, with which divides physical objects into work type and resource type. Jiang and Wu (1992) defined an algebraic operation, named net operation and tried to build large net system via this algebraic operation. The structural properties during the process of net operations were also discussed (Jiang and Wu, 1992). Zeng (2004) extended the concept of synchronous composition to more than two Petri nets, which was used to express the language behavior of a structure-complex Petri net. Another method (Lee-Kwang *et al*., 1987; Suzuki and Murata, 1983) is to decompose a structure-complex net system to a series of structure-simple nets, hoping the original properties of the net can be reflected in the subnets. The method of the sum decomposition and the union decomposition were presented (Wang, 1999, 2001a). However, only the structural properties between the original net system and the subnets were discussed in details while the state and behavior relations, which are more widely used in the process of a real system, were not discussed (Wang, 1999, 2001b). To solve this problem, the decomposition method for petri nets by defining the index

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of places and transitions was given, respectively (Zeng, 2007, 2011; Zeng *et al.*, 2008b) which can decompose a structure-complex Petri net into a set of S-nets (Zeng, 2011; Cui *et al.*, 2011) or T-nets (Wu, 2006). A new decomposition method for Petri net by defining the index of places is given, with which the decomposed subnets are all S-nets (Zeng and Wu, 2002). They also give a deep research on this decomposition method and a necessary and sufficient condition for keeping the state and language invariant between the original system and the subnet system has been obtained (Zeng and Wu, 2004).

In this study, we keep on the research of the decomposition method based on the index of transitions (Zeng, 2011, 2006), especially on the projection relation of the reachable marking sets and the languages between the original system and the subnet systems. A necessary and sufficient condition for keeping the states and languages invariant between the original system and the subnet system has been obtained. Based on the simplified reachable marking graph, an algorithm is given to decide the states and languages invariant.

BASIC CONCEPTS OF PETRI NETS

It is assumed that readers are familiar with the basic concepts of Petri nets (Wu, 2006; Zeng, 2008; Murata, 1989; Peterson, 1981). Some of the essential terminologies and notations about Petri nets used in this study are listed as follows:

A tuple N = {P, T; F} is named as a net iff :

$$P\cap T = \varnothing,\ P\cup T \neq \varnothing \tag{1}$$

$$F\subseteq (P\times T) \cup (T\times P) \tag{2}$$

$$Dom(F)\cup Cod(F) = P\cup T \tag{3}$$

where, $Dom(F) = \{x \in P\cup T|\exists y \in P\cup T: (x, y)\in F\}$ and $Dom(F) = \{x \in P\cup T|\exists y \in P\cup T: (y, x) \in F\}$.

For all $x\in P\cup T$, the set ${}^{\bullet}x = \{y \mid y \in P\cup T \wedge (y, x) \in F\}$ is named as the pre-set of x and the set $x^{\bullet} = \{y \mid y \in P\cup T\wedge(x, y) \in F\}$ is named as the post-set of x.

A Petri net is a 4-tuple $\Sigma = \{P, T; F, M_0\}$, where N = {P, T; F} is a net and M_0: $P\rightarrow Z^+$ (Z^+ is the non-negative integer set) is the initial state of Σ. A state M is reachable from M_0 if there is a transition firing sequence σ such that M_0 [σ>M. We use $R(M_0)$ to represent the set of all reachable states from M_0. Let Σ = (P, T; F, M) be a Petri net. A 3-tuple RMG (Σ) = ($R(M_0)$, E, f) is defined as the reachable marking graph, where: $E = \{(M_i, M_j)|M_i, M_j \in R(M_0), \exists t_k \in t, M_i [t_k>M_j\}$, f: E→T, f $(M_i, M_j) = t_k$, iff M_i $[t_k>M_j$, V is named as the place set of RMG (Σ), E is named as the arc set of RMG (Σ). If f (M_i, M_j), t_k is the side mark of f (M_i, M_j).

