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Research on High Speed Evader vs. Multi Lower Speed Pursuers in Multi Pursuit-evasion Games

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Abstract: In multi pursuit-evasion games the capability of pursuers and evaders is unequal, especially in the maximal velocity capability. Whether can the pursuers in low maximal velocity capture the evader in high maximal velocity? The success capture condition of unequal robot in multi pursuit-evasion games includes n pursuers and single evader is researched. If every robot's vision field of pursuer is infinite, the pursuers can always capture the evader even if the top velocity of the pursuers is lower than the evader when the two necessary conditions are satisfied simultaneity with appropriate strategy. (1) The velocity ratio of the pursuer and evader is higher than the value of sin (π/n) and (2) Position of the evader should be located in the convex polygon which is composed by the position of the multi pursuers and the adjacent Apollonius circles formed by the evader and each pursuer should be intersected or tangent. If the two necessary conditions are not satisfied, the pursuers may capture the evader sometimes but not necessarily. The optimal strategy of the pursuer and evader is designed in success capture conditions. The experiments can also prove that success capture conditions are correct.

Key words: Multi robot system, pursuit-evasion game, Apollonius circles, constraint condition, pursuit-evasion strategy

INTRODUCTION

Multi pursuit-evasion games as a platform that aims to survey the team cooperation and competition among robots, has been a popular issue upon which many artificial intelligence researcher focus on. The problem of two player pursuit-evasion problem was first put forward by Isaacs (1965) and he figured out the pursuit direction and total pursuit time by constructing a partial differential equation and solving the "saddle point" of the equation. After that, the multi pursuit-evasion problem was produced by Vidal *et al.* (2002). To simplify the complexity of solution, the researcher pay more attention to the discrete multi-robot pursuit-evasion problem Pu-Cheng *et al.* (2006), Lazhar *et al.* (2011) and Pu-Cheng *et al.* (2007) put forward a series of solution contained reinforcement learning, contact network and advanced contact network. Recently, researchers tend to focus on continuous multi cooperation pursuit. There are two approaches to solve this problem. The one is decomposed this problem into several two player pursuit problems by Cai *et al.* (2009). The core difficulty is how to decompose it and how to strengthen the cooperation between them and the main solution is based on hierarchical decomposition. Another approach is to use state reduce technique, such as CMAC by Justus *et al.* (2011) and fuzzy technology by Dahmani and Benyettou (2004), Herrero-Perez and Martinez-Barbera (2010), Mehrjerdi *et al.* (2010) and Herrero-Perez and Martinez-Barbera (2010) to transform the continuous state into discrete state in order to regard this problem as a discrete multi pursuit problem.

All research above is based on the precondition that the velocity capable of pursuer is not less than the evader. In this case, it is an important scientific problem that whether the pursuit actions will success when velocity capable of pursuer is less than the evader. Through the experiment, Zhi-Bao *et al.* (2004) discovers that the performance of pursuit success is determined by the number of pursuers. In general situation, the more the number of pursuers, the easier to pursuit the evader.

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V_p, V_e represent the maximum of pursuit-robot and evasion-robot. Set $\lambda = V_p/V_e$, Consider that so much studies for the situation the situation that $\lambda>1$, $\lambda = 1$, we will not to repeat them. The main purpose of this study is to survey under the situation that $\lambda>1$, if it meets some conditions, even thought the evader will take the optimal action, the pursuer still be able to catch up with the evader.

DESCRIPTION OF TWO PLAYER PURSUIT-EVASION PROBLEM

Consider the following evasion dynamic Eq. 1 which represents only two participants p, e in it:

$$\dot{x}_p = v_p \cos\theta_p, \dot{y}_p = v_p \sin\theta_p$$
$$\dot{x}_e = v_e \cos\theta_e, \dot{y}_e = v_e \sin\theta_e \quad (1)$$

Here, θ represents the movement direction of robot, set the initial position is (x_{p0}, y_{p0}), (x_{e0}, y_{e0}) and P (x_p, y_p), E(x_e, y_e) is the position of pursuit and evasion. v_p, v_e is the rate of pursuit and evasion and $v_p \in [0, V_p]$, $v_e \in [0, V_e]$. Define ε as a small given real number, we can regard it as the capture radius. If the distance between pursuer and evader less than ε, d (P (x_p, y_p), E $(x_e, y_e)) \le \varepsilon$, we consider that the evader is captured. d $(\bullet,\bullet)$ is a paradigm in Hilbert space.

As Fig. 1 showed, M (x_m, y_m) is a random point. If λ = MP/ME ($\lambda<1$), then the locus of the circle form an Apollonius circle that the characteristic as follows. The center of circle must be in the line PE, point C and D are the intersection point of the Apollonius circle and Line PE and CP/CE = λ, DP/CE = λ. We can figure out the coordinate of circle center is:

$$O\left(\frac{x_p - \lambda^2 x_e}{1-\lambda^2}, \frac{y_p - \lambda^2 y_e}{1-\lambda^2}\right)$$

through the plane geometry knowledge, the radius of the circle as the Eq. 2:

$$r = \lambda\sqrt{(x_p - x_e)^2 + (y_p - y_e)^2}/(1-\lambda^2) \quad (2)$$

From Eq. 2, we can find that the smaller λ which represents the evader's rate is faster than the pursuer, the smaller the radius of Apollonius circle and the evader has a more widely path to escape. Therefore, the pursuer brings greater difficulty of the pursuit action which is consistent with the actual situation.

