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ITJ

ISSN 1812-5638

# INFORMATION TECHNOLOGY JOURNAL

**ANSI***net*

Asian Network for Scientific Information  
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

## Study on Discrete Spectrum Correction Technology of RLC Measuring Instrument

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**Abstract:** RLC measuring principle based on discrete spectrum analysis is proposed. RLC is abbreviations of resistance, inductance and capacitance. The value of complex impedance is obtained by gathering sinusoidal voltage and sinusoidal current flowing through impedance measuring synchronously and taking FFT to the signal gathered. By researching on the source of errors in RLC measurement based on the analysis of discrete spectrum, discrete spectrum correction technology which can reduce errors in RLC measurement is proposed. Discrete spectrum correction technology is made of frequency correction, amplitude correction and phase correction. The precision of RLC measuring instrument is improved greatly by using ratio formula correction method based on rectangular window.

**Key words:** Discrete spectrum correction, RLC measuring instrument, impedance measurement, spectrum analysis, error

### INTRODUCTION

Impedance measurement generally is the measurement of the parameters such as resistance, capacitance, inductance, conductivity, loss angle, value of related Q and so on. Resistance represents the power loosed in circuit, capacitance and inductance represent storage of electric field power and magnetic field power.

There are many methods for impedance measurement. The common basic methods are bridge method resonance method and voltammetry. Bridge method has much higher measuring accuracy but it has to adjust balance repeatedly, long measuring time and it is difficult to realize fast automatic measurement. Resonance method requires excitation signal which has much higher frequency and it is difficult to satisfy the requirement of high accuracy. And the test speed of resonance method is difficult to be improved, since the test frequency is non-fixing. The measuring principle of voltammetry is originated from the definition of impedance. If the current flowing through the impedance measuring is known and the voltage of the

impedance measuring is measured, then the impedance measuring can be got by the ratio. In order to realize the method, the instrument must have the capacity of vector measurement and division operation. The traditional voltammetry depends on phase sensitive detector and need two sinusoidal reference signals which are orthogonal, so it complicates the measurement circuit and increases measurement cost (Li and Zhang, 2011).

The traditional voltammetry is improved in this study. The principle of RLC measuring instrument based on discrete spectrum analysis is proposed and the measuring accuracy is improved greatly by using discrete spectrum correction technology.

### PRINCIPLE OF RLC MEASURING INSTRUMENT BASED ON DISCRETE SPECTRUM ANALYSIS

Block diagram of RLC measuring instrument based on discrete spectrum analysis is shown in Fig. 1. Sine signal  $(x)t$  is input in the circuit which is made of impedance

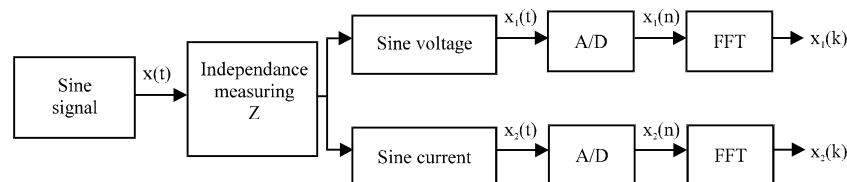


Fig. 1: RLC measuring instrument based on discrete spectrum analysis

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testing, the sine voltage  $x_1(t)$  and the sine current  $x_2(t)$  of impedance testing can be obtained. Then the sine voltage  $x_1(n)$  and the sine current  $x_2(n)$  can be got after synchronous acquisition of them. Then the sampling signal  $X_1(k)$  and  $X_2(k)$  can be obtained by taking FFT to  $x_1(n)$  and  $x_2(n)$ . Impedance measuring  $Z$  can be got because  $X_1(k)$  and  $X_2(k)$  are known.

Suppose sinusoidal excitation signal  $x(t) = A\sin(\Omega t)$ , after it flows through linear network which is made of impedance measuring  $Z$ , sinusoidal quantity of voltage and sinusoidal quantity of current of impedance measuring  $Z$  are represented, respectively as follows:

$$x_1(t) = A_1\sin(\Omega t + \varphi_1) \quad (1)$$

$$x_2(t) = A_2\sin(\Omega t + \varphi_2) \quad (2)$$

In order to measure impedance  $Z$ , we need to gather  $x_1(t)$  and  $x_2(t)$  synchronously. Sampling signal  $x_1(n)$  and  $x_2(n)$  are given by:

$$x_1(n) = x_1(nT) = A_1\sin(\Omega nT + \varphi_1) \quad (3)$$

$$x_2(n) = x_2(nT) = A_2\sin(\Omega nT + \varphi_2) \quad (4)$$

Impedance measuring  $Z$  can be expressed as  $z = |z|e^{j\varphi_z}$ . where,  $|Z| = A_1/A_2$ ,  $\varphi_z = \varphi_1 - \varphi_2$ . In order to know the values of  $|z|$  and  $\varphi_z$ , it has to take FFT to  $x_1(n)$  and  $x_2(n)$ , then  $X_1(k)$  and  $X_2(k)$  can be got by:

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n) W_N^{kn} = |X_1(k)| e^{j\varphi_1(k)}, \quad k = 0, 1, \dots, N-1 \quad (5)$$

$$X_2(k) = \sum_{n=0}^{N-1} x_2(n) W_N^{kn} = |X_2(k)| e^{j\varphi_2(k)}, \quad k = 0, 1, \dots, N-1 \quad (6)$$

We can get the following formulas based on the principle of spectrum analysis by taking FFT to continuous signal:

$$|Z| = \frac{A_1}{A_2} = \frac{|X_1(f)|}{|X_2(f)|} \quad (7)$$

$$\varphi_z = \varphi_1 - \varphi_2 = \varphi_1(f) - \varphi_2(f) \quad (8)$$

Equation 7-8 are the formulas to measure RLC by using discrete spectrum analysis.

### DISCRETE SPECTRUM CORRECTION TECHNOLOGY IN RLC MEASURING INSTRUMENT

**Errors in RLC measuring instrument using discrete spectrum analysis:** When using Eq. 7-8 to measure RLC,

frequency, amplitude and phase of harmonic signal after FFT may all have large errors, which lead to low measurement precision in measuring RLC. The reasons of the errors in RLC measuring instrument using discrete spectrum analysis can be found from the discusses about frequency, amplitude and phase:

- **Reasons causing frequency errors:** Adding window in time domain and frequency domain discretization are the main reasons which cause frequency errors (Santamria *et al.*, 2000). Harmonic signal which has one frequency with adding window in time domain lead to that the spectrum of harmonic signal change form one frequency component in theory into continuous frequency component in a section (Zhu and Youlun, 2001). After frequency domain discretization, when the frequency on the peak of the continuous spectrum after adding window is not equal to the corresponding frequency in discrete spectrum line which will cause frequency errors
- **Reasons causing amplitude errors:** Adding window in time domain and frequency domain discretization are the main reasons cause amplitude errors. The maximum amplitude relative error with rectangular window is 36.3%. The maximum amplitude relative error with Hanning window is 15.1%. The maximum amplitude relative error with Hamming window is 18.3%
- **Reasons causing phase error:** The main reasons which cause phase errors are symmetrical window function translate  $T/2$  and frequency domain discretization. Phase errors is in proportion to frequency errors (Zhu *et al.*, 2002). The larger the frequency error is, the larger the phase error is. When the maximum frequency error is , the maximum phase error is  $\pm 90^\circ$

### Discrete spectrum correction technology based on ratio correction method

**Frequency correction:** Suppose frequency spectrum function of normalization window function is  $W_1(f)$ . All amplitudes are positive, so windowing spectrum modular function is equal to windowing spectrum function totally (Jiao and Kang, 2003),  $W_1(f)$  is symmetrical to y axis as shown in Fig. 2b. Obviously, the center of main lobe is origin of coordinate.

The definition of Frequency correction is to know the value of  $\nabla f^l$ .  $\forall \nabla f^l$  ( $-1 \leq \nabla f^l \leq 0$ ) shown in Fig. 2, the amplitude of windowing spectrum function is  $W_1(\nabla f^l)$ , then the amplitude of the corresponding  $k$  spectrum line shown in Fig. 2a is  $y_k$  and  $y_k = AW_1(\nabla f^l)$ . The amplitude of windowing spectrum function is  $W_1(\nabla f^l + 1)$  for  $(\nabla f^l + 1)$ ,