DECOMPOSITION OF PETRI NET BASED ON THE INDEX OF TRANSITIONS

Here, we introduce the method to decompose a structure-complex Petri net based on an index function defined on the transition set (Zeng, 2007, 2011).With this method, a structure-complex Petri net can be decomposed into a set of structure-simple nets such that $|p^{\bullet}|\leq 1$ and $|{}^{\bullet}p| \leq 1$ for all places.

Definition 1 (Zeng, 2006): Let $\Sigma = (P, T; F, M_0)$ be a Petri net. A function f_T: T→ {1, 2, 3,..., k} is an index function of T, iff $\forall t_1, t_2\in T$, if $t^{\bullet}_1\cap t^{\bullet}_2 \neq \varnothing$ or $f_T(t_1) \neq f_T(t_2)$. $f_T(t)$ is named as the index of the transition t.

Definition 2 (Zeng, 2006): Let $\Sigma = (P, T; F, M_0)$ be a Petri net and f_T: T→ {1, 2, 3,..., k} be the index function of T. Petri net $\Sigma_i = (P_i, T_i; F.M_{0i})$ (i∈{1, 2, 3,..., k}) is a decomposed net of Σ based on f_T iff Σ_i satisfies the following conditions:

$$T_i = \{t\in T|\ f(t) = i\};\ (i\in\{1, 2...k\})$$

$$P_i = \{p\in P|\ \exists t\in T,\ p\in {}^{\bullet}t\cap t^{\bullet}\}(i\in\{1, 2...k\})$$

$$F_i = \{(P_i\times T_i) \cup (T_i\times P_i)\} \cap F,\ (i, \in \{1, 2, 3...k\})$$

$$M_{0i} = \Gamma_{P\rightarrow Pi} M_0$$

Definition 3 (Wu, 2006): A Petri net $\Sigma = (P, T;F, M_0)$ is a T-Net iff $\forall p\in P$ such that $|{}^{\bullet}p|\leq 1$ and $|p^{\bullet}|\leq 1$.

Theorem 1 (Zeng, 2006): Let $\Sigma_i = (P_i, T_i; F.M_{0i})$ (i∈ {1, 2, 3... k}) be the decomposed net of $\Sigma = (P, T; F, M_0)$ based on the function index of f_T iff:

$$\forall i, j \in \{1, 2, 3\cdots k\}, i \neq j, T_i \cap T_j = \varnothing \text{ and } \bigcup_{i=1}^{k} T_i = T$$

For all k>1, if ∀i∈{1, 2, 3... k},∃j{1, 2, 3...k}, i ≠ j, then,

$$P_i\cap P_i \neq \varnothing \text{and } \bigcup_{i=1}^{k} P_i = P$$

Theorem 1 indicates that a Petri net can be decomposed into a set of structure-simple subnets, named T-Nets. A Polynomial-time Decomposition Algorithm was given (Zeng, 2011).