Theorem 1: When pursuer and evader have a constant velocity and pursuer's velocity is slower than evader, the pursuit range is an Apollonius circle.

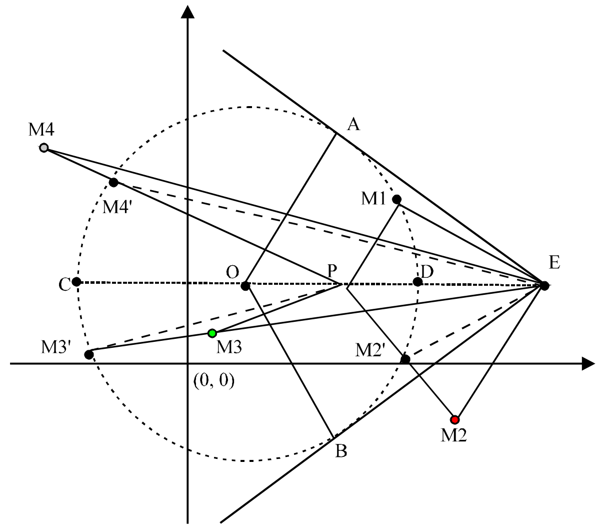

Fig. 1: Apollonius circle

In other words, only if the path of evader is included in the Apollonius circle, the pursuer is possible to catch it and the catch point must locate in the Apollonius circle.

Proof: As the Fig. 1 show, M1 is a random point in Apollonius circle, if the direction of evader E is $\overrightarrow{EM1}$ set the direction of pursuer P is $\overrightarrow{PM1}$, then:

$$|\overrightarrow{PM1}|/|\overrightarrow{EM1}| = \lambda$$

- Set after t_1 second the evader reaches M1, then $V_e t_1 = |\overrightarrow{EM1}|$
- We also know that $\lambda = V_p/V_e$
- So:

$$V_p t_1 / V_e t_1 = V_p/V_e = \lambda = |\overrightarrow{PM1}|/|\overrightarrow{EM1}|, V_p t = |\overrightarrow{PM1}|$$

Then, the pursuer will catch the evader at point M1.

If the movement direction of evader is out of $\angle$AEB, then the evasion track of evader is not in the Apollonius Circle, set M2 is random point on the movement direction of evader, $\overrightarrow{PM2}$ is the pursuer direction, M2' is the intersection point of PM2 and Apollonius circle, so: $|\overrightarrow{PM2'}|/|\overrightarrow{EM2'}| = \lambda$:

- Set t_2' is time that pursuer reaches M2', then $V_p t_2' = |\overrightarrow{PM2'}|$
- So: $V_e t_2' = |\overrightarrow{EM2'}|$

Set pursuer uses t_{p2} reaches and evader use t_{e2} reaches, then:

$$t_{e2} = \frac{|\overrightarrow{EM2}|}{V_e} < \frac{|\overrightarrow{EM2'}| + |\overrightarrow{M2'M2}|}{V_e} = \frac{|\overrightarrow{EM2'}|}{V_e} + \frac{|\overrightarrow{M2'M2}|}{V_e} = t_2' + \frac{|\overrightarrow{M2'M2}|}{V_e} = \frac{|\overrightarrow{PM2'}|}{V_p} + \frac{|\overrightarrow{M2'M2}|}{V_e} < \frac{|\overrightarrow{PM2'}|}{V_p} + \frac{|\overrightarrow{M2'M2}|}{V_p} = \frac{|\overrightarrow{PM2'}| + |\overrightarrow{M2'M2}|}{V_p} = t_{p2}$$

Therefore, in this situation the pursuer is not able to catch the evader.

If the movement direction of pursuer in the range of ∠AEB M3 is a random point that on the movement direction of evader and in the Apollonius circle, M3' is a intersect point of the track of evader and the Apollonius circle, if the direction of pursuer is $\overrightarrow{PM3}$, then $|\overrightarrow{PM3'}|/|\overrightarrow{EM3'}| = \lambda$, we presume that the pursuer costs t_{p3} second reach M3 and the evader costs t_{e3} reach M3, then:

$$t_{p3} = \frac{|\overrightarrow{PM3}|}{V_p} < \frac{|\overrightarrow{PM3'}|}{V_p} = \frac{|\overrightarrow{EM3'}|}{V_e} = t_{e3}$$

In other words, in the circle, even if pursuer and evader both take optimal strategy, the pursuer still will catch the evader.

If the movement direction of evader in the range of ∠AEB, M4 is a random point which on the track of evader movement but outer the Apollonius circle, if evader want to take the fastest strategy to reach M4', it must pass through the Apollonius Circle, in this process, it will be captured.

