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Interference Analysis of Modulation/Demodulation Filter Banks in Generalized Multi-carrier Systems

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Abstract: The generalized multi-carrier (GMC) is a new type of broadband wireless multiple access technique, where the modulation and demodulation filter bank played an important role. This study investigated the interference of the GMC system with the modulation/demodulation filter bank and the close-form interference expression was obtained in terms of the inter-carrier interference and the inter-symbol interference. Moreover, three different kinds of prototype filters, named Rcosine, Remez and Nyq2, were simulated to show the influence of filter type on the interference, where both the infinite precision calculation and the finite precision computation were employed. The result shows that the interference performance would be limited by the filter delay, the quantization word length and the filter type.

Key words: Generalized multi-carrier system, filter bank, prototype filter, Nyquist filter, interference

INTRODUCTION

With the further development of the economic, mobile communication technologies have attracted much attention (Meng *et al.*, 2008; Ruan *et al.*, 2012; Wu and Niu, 2005). Moreover, extensive attention has been paid to multi-carrier technology because of its natural advantage in spectrum efficiency, resistance to frequency selective fading, support for a variety of business and some other aspects. Generalized Multi-Carrier (GMC) is a new type of broadband wireless multiple access transmission technology proposed by future project in this context (You *et al.*, 2005; 3GPP, 2008), where the modulation and demodulation filter bank played an important role (Hua *et al.*, 2004). Note that G.B. Giannakis had done some other GMC applications-related research (Giannakis *et al.*, 2000; Wang and Giannakis, 2001).

The traditional GMC system used a Root Square Raised Cosine (RRC) filter (Joos, 2010; Hua *et al.*, 2012) of length 217 to meet the floating-point performance requirements but whether it is the best choice, as well as the filter interference performance has not been reported in the literature, which is disadvantage for the practical application. Accordingly, this study presents a novel analysis method of the interference caused by the GMC filter bank, which had not been proposed by previous literature. Conventionally, people assumed that the prototype filter met Nyquist (Nyq) conditions for simplicity and also neglected the problem of finite-word-

Length (FWL) quantization. Hence, this study gives the interference analysis considering both the FWL effect and the non-ideal Nyq influence, which must provide a reference for the real-world application of the GMC system.

Starting from the time domain expression of the received signal, this study introduces the effect of filter bank to make a specific analysis of the interference. Moreover, the influence of the prototype filter is investigated, which will provide a reference for the prototype filter selection. The numerical computation is done for three prototype filters, i.e., the RRC filter, the Nyq2 filter (Harris *et al.*, 2005; Harris, 2004; Farhang-Boroujeny, 2008) and the equiripple filter designed by the Remez algorithm (Jinno *et al.*, 2010). The quantization result turns out that the quantization mainly affects the Inter-Carrier Interference (ICI) and the choice of the optimal prototype filter is limited by the quantization word length and the filter delay.

GMC STRUCTURE

The structure of the Generalized Multi-Carrier (GMC) system is shown in Fig. 1, where the subcarrier number is M and $y_k(m)$ denotes the DFT (discrete fourier transform) of the input $x_k(m)$ at the k th subcarrier. After the N -fold interpolation, $y_k(m)$ turns into $\tilde{y}_k(m)$ and convolves with the sub-channel transmitted filter $h_k(n)$, resulting in the subcarrier signal:

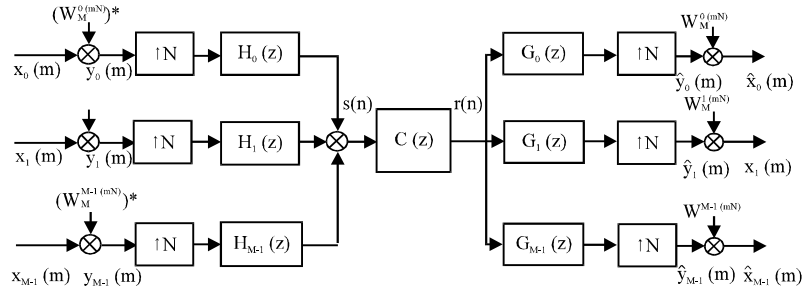


Fig. 1: The structure of GMC modulation and demodulation filter bank, ↓: Down sampling, ↑: Up sampling, E{•}: Expectation operation ⊗: Convolution product, (.)*: Conjugate operation

$$s_k(n) = \tilde{y}_k(m) \times h_k(m) \quad (1)$$

Generally, $g_l(n) = h_l(n)$, resulting in:

Then the multicarrier transmitted signal $s(n)$ can be derived as following:

$$[h_k(n) \times g_l(n)] = [h_k(n) \times h_l(n)] = \left(W_M^{l(n-\frac{L-1}{2})} \right)^* R_{k,l}(n) \quad (5)$$

$$s(n) = \sum_{k=0}^{M-1} \tilde{y}_k(m) \times h_k(m) \quad (2)$$

with $W_M = \exp(-j2\pi/M)$ and:

$$R_{k,l}(n) = \sum_{l=-\infty}^{\infty} h(l)h(n-l) \exp\left(\frac{j2\pi}{M}(k-l)\left(1-\frac{L-1}{2}\right)\right) \quad (6)$$