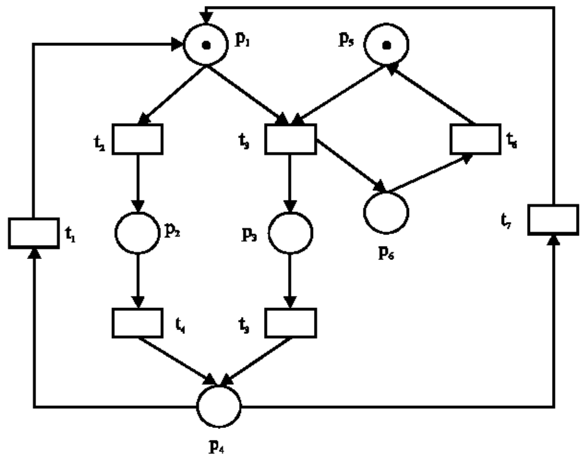

Fig. 1: A Petri net example

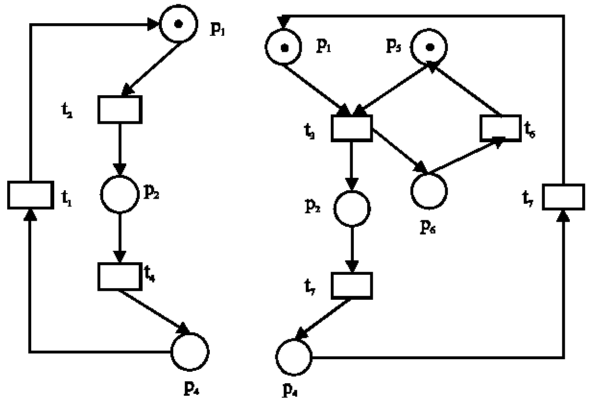

Fig. 2: Two decomposed subnets

Theorem 2 (Zeng, 2006): Let $\Sigma_i = (P_i, T_i; F.M_{0i})$ ($i\in \{1, 2, 3... k\}$) be the decomposed net of $\Sigma = (P, T; F.M_0)$ based on the function index of f_T iff:

For all Σ_i ($i \in \{1, 2, 3... k\}$), $\forall p \in P_i$, $|{}^{\bullet}p| \leq 1$ and $|p^{\bullet}| \leq 1$

Definition 4: Let $\Sigma_i = (P_i, T_i; F.M_{0i})$ ($i\in \{1, 2, 3... k\}$) be the decomposed net of $\Sigma = (P, T; F.M_0)$ based on the index function of transitions and $R(M_0)$ be the state set of Σ and $R(M_{0i})$ be the state set of Σ_i, respectively. A projection $\Gamma_{P\to Pi}$: $R(M_0)\to R(M_{0i})$($i = 1, 2, ... k$) such that $\Gamma_{P\to Pi}(M)$ is the states which are obtained from M by deleting all the places that do not belong to P_i. $\Gamma_{P\to Pi}$ is denoted as the projection function from $R(M_0)$ to $R(M_{0i})$ and $\Gamma_{Pi\to P}$ is named as the inverse projection of $\Gamma^{-1}_{P\to Pi}$, ($i = 1, 2, ...k$).

Theorem 3: Let $\Sigma_i = (P, T; F.M_{0i})$ ($i\in \{1, 2, 3... k\}$) be the decomposed net of $\Sigma = (P, T; F.M_0)$ based on the index of transitions, such that:

$$\Gamma_{P\to Pi}(R(M_0))\subseteq R(M_{0i}), (i\in\{1, 2, 3, ..k\})$$

$\Gamma_{T\to Ti}(L(\Sigma)) \subseteq L(\Sigma_i)$, ($i\in\{1, 2, 3... k\}$) ($\Gamma_{T\to Ti}(\sigma)$ represents the projection of σ on T_i)

A Petri net example is shown in Fig. 1. The index function f_T: $T\to\{1, 2... k\}$ is defined as: $f(t_1) = f(t_2) = f(t_4) = 1$; $f(t_3) = f(t_5) = (t_6) = f(t_7) = 2$

Obviously, f_T satisfies Definition 2. Using the decomposition method based on the index of transitions, the Petri net shown in Fig. 1 can be divided into two subnets Σ_1 and Σ_2, which are illustrated in Fig. 2.

NECESSARY AND SUFFICIENT DECOMPOSITION CONDITIONS FOR KEEPING THE PROPERTY INVARIANT

It can be known from Theorem 3 that the projection of the original net system on the subnet systems is only a subset of the subnet's states and behaviors. The subnets add some unnecessary states and behaviors while keeping the properties of the original Petri net. Here, we present the necessary and sufficient conditions for keeping the property invariant while the decomposition.

Firstly, the definition of the property invariance while the decomposition is given.

Definition 5: Let $\Sigma_i = (P_i, T_i; F.M_{0i})$ ($i\in \{1, 2, 3... k\}$) be the decomposed net of $\Sigma = (P_i, T_i; F.M_0)$ based on the index of transitions, If:

(1) $\Gamma_{T\to Ti}(L(\Sigma)) = L(\Sigma_i)$ ($i\in\{1, 2, 3... k\}$), then the decomposed subnets keep the behavior property of the original net
(2) $\Gamma_{P\to Pi}\, R(M_0) = R(M_{0i})$, ($i\in\{1, 2, ... k\}$), then the decomposed subnets keep the state property of the original net

If (1) and (2) are satisfied together, the decomposed subnets keep the property invariant of the original net.