CONSTRAINT CONDITION OF SUCCESS IN MULTI PURSUIT_EVASION GAME WITH ONE EVADER AND MULTI PURSUERS

Besides that the velocity factor, in multi pursuit-evasion games, the sense radius of robot is also important for the pursuit. In the condition that the radius of pursuer and evader both are very small, the pursuers can ambush through communicating to form a surround circle to achieve the goal of capture. In this study, we only consider the condition that the pursuit radius is infinity. In other words, the pursuer's teams can't use the evader's sense limitation and team communication advantage to realize the success pursuit.

First necessary conditions for the success capture: We can safely draw a conclusion that if there are more than one pursuers and only one evader, smaller λ leads to smaller radius of Apollonius circle, then it is necessary to use more pursuers to catch the evader and form a surround circle in order to ensure to success.

According to theorem 1, when more than one pursuer catch only one evader, in the last phase of pursuit, if the escape track of evader must pass through the Apollonius circle, then if the pursuit take correct strategy, it must be able to catch the evader. If it exists a gap between adjacent Apollonius circles of evader and each pursuer and the evader takes the optimal strategy, it must be able to get away.

As Fig. 2 showed. E represents evader, P_1, P_2, ..., P_n represents the pursuers, O_1, O_2, ..., O_n are the circle center of Apollonius circle which is formed by P_1, P_2, ..., P_n and E. If we want to ensure there is no evasion track, then the circle O_1, O_2, ..., O_n must tangent to each other (or intersect each other), We can find out that in the condition of same number of pursuers, compared with the tangent, the intersect needs larger radius, in other words, it needs larger radio of pursuer's velocity to evader's velocity. In this study, we focus on the minimum radio of pursuer to evader, so we select the tangent condition.

Set P_{t1} is the tangent point between circle O_1 and circle O_2, P_{ti} is the tangent point between circle O_{i-1} and circle O_i. α_1, β_1 represents $\angle P_{t1}EO_2$ and $\angle O_1EO_2$, α_i, β_i represent $\angle P_{ti}EO_i$ and $\angle O_{i-1}EO_i$, then $\beta_1 + \ldots + \beta_i + \beta_n = 2\pi$, in this study, we restrict the pursuers are homogeneous and they have the same maximal velocity. Here, $\beta_1 = \ldots = \beta_i = \ldots = \beta_n$, then:

$$\alpha_1 = \beta_1 / 2 = \frac{\pi}{n} \tag{3}$$

Satisfied:

$$\sin\alpha_1 = \frac{O_2P_{t1}}{EO_2}$$

bring the coordinate of circle and Eq. 2, we get:

$$\sin\alpha_1 = \frac{\lambda\sqrt{x_2^2 + y_2^2}/(1-\lambda^2)}{\sqrt{(\frac{x_2}{1-\lambda^2})^2 + (\frac{y_2}{1-\lambda^2})^2}} = \lambda$$

Bring into Eq. 3, there:

$$\sin\frac{\pi}{n} = \lambda = \frac{V_p}{V_e}$$

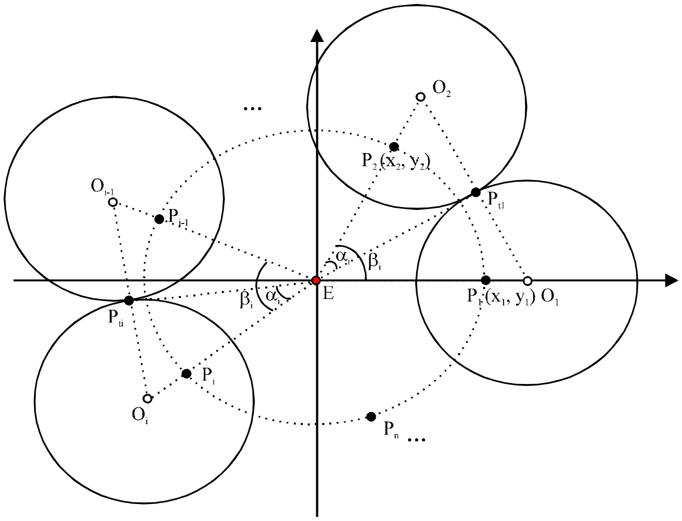

Fig. 2: Pursuit-Evasion Apollonius circle with n pursuers and one evader

In other words, the radio of maximal pursuer's velocity to evader's maximal velocity connects with the number of pursuer, more pursuer means lower requirement of pursuer's velocity.

So the first necessary condition for the success capture is:

$$\frac{V_p}{V_e} \geq \sin \pi / n$$

Second necessary conditions for the success capture: If satisfied the first necessary condition, must the pursuer be able to catch the evader? The answer is not. Figure 3 is an illustration that contains five pursuers and four evaders in different position. In this Figure, point P1, P2... P5 represents the position of five pursuers, they construct a pentagon, E1 is inner of the pentagon, E2 locates on the line of edge, E3 is located outside of the pentagon and its foot of perpendicular to the nearest edge is on the nearest edge. E4 is outer of the pentagon and its foot of perpendicular to the nearest edge is on the extension line of nearest edge.