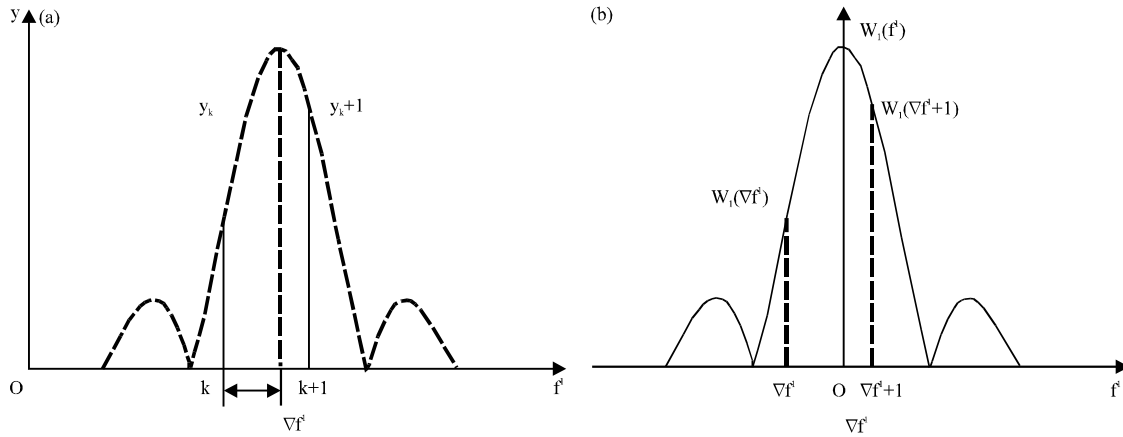


Fig. 2(a-b): Correction principle of ratio method (a) Frequency spectrum modular function of harmonic signal with normalization window and (b) Frequency spectrum modular function of normalization window function

the amplitude of the corresponding spectrum line is  $y_{k+1}$  and  $y_{k+1} = AW_1(\nabla f^l + 1)$ . Since, the function of  $W_1(\nabla f^l)$  is known,  $v$  is the ratio function between  $W_1(\nabla f^l)$  and whose  $W_1(\nabla f^l + 1)$  interval is 1.  $v$  can be described as:

$$v = F(\nabla f^l) = \frac{W_1(\nabla f^l)}{W_1(\nabla f^l + 1)} = \frac{y_k}{y_{k+1}} \quad (9)$$

where,  $v$  is the function of  $\nabla f^l$ , its value is  $y_k/y_{k+1}$ . And  $v$  is the ratio of the maximum and the secondary maximum value of amplitude spectrum which is got by taking DFT to harmonic signal with adding window.

The inverse function of Eq. 9 is defined as:

$$\nabla f^l = g(v) \quad (10)$$

So we can get the frequency correction quantity  $\nabla f^l$  which is normalized by window length. This method is defined as ratio correction formula method.

The real frequency after correction is:

$$f_0 = (k - \nabla f^l) \frac{f}{N} \quad (11)$$

**Amplitude correction:** Suppose frequency spectrum modular function of normalization window function is  $W_1(\nabla f^l)$ , from Fig. 2, we can see that main lobe function of frequency spectrum of normalization harmonic signal with adding window can be described as:

$$y = AW_1(\nabla f^l) = AW(f^l - f_0^l) \quad (12)$$

It is the convolution result between frequency spectrum of harmonic signal and frequency spectrum of normalization window function. where,  $A$  is the real amplitude. The corresponding center of main lobe is  $f_0^l$ . Substitute  $y = y_k$ ,  $f^l = k$  in Eq. 12, the new formula is shown as follows:

$$y_k = AW_1(k - f_0^l) \quad (13)$$

where,  $k - f_0^l = \nabla f^l$ , so amplitude correction formula can be got by:

$$A = \frac{y_k}{W_1(\nabla f^l)} \quad (14)$$

**Phase correction:** Phase function of normalization harmonic signal with symmetrical window after being taken FFT can be described as:

$$\phi = \theta - \pi(f^l - f_0^l) \quad (15)$$

where,  $\theta$  is the real phase.

Substitute  $\phi = \phi_k$  and  $f_1 = k$  in Eq. 15, we can get the Eq. 16:

$$\phi_k = \theta \pi(k - f_0^l) \quad (16)$$

where,  $k - f_0^l = \nabla f^l$ , the phase correction formula can be given by:

$$\theta = \phi_k + \pi(k - f_0^l) = \arctan\left(\frac{I_k}{R_k}\right) + \pi \nabla f^l \quad (17)$$

**RATIO FORMULA CORRECTION METHOD  
BASED ON RECTANGULAR WINDOW**

Now analyze the ratio formula correction method through rectangular window, on the basis of correction formula of frequency, amplitude and phase discussed above.