From the second conclusion of Theorem 1, we know that if there are more than one decomposed subnets, for any Σ_i, there necessarily exists another subnet Σ_j ($i \neq j$), such that $P_\Delta \neq \varnothing$, where $P_\Delta = P_i \cap P_j$ and P_i and P_j is the place set of Σ_1 and Σ_2, respectively.

Theorem 4: Let $\Sigma_i = (P_i, T_i; F.M_{0i})$ ($i\in \{1, 2, 3... k\}$) be the decomposed net of $\Sigma = (P, T; F.M_0)$ based on the index of transitions. For all $i, j\in\{1, 2, 3... k\}$($i \neq j$), if $P_\Delta \neq \varnothing$ such that $P_\Delta = P_i \cap P_j$, $\Gamma_{P\to Pi}\, R(M_0) = R(M_{0i})$, ($i\in\{1, 2, ... k\}$), iff $\forall i, j\in\{1, 2, 3...k\}$ ($i \neq j$), $\Gamma_{Pi\to P\Delta}(R(M_{0i})) = \Gamma_{pj\to P\Delta}(R(M_{0j}))$.

Proof:

(1) (⇒)

Firstly, we prove $\Gamma_{pi\to p\Delta}$ $(R(M_{0i})) \subseteq \Gamma_{pj\to P\Delta}$ $(R(M_{0j}))$.

For $\forall M_{\Delta i} \in \Gamma_{Pi\to P\Delta}$ $(R(M_{0i}))$, then $\exists M_i \in R$ (M_{0i}),

such that $\Gamma_{Pi\to P\Delta}(M_i) = M_{\Delta i}$, because $\Gamma_{Pi\to P\Delta}R(M_0) = R(M_{0i})$ $(i = \{1, 2, ...k\})$

and then for $M_i \in R(M_{0i})$, $\exists M \in R(M_0)$,

such that satisfies $\Gamma_{pi\to p\Delta}(M) = M_i$,

so, $M_{\Delta i} = \Gamma_{pi\to p\Delta}$ $(\Gamma_{P\to Pi}$ $(M)) = \Gamma_{P\to P\Delta}$ $(M) \in \Gamma_{P\to P\Delta}$ $(R(M_{0j}))$.

Therefore, $\Gamma_{P\to P\Delta}$ $(R(M_{0j})) \subseteq \Gamma_{Pj\to P\Delta}$ $(R(M_{0j}))$

Similarly, we can prove that $\Gamma_{pi\to p\Delta}$ $(R(M_{0i})) \subseteq \Gamma_{Pj\to P\Delta}$ $(R(M_{0j}))$.

As a result, $\Gamma_{Pi\to P\Delta}$ $(R(M_{0i})) = \Gamma_{Pj\to P\Delta}$ $(R(M_{0j}))$.

(2) (⇐)

$\forall M_i \in R$ (M_{0i}) denote $M_{\Delta j} = \Gamma_{Pj\to Pi}$ $(R(M_{0j}))$.

Since, $\Gamma_{pi\to p\Delta}(R(M_{0i})) = \Gamma_{Pj\to P\Delta}(R(M_{0j}))$, $M_{\delta i} \in \Gamma_{pj\to P\Delta}$ $(R(M_{0j}))$,

that is to say $\exists M_j \in R(M_{0j})$, such that $\Gamma_{pj\to P\Delta}$ $(M_j) = M_{\Delta i}$.

Denote $M = \{M° | M° \in P^* \wedge \Gamma_{P\to Pj}(M°) = M_i \wedge \Gamma_{P\to Pj}(M°) = M_j\}$.

Based on the Definition 4, we can obtain that $M \in \Gamma^{-1}_{P_i\to P}(R(M_{0i})) \cap \Gamma^{-1}_{P_j\to P}(R(M_{0j})) = R(M_0)$. As $\Gamma_{P\to Pi}$ $(M) = M_i \in R(M_{0i})$ and $\Gamma_{P\to Pj}(M) = M_j \in R(M_{0j})$, $R(M_{0i}) \subseteq \Gamma_{P\to Pi}$ $(R(M_0))$.