Obviously, because of the pursuit's pace lower then the evasions, if E2 along with the direction of vertical line of P1P2, E3 along with the direction of vertical line of P3P4, E4 along with the direction of P5E4 to escape, the evader must be able to get away.

Consider E1, the pursuer is not necessarily to catch the evader; it depends on the specific position of E1. As Fig. 4 shows, although E is inner of the pentagon but the adjacent Apollonius circles which are formed by P_1 and E and by P4 and E are not tangent or intersect to each other, in other word, it exists a gap, so the evader E can also find an escape track just following the arrowhead direction showed as Fig. 4.

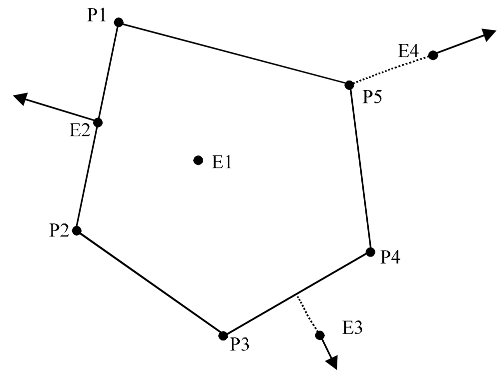

Fig. 3: Different position of multi evasion and pursuit

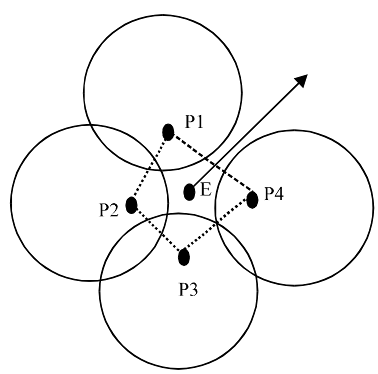

Fig. 4: The Evader in the inner of the pentagon can also escape

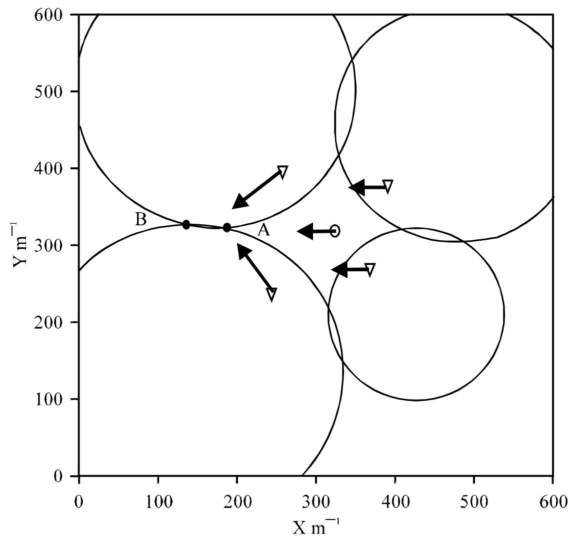

Fig. 5: Normal algorithm of pursuit strategy

In Fig. 5, if the evader is inner of the pentagon and all adjacent Apollonius circles which are formed by each pursuers and evader are tangent or intersect to each other, then the evader can not find an escape track in real time. So the pursuers keep such situation and compress the surround circle gradually. Finally, the evader should be captured.

So, the second necessary condition for the success capture is: the evader should be inner of the pentagon constructed by pursuers and all adjacent Apollonius circles which are formed by each pursuers and evader should be tangent or intersect to each other.

THE OPTIMAL STRATEGY OF EVADER AND PURSUER

Evader's strategy mainly involves two approaches. One is to escape from the closest pursuer; another is according the weight of direction to select the track to escape as described by Vidal *et al*. (2002) . However, both of these are the simple strategy, not the optimal strategy. Obviously, if the evader is outer of the convex polygon of pursuer, we can get the optimal strategy. In the convex polygon, if the evader wants to succeed in the condition that adjacent Apollonius circles are not tangent or intersect to each other, it can to select the gap area to escape. If there is no gap of each Apollonius circles, it should move to the furthest intersection point of Apollonius circle to extend the capture time. They will possibly success to escape if the pursuit does not take the optimal strategy.

To the pursuers, their goal is to surround the circle whatever the evader in the capture area, lock them in the convex polygon, then keep and try to ensure all adjacent Apollonius circle can intersect or tangent. Obviously, if every pursuer take the directly strategy, it is easily lead to the Apollonius Circle cannot intersect or tangent. In this study, we put forward a pursuit algorithm based on the study by Shiyuan and Zhihua (2010). As is showed in Fig. 5, in the condition of success capture, we select the furthest intersect point from many adjacent intersect points of Apollonius circle (This point is the point that the pursuer cost longest time to capture the evader), the pursuer strategy that constitute this point corresponding to the 2 pursuers toward the direction of intersect point in order to compress evader's escape space. The strategy of other pursuers is maintaining the movement direction as evader movement direction, try to press the evader, according to the radius of Apollonius circle and the distance between them is proportional, the radius of Apollonius circle will become larger and larger, in order to intersect to adjacent circle. If the situation cannot meet the success condition, there are at least one gap existed, the strategy of the two pursuers that near the gap is to make up the gap, other's strategy are to maintain the direction of evader and press it.