The definition of discretization rectangular window is described as:

$$\omega(n) = 1, \quad n = 0, 1, 2, \dots, N-1 \quad (18)$$

The modular function in main lobe after normalized by window length is given by:

$$W_1(k) = \frac{\sin(pk)}{\sin(pk/N)} \quad (19)$$

$k \in [-1, +1]$ , if  $N \gg 1$ ,  $1/N \rightarrow 0$ , then the simplified condition is:

$$\sin\left(\frac{kp}{N}\right) \approx \frac{kp}{N} \quad (20)$$

From the above simplified condition, the frequency spectrum modular function normalized by window length in which  $k$  is replaced by  $f^1$  can be described as:

$$W_1(f^1) = \frac{\sin(pf^1)}{pf^1} \quad (21)$$

The ratio correction function according to Eq. 21 can be represented as:

$$v = F(\nabla f^1) = \frac{W_1(\nabla f^1)}{W_1(\nabla f^1 + 1)} = \frac{\sin(\pi \nabla f^1)}{\pi \nabla f^1} \cdot \frac{\pi \nabla f^1 + 1}{\sin[\pi \nabla f^1 + 1]} = -\frac{\nabla f^1 + 1}{\nabla f^1} = \frac{y_k}{y_{k+1}} \quad (22)$$

The normalization frequency correction according to Eq. 22 can be got by:

$$\nabla f^1 = -\frac{1}{1+v} \quad (23)$$

Equation 22 can be expressed as following directly:

$$\nabla f^1 W_1(\nabla f^1) + (\nabla f^1 + 1) W_1(\nabla f^1 + 1) = 0 \quad (24)$$

Equation 24 indicates that take two dots  $p_1(f^1_1, y_1)$  and  $p_2(f^1_2, y_2)$  arbitrarily from the curve which is represented by Eq. 21, if  $|f^1_2 - f^1_1| = 1$  and the two dots are in main lobe, it is like the situation that spectrum line is sampled, as shown in Fig. 3. Red heart theorem about discrete spectrum correction using harmonic signal with

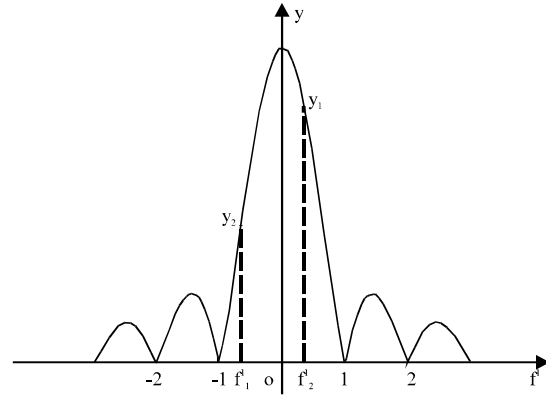


Fig. 3: Barycenter theorem graph of rectangular window

rectangular window can be got: barycenter of the two adjacent spectrum line in main lobe of amplitude spectrum is the center of main lobe, in corresponding amplitude spectrum, the frequency in barycenter is the real frequency of harmonic signal normalized by window length.

The correction frequency can be got by substituting the normalization frequency correction in Eq. 11.

Amplitude correction formula of rectangular window is got by substituting Eq. 23 in 14.

$$A = \frac{\pi \nabla f^1 y_k}{\sin(\pi \nabla f^1)} \quad (25)$$

The phase correction formula is obtained by substituting Eq. 23 in 17:

$$\theta = \phi_k + \pi(k - f^1_0) = \arctan\left(\frac{I_k}{R_k}\right) + \pi \nabla f^1 \quad (26)$$

**SIMULATIONS**

A cosine signal whose frequency is 143.2 Hz and phase is  $10^\circ$  is generated by computer. The sampling frequency is 1024 Hz, after taking FFT with 1024, the frequency resolution is 1 Hz, the accuracy amplitude of unilateral amplitude spectrum is 1, so that the correction error is easy to be observed. Analysis results and correction results are shown in Table 1 and 2. Analysis results of discrete spectrum of frequency signal are shown in Table 1. Correction results of rectangular window frequency spectrum are shown in Table 2. From the data in Table 1 and 2, we can see that the measuring precision is improved and the effect is obvious by using spectrum correction technology to implement RLC measurement:

$$y(t) = \cos(2\pi 143.2t + 10\pi/180)$$

Table 1: Analysis results of discrete spectrum of frequency signal

Window function	Frequency (Hz)	Amplitude	Phase
Rectangular window	143	0.9473525	10.845341

Table 2: Correction results of rectangular window frequency spectrum

Window function	Frequency (Hz)	Amplitude	Phase
Rectangular window	143.189621	1.005797	10.005543

### CONCLUSION

Merits and drawbacks of the common RLC measuring instrument are summarized in this study. The principle of RLC measuring instrument based on voltammetry is proposed. The accuracy of RLC measurement is improved by using discrete spectrum correction technology. Significance and value of discrete spectrum correction technology used in RLC measuring instrument is proved by simulation analysis.

### ACKNOWLEDGMENTS

This study is supported by the science and technology project of Heilongjiang province (GZ11A401) and the research project about science plan of Heilongjiang Province Education Department (12521462).

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