Similarly, we can prove that R $(M_{0j}) \subseteq \Gamma_{P\to Pj}$ $(R(M_0))$.

Thus, $R(M_{0i}) \subseteq \Gamma_{P\to Pi}$ $(R(M_0)$, $(i = 1, 2, ...\ k))$.

On the other hand, $\Gamma_{P\to Pi}$ $R(M_{0i}) \subseteq R(M_{0i})$, $(i = \{1, 2, ...k\})$.

So, $\Gamma_{P\to Pi}$ $R(M_0) \subseteq R(M_0)$, $(i = \{1, 2, ...k\})$.

According to (1) and (2), the theorem is proved.

Theorem 4 shows that in order to keep the state property of the decomposed subnets during the decomposition based on the index of transitions, the projection of subnets on the public intersection must be equal with each other, vice versa. Theorem 5 presents the conditions to keep the behavior property during the process of decomposition.

Theorem 5: Let $\Sigma_i = (P_i, T_i; F.M_{0i})$ $(i\in \{1, 2, 3...\ k\})$ be the decomposed net of $\Sigma = (P, T, F, M_0)$ based on the index of transitions.

$\Gamma_{T\to Ti}$ $(L(\Sigma)) = L(\Sigma_i)$, iff $\Gamma_{P\to Pi}$ R $(M_0) = R$ (M_{0i}), $(i\in\{1, 2, 3...k\})$

Proof:

(⇒)Firstly, we prove $\Gamma_{P\to Pi}$ R $(M_0) \subseteq R$ (M_{0i}).

$\forall M_i \in R(M_{0i})$, let M_{0i} $[\sigma_i > M_i$, then $\sigma_i \in L(\Sigma_i)$,

because $\Gamma_{T\to Ti}$ $(L(\Sigma)) = L(\Sigma)$ and $\exists \sigma \in L$ (Σ) such that $\Gamma_{T\to Ti}$ $(\sigma) = \sigma_i$.

If, we denote M_0 $[\sigma > M$, the corresponding vector of σ is X, then $M = M_0 + A^T X$, $M_i = M_{0i} + A^T_i X_i$, where X_i is the corresponding vector of σ_i. Since, $M_{0i} = \Gamma_{P\to Pi}$ (M_0), $\forall p \in P$ such that $M(p) = M_0(p) + \sum_{t\in {}^{\bullet}p\cap T} \#(t/\sigma) - \sum_{t\in p^{\bullet}\cap T} \#(t/\sigma)$ where, $\#(t/\sigma)$ demonstrates the times of transition t appearing in σ.

Thus,

$$\Gamma_{P\to P_i}(M(p)) = \Gamma_{P\to P_i}(M_0(p)) + \Gamma_{\Sigma\to\Sigma_i}\Big(\sum_{t\in {}^{\bullet}p\cap T} \#(t/\sigma) - \sum_{t\in p^{\bullet}\cap T} \#(t/\sigma)\Big)$$

This is to say $\Gamma_{P\to P_i}(M(p)) = M_{0i}(p) + \sum_{t\in {}^{\bullet}p\cap T} \#(t/\sigma) - \sum_{t\in p^{\bullet}\cap T} \#(t/\sigma)$

Since $p \in P_i$, $\Gamma_{P\to Pi}$ $(M) = M_i \in R(M_{0i})$,

that is to say $\Gamma_{P\to Pi}$ R $(M_0) \subseteq R$ (M_{0i}), $(i\in\{1, 2, 3...k\})$.

On the other hand, R $(M_{0i}) \subseteq \Gamma_{P\to Pi}$ $(R$ $(M_0))$, $(i\in\{1,2,3...k\})$ can be easily proved while $\Gamma_{T\to Ti}(L(\Sigma)) = L(\Sigma_i)$.

Thus, $\Gamma_{P\to Pi}$ R $(M_0) = R$ (M_{0i}).

Similarly, we can prove that: (⇐)

As a result, the theorem is proved.