The pursuit algorithm as follows:

```
Direction pursuers get pursuers direction (Point pursuers, direction evader,
set cross points)
//Point Pursuers locates all pursuers
Direction evader Is the direction of the evader movement
Set crosspoints means the set of the intersect point of all Apollonius circles
//Direction Pursuers is the return of the function, means the Pursuers
movement direction
IF can Capture = = TRUE        //the capture condition is satisfied
    Furthest crosspoint = Get furthest crosspoint (set cross points)
    FOR i=1 to Num of Pursuers
      IF (Pursuers[i] = Producer of furthest crosspoint ())
            Direction pursuers [I] = Get direction (Pursuers [I], furthest
            crosspoint);
       ELSE
          Direction Pursuers [i]= Direction evader
ELSE
   FOR i = 1 to Num of Pursuers
            IF (Cross point exist (Pursuers [I], pursuers ([I+1] % (Num of
            pursuers-1)) = = FALSE)
                 Direction pursuers [I]=get direction (pursuers [I], pursuers
                 ((I+1) % (num of pursuers-1))
            ELSE
            Direction pursuers [I] = Direction evader
```

In the real condition, if the maximum ratio of pursuer velocity to evader velocity close to π/n, the Apollonius circle of the pursuer which moves toward the furthest point is very difficult to intersect with the adjacent Apollonius circle of pursuit, so it easy produce a gap and the gap will destroy the condition of success capture,

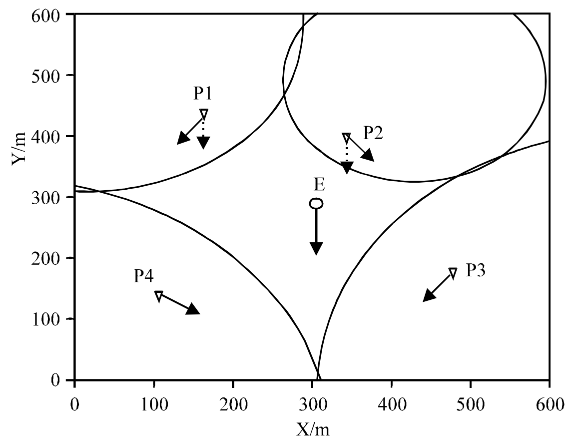

Fig. 6: Revised algorithm of pursuit strategy

provide a space for evasion to escape. As Fig. 6 shows, if pursuit P2 and P3 take the pursuit strategy which are represented by the green broken lines, their Apollonius circle are not able to intersect with each other and form a gap. Therefore, in this study, we revise the algorithm above to that, when the ratio of pursuit velocity to evasion velocity close to the threshold condition, some pursuers will move toward the furthest intersection point, meanwhile the adjacent pursuers transforms the former strategy that moves toward the evasion with the parallel direction to move toward the evader with the parallel but maintain a certain angle outside. As Fig. 6 shows, we take the strategy which is represented by red real line and it can enlarge the distance between itself and evasion and enlarge the radius of Apollonius circle in order to maintain the Apollonius circle of pursuit can meet the success pursuit condition.

Now, the pursuit strategy as follows:

```
Direction pursuers get pursuers direction (Point pursuers, direction evader, set cross points)
----------------------------------------------------------------
//Point Pursuers locates all pursuers
  Direction Evader is the direction of the evader movement
  Set cross points means the set of the intersect point of all Apollonius circles
      //Direction pursuers is the return of the function, means the pursuers movement direction
----------------------------------------------------------------
IF can Capture = TRUE       //the capture condition is satisfied
   Furthest crosspoint = Get furthest crosspoint (set crosspoints)
   FOR i =1 to num of pursuers
      IF (Pursuers [i] = = Producer of furthest crosspoint ())
           Direction pursuers [i] = Get direction (pursuers [i], furthest crosspoint);
      ELSEIF (Pursuers [i] = = Neighbour of producer of furthest crosspoint ()
                 &&Pursuers != Producer of furthest crosspoint())
           Revise angle = Get angle (direction evader, get direction (pursuers [i], evader));
           Direction pursuers [i] = DirectionEvader+ReviseAngle/2;
   ELSE
           Direction pursuers [i] = Direction evader;
----------------------------------------------------------------
Algorithm continue:
----------------------------------------------------------------
ELSE
   FOR i =1 to Num of Pursuers
      IF (Cross point exist (pursuers [i], pursuers ([i+1] % (num of pursuers-1) = = FALSE)
           Direction pursuers [i] = Get direction (Pursuers [i], pursuers ((I+1) % (Num of pursuers-1));
           Direction pursuers [i+1] = Get direction (pursuers ((I+1) % (Num of Pursuers-1), Pursuers[i]);
   ELSEIF (Pursuers [i] = = Neighbour of producer of furthest crosspoint ()
                 && Pursuers != Producer of furthest crosspoint ())
      Revise angle = Get angle (direction evader, get direction (pursuers [i], evader));
      Direction pursuers [i] = Direction evader +revise angle/2;
   ELSE
           Direction pursuers [i] = Direction evader;
----------------------------------------------------------------
```

EXPERIMENT AND ANALYSIS

The experiments are implemented in simulation way and every period is 100 ms. Pursuer and evader can observe each other's position and action in real-time, without the actual turning angle constraints, robots are able to turn instantaneous, in other words, the turning action cost no time slice. According to many times observation, if the evasion lay outside the convex polygon constructed by the pursuer, the distance between evader and pursuer will become more and more far, so we do not give this condition. And the definition of failure action is the evasion escape outside of the convex polygon.

Experiment 1: To satisfy the necessary condition 1 but not to satisfy the necessary condition 2

One random example: The initial position of every robot as follows:

P1:(239.1, 366.3); P2:(353.5, 330.1); P3:(349.5, 242.6); P4: (213.5, 259.4); E: (311.8, 274.4)

Set the velocity of evader is Ve = 4 m s^{-1}, the velocity of pursuer is Vp = 3 m s^{-1}, there are 4 pursuers, so:

$$\frac{V_p}{V_e} = \frac{3}{4} = 0.75 \geq \sin\frac{\pi}{4} = 0.707$$

satisfy the condition 1.

Although, the initial position of evader is in the convex polygon, the adjacent Apollonius circle are unable to intersect; it cannot meet the condition 2. As Fig. 7a shows the strategy of evader is move toward the gap that formed by these two pursuers and we take the revised strategy. Figure 7b is a screen shot in the 500 cycles of pursuit process. And in Fig.7c, the evader locates outside the convex polygon, so the pursuit is failure.

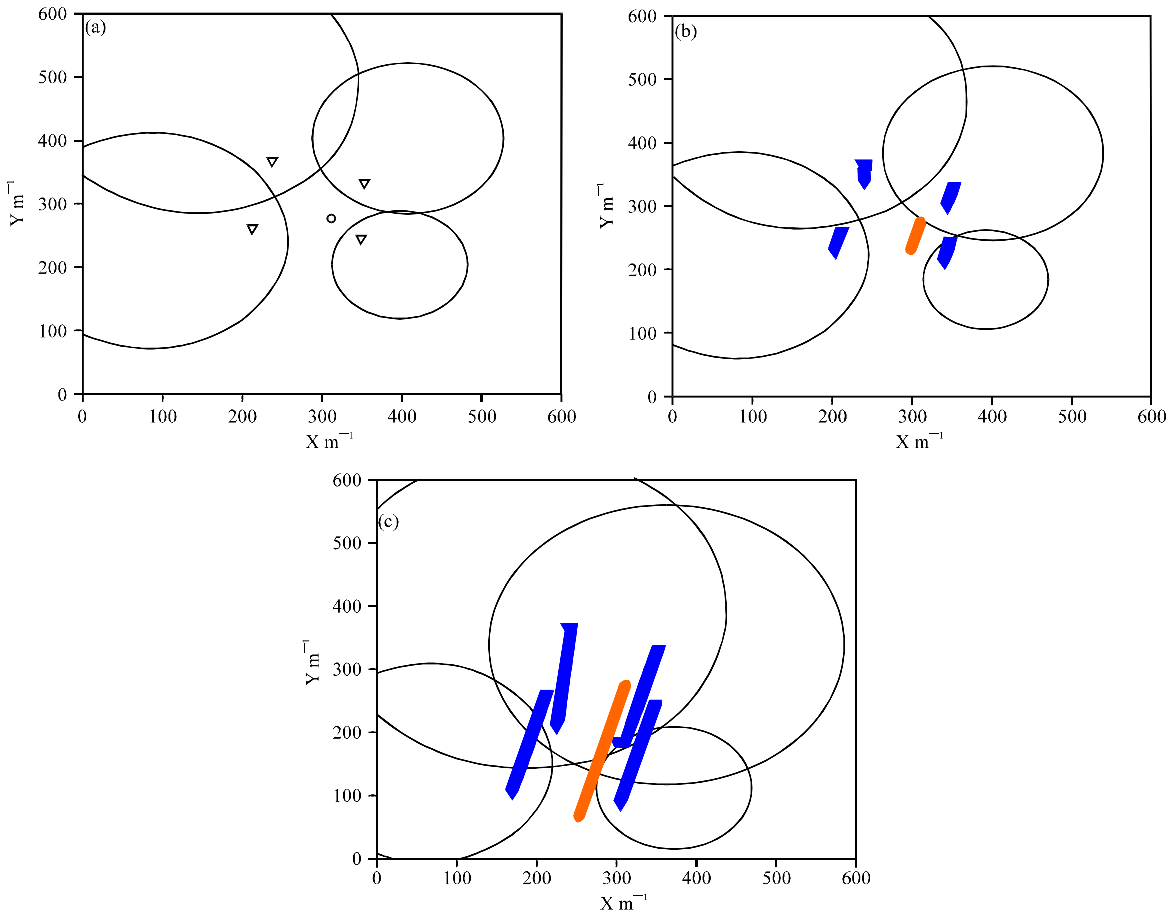

Fig. 7(a-c): Pursuit process of condition that satisfy the necessary condition 1 but not satisfy the condition 2; (a) Evader move towards gap, (b) Screen shot and (c) Evader locates outside