From Theorem 4 and 5, Corollary 1 can be obtained.

Corollary 1: Let $\Sigma_i = (P_i, T_i; F.M_{0i})$ $(i\in \{1, 2, 3...\ k\})$ be the decomposed net of $\Sigma = (P, T, F, M_0)$ based on the index of transitions, $\forall i, j \in \{1, 2, 3, ...k\}$, $(i \neq j)$, if $P_\Delta \neq \varnothing$, $\Gamma_{T\to Ti}$ $(L(\Sigma)) = L$ (Σ_i) iff $\Gamma_{Pi\to P\Delta}(R(M_{0i})) = \Gamma_{Pj\to P\Delta}(R(M_{0j}))$.

A DECISION ALGORITHM FOR DECOMPOSITION PROPERTY INVARIANCE

Here, we have presented the sufficient and necessary conditions for keeping the state and behavior property during the process of decomposition. Here, an algorithm to decide the property invariant is given based on the simplified reachable marking graph.

Definition 6: Let $RMG(\Sigma_i) = \langle V_i, E_i; f_i \rangle (i = 1, 2, ...k)$ be the reachable marking graph of $\Sigma_i = (P_i, T_i; F.M_{0i})$ $(i\in \{1, 2, 3...\ k\})$, the subnets decomposed from Petri net Σ based on the index of transitions. $SRMG_{P\Delta}(\Sigma_i) = \Gamma_{pi\to P\Delta}$ $(RMG(\Sigma_i))$ is defined as the simplified reachable marking graph of RMG (Σ_i) based on the place set P_Δ, where $SRMG_{P\Delta}(\Sigma_i) = \langle V_{i\Delta}, E_{i\Delta}; f_{i\Delta} \rangle$, if the following conditions are satisfied:

$$\forall v_{i\Delta} \in V_{i\Delta}, v_i \in V_i \text{ such that } v_{i\Delta} = \Gamma_{P_i\to P_\Delta} v_i$$

$$\forall e_{i\Delta} \in E_{i\Delta}, e_{i\Delta} = \{(v_{x\Delta}, v_{y\Delta}) \mid v_{x\Delta}, v_{y\Delta} \in V_{i\Delta}\}$$

$$f_{i\Delta} : E_{i\Delta} \to T_{i\Delta}, T_{i\Delta} = \{t_{i\Delta} \mid t_{i\Delta} = {}^{\bullet} p_{i\Delta} \cup p_{i\Delta}{}^{\bullet}, p_{i\Delta} \in P_\Delta\}$$

Definition 7 given a formal definition of isomorphism between two reachable marking graphs of Petri nets based on the public place set $P_\Delta \neq \varnothing$.

Definition 7: Let $RMG(\Sigma_i) = \langle V_i, E_i; f_i \rangle (i = 1, 2, ...k)$ be the reachable marking graph of $\Sigma_i = (P_i, T_i; F.M_{0i})$ $(i\in \{1, 2, 3...\ k\})$, the subnets decomposed from Petri net Σ based on the index of transitions and $SRMG_{P\Delta}(\Sigma_i)$ be the simplified reachable marking graph of $RMG(\Sigma_i)$ based on the place set P_Δ, where, $SRMG_{P\Delta}(\Sigma_i) = \langle V_{i\Delta}, E_{i\Delta}; f_{i\Delta} \rangle$. RMG (Σ_i) and RMG (Σ_j) is isomorphic on P_Δ iff:

$$|V_{i\Delta}| = |V_{j\Delta}| \text{ and } \forall v_{i\Delta} \in V_{i\Delta}, \exists v_{j\Delta} \in V_{j\Delta} \text{ such that } v_{i\Delta} = v_{j\Delta}$$

$$|E_{i\Delta}| = |E_{j\Delta}|$$

Denote $\Phi_{P\Delta}$ $(RMG(\Sigma_i)) \cong \Phi_{P\Delta}$ $RMG(\Sigma_j)$ if $RMG(\Sigma_i)$ and $RMG(\Sigma_j)$ are isomorphic on P_Δ.