Under this situation, the random position and velocity and 1000 sets of experiment data, the evader always moves toward the gap, the possibility of pursuit is 100%.

Experiment 2: To satisfy the necessary condition 2 but not to satisfy the condition 1: It is easily to satisfy the condition that the evader maintain in the convex polygon but if it is required to maintain the near Apollonius circle are intersect with each other or tangent to each other, it will satisfy the condition 1 that the ratio of pursuer and evader velocity greater than or equal to sin π/n, we observe 3000 thousand sets of experiment date and find that the ratio less than sin π/n have no one satisfy the condition that the Apollonius circle are intersect with each other. According this, we can prove the conclusion that to ensure the success of pursuit action, the relation between pursuit and evasion must satisfy:

$$\frac{V_p}{V_e} \geq \sin \pi / n$$

is correct.

Experiment 3: To satisfy both the necessary condition 1 and 2: One random example: the initial positions of every robot are as follows:

P1:(197.5, 209.5); P2:(381.1, 108.0);
P3:(435.7,299.5);P4:(357.7.9,579.4);P5(81.4,370.3)
E: (286.1, 288.7)

Set the velocity of evader is Ve = 4 m sec^{-1}, the velocity of pursuer is Vp = 2.8 m s^{-1}, there are 5 pursuers, then:

$$\frac{V_p}{V_e} = \frac{2.8}{4} = 0.7 \geq \sin\frac{\pi}{5} = 0.587$$

satisfy the condition 1 and the initial position of evasion locate in the convex polygon and the Apollonius circle are intersect with each other which satisfies the condition 2. As Fig. 8a showed the evader moves toward the furthest direction of Apollonius circle and the pursuit take the normal pursuit strategy. Figure 8b is the screen shot of the 823 cycles during the pursuit process; Fig. 8c shows that the pursuit process is success.