Theorem 6: Let $RMG(\Sigma_i) = \langle V_i, E_i; f_i \rangle (i = 1, 2, ...k)$ be the reachable marking graph of $\Sigma_i = (P_i, T_i; F.M_{0i})$ $(i\in \{1, 2, 3...\ k\})$, the subnets decomposed from Petri net Σ based on the index of transitions. If $P_\Delta \neq \varnothing$, $\Gamma_{pi\to P\Delta}$ $(R$ $(M_{0i})) = \Gamma_{Pj\to P\Delta}$ $(R$ $(M_{0j}))$ iff $\Phi_{P\Delta}$ $((RMG(\Sigma_i)) \cong \Phi_{P\Delta}$ $(RMG(\Sigma_j))$.

Proof: The conclusion can be easily obtained based on Definition 6 and 7, and Theorem 5 and 6.

Based on the Theorem 5 and 6, Corollary 2 and 3 can be obtained.

Corollary 2: Let $\Sigma_i = (P_i, T_i; F.M_{0i})$ ($i\in \{1, 2, 3... k\}$) be the decomposed net of $\Sigma = (P, T, F, M_0)$ based on the index of transitions and RMG(Σ_i) be the reachable marking graph of Σ_i $\Gamma_{pi\to P\Delta}$ (R (M_{0i})) = $\Gamma_{Pj\to P\Delta}$ (R (M_{0j})) iff $\Phi_{P\Delta}$ (RMG(Σ_i)) ≅ $\Phi_{P\Delta}$ (RMG(Σ_j)) for $\forall i, j\in\{1, 2, 3...k\}$ such that $P_\Delta \neq \varnothing$.

Corollary 3: Let $\Sigma_i = (P_i, T_i; F.M_{0i})$ ($i\in \{1, 2, 3... k\}$) be the decomposed net of $\Sigma = (P, T, F, M_0)$ based on the index of transitions and RMG(Σ_i) be the reachable marking graph of Σ_i $\Gamma_{pi\to P\Delta}$ (R (M_{0i})) = $\Gamma_{Pj\to P\Delta}$ (R (M_{0j})) iff $\Phi_{P\Delta}$ (RMG(Σ_i)) ≅ $\Phi_{P\Delta}$ (RMG(Σ_j)) for $\forall i, j\in\{1, 2, 3...k\}$ such that $P_\Delta \neq \varnothing$.

According to Corollary 2 and 3, Algorithm 1 presents an algorithm to decide the property invariant of the decomposed subnets based on the simplified reachable marking graph.

Algorithm 1: Let $\Sigma_i = (P_i, T_i; F.M_{0i})$ ($i\in \{1, 2, 3... k\}$) be the decomposed net of $\Sigma = (P, T, F, M_0)$ based on the index of transitions and RMG(Σ_i) be the reachable marking graph of Σ_i.

Input: P_i and RMG (Σ_i)
Output: If the properties of the decomposed subnet are invariant, then output True, otherwise output False.
Step 1: mark←True

Step 2:

```
        For i←1 to k
                For j←i+1 to k
                        If PΔ≠∅, then
                If RMG(Σi) and RMG(Σj) is
                not isomorphic on PΔ, then mark←False
                                Endif
                        Endif
                j←j+1
                Endfor
        i←i+1
Endfor
```

Step 3: returu mark.

AN EXAMPLE

Here, the example shown in Fig. 1 is used to verify the decision algorithm shown in Algorithm 1. The reachable marking graphs of the decomposed subnets are shown in Fig. 3, respectively.

- In RMG(Σ_1), $P_1 = \{p_1, p_2, p_4\}$, $M_0 = (1, 0, 0,)$, $M_1 = (0, 1, 0)$, and $M_2 = (0, 0, 1)$
- In RMG(Σ_2), $P_2 = \{p_1, p_2, p_3, p_4, p_5, p_6\}$, $M_0 = (1, 0, 0, 1, 0)$, $M_1 = (0, 1, 0, 0, 1)$, $M_2 = (0, 0, 1, 0, 1)$, $M_3 = (0, 1, 0, 1, 0)$, $M_4 = (1, 0, 0, 0, 1)$, $M_5 = (0, 0, 1, 0, 1)$

Since, $P_\Delta = \{p_1, p_4\}$, the simplified reachable marking graphs corresponding to the decomposed subnets is given in Fig. 4, where:

- In $SRMG_{P\Delta}$ (Σ_1), $M_0^i = \Gamma_{Pi\to P\Delta}(M_0) = (1, 0)$, $M'_1 = \Gamma_{pi\to P\Delta}(M_1) = (0, 0)$, and $M'_2 = \Gamma_{pi\to P\Delta}(M_2) = (0, 1)$
- In $SRMG_{P\Delta}$ (Σ_2), $M'_0 = \Gamma_{P2\to P\Delta}(M_0) = \Gamma_{P2\to P\Delta}(M_4) = (1, 0)$, $M'_1 = \Gamma_{P2\to P\Delta}(M_1) = \Gamma_{P2\text{-}P\Delta}(M_3) = (0, 0)$, and $M'_2 = \Gamma_{P2\to P\Delta}(M2) = \Gamma_{P2\to P\Delta}(M_5) = (0, 1)$

From $SRMG_{P\Delta}$ (Σ_1) and $SRMG_{P\Delta}$ (Σ_2) in Figure 4, we can see:

$$|V_1| = |V_2| \text{ and } \forall v_i \in V_1, \exists v_j \in V_2, \text{ such that } v_i = v_j$$

$$|E_1| = |E_2|$$

Thus, RMG(Σ_1) and RMG(Σ_2) is isomorphic on $P_\Delta = \{P_1, P_4\}$. It indicates that the decomposed subnets

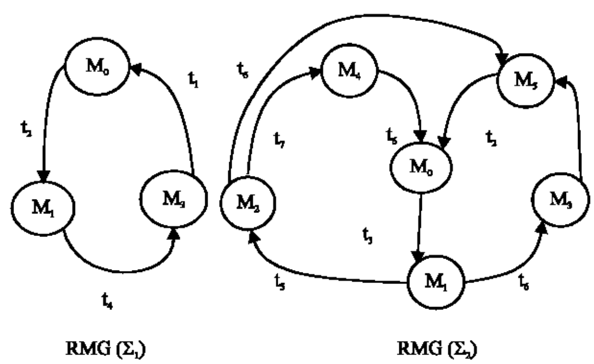

Fig. 3: The reachable marking graphs of the decomposed subnets

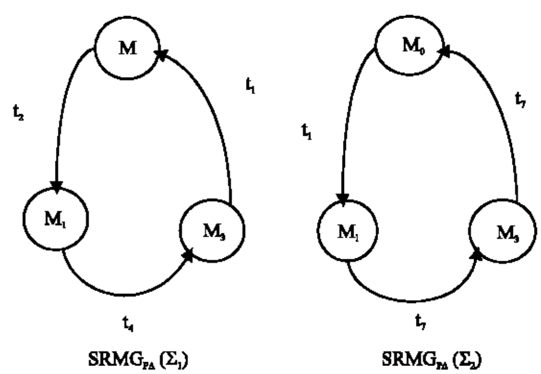

Fig. 4: The simplified reachable marking graphs

keep the state and the behavior property of the original Petri net.

CONCLUSIONS

In order to analyze properties of structure-complex Petri nets, the decomposition based on the index of transitions is a very convenient and useful approach. With this decomposition method, a structure-complex net system is decomposed to a set of structure-simple subnets, named T-Nets. There is a projection relation of the reachable marking sets and languages between the original net system and the subnet systems. However, some unnecessary states and languages are also added in the subnet systems. This study presents the deep research results on the decomposition method of Petri nets based on the index of transitions. The necessary and sufficient conditions for keeping the property invariant between the original system and the subnet systems are obtained. At the same time, a decision algorithm for the property invariant decomposition of Petri net based on the simplified reachable marking graph has been presented. The conclusions and algorithms proposed in this paper have contributions for Petri nets used to model and analyze many real physical systems.

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