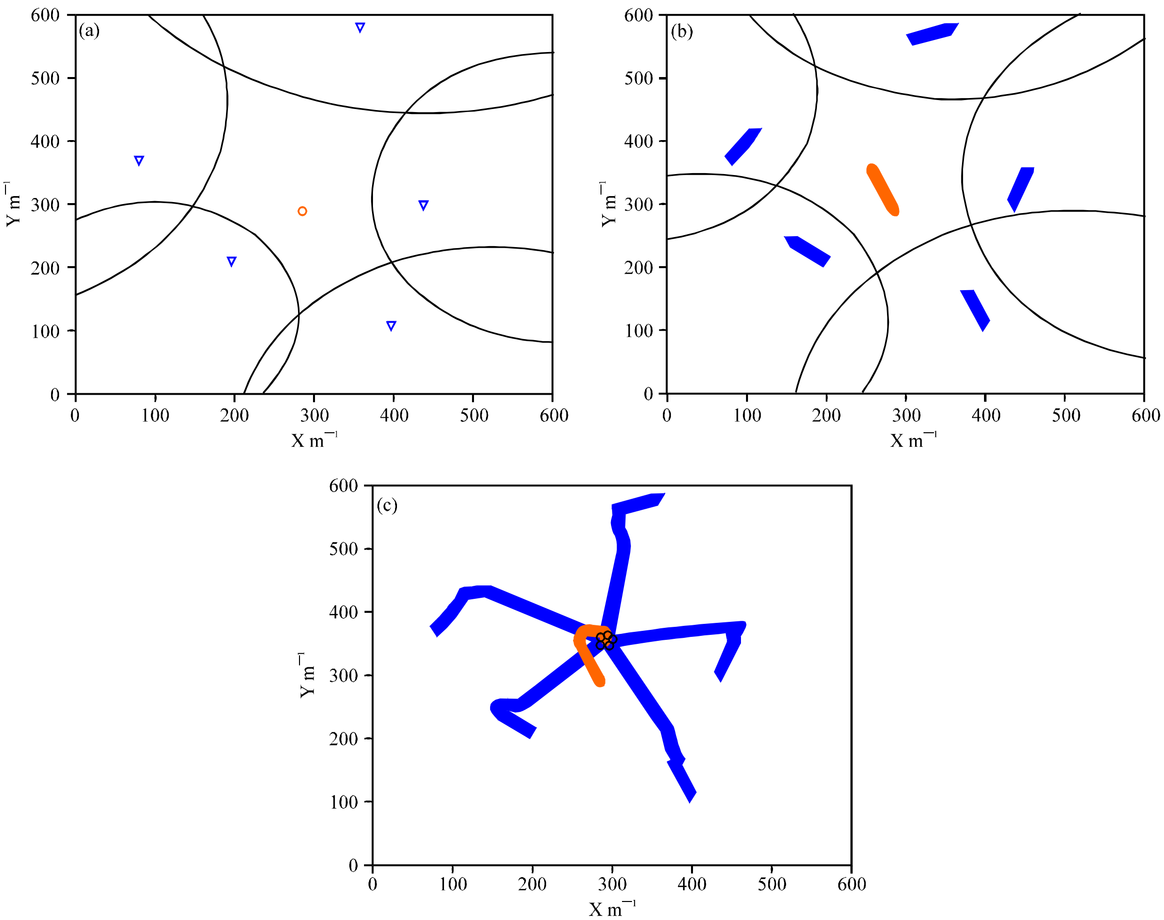

Fig. 8(a-c): Pursuit process of satisfy both the necessary condition 1 and 2; (a) Evader move towards furthest direction, (b) Screen shot and (c) Pursuit process

In the experiments, the pursuers' number is four, the velocity of evader is 4.0 m s^{-1}, the velocity range of pursuit from 2.9 to 3.9 m sec^{-1} and set 0.1 intervals, velocity of each set is selected from 1000 sets which satisfy the condition, Table 1 is the result of normal strategy and Table 2 is the revised strategy. In the table, number is the number of pursuers; sample size is the samples that satisfy the condition. Success ratio is the pursuit success possibility.

From Table 1 and 2, we can conclude that when the velocity of pursuit is 2.9, the rate of pursuit to evasion is 0.725, slightly greater than the threshold condition 0.707, the failure possibility of normal algorithm is higher; the possibility of success from 1000 sets of random experiment is 12.4%. However, the success possibility of revised algorithm is improved and reaches 92.7%. It does not reach 100% because, the pursuer strategy is not enough delicate. From the analysis of failure experiment data, we find that if the nearest intersection point does not select the closest point but select a particular point between A, B according the specific situation, as Fig. 5

Table 1: Result of normal algorithm based on the initial condition

Vp (m sec^{-1})	Ve (m sec^{-1})	Number	Sample size	Success ratio (%)
2.9	4	3	1000	12.4
3.0	4	3	1000	34.4
≥3.1	4	5	9000	100

Table 2: Result of revise algorithm based on the initial condition

Vp (m sec^{-1})	Ve (m sec^{-1})	Number	Sample size	Success ratio (%)
2.9	4	3	1000	85.6
3.0	4	3	1000	99.8
≥3.1	4	5	9000	100

showed, the pursuit performance can be greatly improved. So we find out that the simple strategy can be advanced by some machine learning algorithms such as reinforcement learning algorithms in order to achieve the completely success. In the situation that the velocity of pursuit is greater than 3.1, whatever we take the normal strategy or the revised strategy, the success possibility both can reach 100% from 9000 sets of random experiment. We also implement the experiments in three or five pursuers and one evader. Table 3 shows the success possibility of three pursuers to eight pursuers to catch one evasion under revised algorithm; the result is similar

Table 3: Different number of pursuits satisfied the pursuit condition under revised algorithm

Number	Threshold speed	Vp (m sec^{-1})	Sample size	Success ratio (%)
3	3.46	3.5	1000	86.7
3		3.6	1000	98.8
3		≥3.7	3000	100
4	2.83	2.9	1000	87.3
4		3.0	1000	98.1
4		≥3.1	9000	100
5	2.35	2.4	1000	88.9
5		2.6	1000	99.2
5		≥2.8	6000	100
6	2.0	2.1	1000	99.7
6		2.2	1000	100
6		≥2.4	9000	100
7	1.74	1.8	1000	98.7
7		2.0	1000	100
7		≥2.2	10000	100
8	1.53	1.6	1000	97.3
8		1.7	1000	100
8		≥1.8	11000	100

to the result of four pursuers and one evader. We can get the conclusion that constraint necessary condition is correct.

CONCLUSION

In this study, we conclude that in the condition that every pursuit-evasion robot has open view and unlimited pursuit space and more than one pursuit take part in the action, even though the maximum velocity of pursuit maybe lower than the evasion's, only if it satisfies that the ratio of pursuer to evader maximum velocity greater than $\sin \pi/n$, the evader locate in the convex polygon and all adjacent Apollonius circles which are formed by each pursuers and evader should be tangent or intersect to each other. The pursuers should catch the evader. The result of experiments prove that under the higher velocity, this theory is completely correct, when the velocity closes to threshold condition, if the pursuer algorithm near reasonable, it also can catch the evader. Using some machine learning algorithms to construct a more complete algorithm to ensure the success possibility of pursuer action, when it satisfies the pursuit-evasion condition, is our further work. Otherwise, if the pursuers are heterogeneous, for example they have different velocity, pursuit constraints is another research direction.

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