Since this study focuses on the study of the interference caused by the filter bank, it is reasonable to assume that the channel is invariant to simplify analysis, i.e., $c(n) = d(n)$ leading to $r(n) = s(n)$ in the receiver.

where, L denotes the prototype filter length. According to Eq. 5, $|h_k(n) \times g_l(n)|^2 = |R_{k,l}(n)|^2$ and it only relates to $|k-l|$, i.e., without accounting for the down-sampling, the interference of other sub-channels only depends on the channel offset. Substituting Eq. 5 into 4, Eq. 4 can be easily rewritten as:

At the receiver, $r(n)$ is passed through the sub-channel filter $g_l(n)$ to demodulate the l th sub-channel, then the signal before down-sampling can be written as:

$$\begin{aligned} \bar{y}_l(n) &= s(n) \times g_l(n) = \sum_k \tilde{y}_k(m) \times h_k(m) \times g_l(n) \\ &= \tilde{y}_l(m) \times h_l(m) \times g_l(n) + \sum_{k \neq l} \tilde{y}_k(m) \times h_k(m) \times g_l(n) \end{aligned} \quad (3)$$

$$\bar{y}_{k,l}(n) = \sum_{m=-\infty}^{\infty} y_k(m) \left[\left(W_M^{k(n-mN-\frac{L-1}{2})} \right)^* R_{k,l}(n-mN) \right] \quad (7)$$

From Eq. 3, the signal is divided into the desired signal of the l th sub-channel and the interference coming from sub-channels.

Next mN is used to replace n in Eq. 7 to operate an N -fold decimation, which leads to:

PROPOSED INTERFERENCE ANALYSIS

$$\hat{y}_{k,l}(m) \triangleq \bar{y}_{k,l}(mN) = \sum_{m=-\infty}^{\infty} y_k(m) \left[\left(W_M^{k(mN-mN-\frac{L-1}{2})} \right)^* R_{k,l}(mN-mN) \right] \quad (8)$$

Interference derivation: In Eq. 3, as $\tilde{y}_k(m)$ can be written into:

Combining Eq. 8 and 3, one can get the output of GMC by adding all sub-channel signals:

$$\sum_{m=-\infty}^{\infty} y_k(m) \delta(n-mN)$$

resulting in:

$$\begin{aligned} \bar{y}_{k,l}(n) &\triangleq \tilde{y}_k(m) \times h_k(m) \times g_l(n) \\ &= \left[\sum_{m=-\infty}^{\infty} y_k(m) \delta(n-mN) \right] \times [h_k(n) \times g_l(n)] \end{aligned} \quad (4)$$

$$\begin{aligned} \hat{y}_l(m) &= \hat{y}_{l,l}(m) + \sum_{k \neq l} \hat{y}_{k,l}(m) \\ &= \sum_{m=-\infty}^{\infty} y_l(m) \left[\left(W_M^{l(mN-mN-\frac{L-1}{2})} \right)^* R_{l,l}(mN-mN) \right] \\ &\quad + \sum_{k \neq l} \sum_{m=-\infty}^{\infty} y_k(m) \left[\left(W_M^{k(mN-mN-\frac{L-1}{2})} \right)^* R_{k,l}(mN-mN) \right] \end{aligned} \quad (9)$$

In Eq. 9, the received signal is divided into two parts, i.e., the signal of the l th sub-channel and the ICI coming from other sub-channels. In fact, the former can be further divided into two parts: the desired signal and the Inter-Symbol Interference (ISI) of the l th sub-channel. Note that the ISI is related to the prototype filter and it will disappear if the filter is the Nyquist filter.

In order to derive expression of the ISI, one can decompose the signal of the l th sub-channel as follows:

$$\hat{y}_l(m') = y_l(m' - 2D) \left(W_M^{j(2DN - \frac{L-1}{2})} \right)^* R_{l,1}(2DN) + \sum_{m \neq m' - 2D} y_l(m) \left[\left(W_M^{j(m'N - mN - Ld)} \right)^* R_{l,1}(m'N - mN) \right] \quad (10)$$

where, $D = (L-1)/2N$ and D denotes the filter delay in terms of the symbol rate. Since one filter causes the delay of DN , the total delay of GMC systems is $2DN$, thus, $\hat{y}_l(m' - 2D)$ should be used instead of $\hat{y}_l(m')$ in the demodulation. Obviously, the last term in Eq. 10 is the ISI.

Power calculation: Here first three assumptions are provided:

- Symbols inside one sub-channel are independent
- Symbols between different sub-channels are independent
- The power of each sub-channel is normalized, viz.,

$$\sigma_y^2 = E[|y_k(m')|^2] = 1$$

Then the average power of $\hat{y}_l(m')$ can be calculated as:

$$\sum_{m \neq m' - 2D} |R_{l,1}(m'N - mN)|^2 + \sum_{k \neq l} \sum_m |R_{k,1}(m'N - mN)|^2 \quad (11)$$

where, the first term is the power of the desired signal, the second term denotes the ISI power and the third term represents the ICI power. Moreover, if the assumption 3 does not exist, one can divide $E[|\hat{y}_k(m')|^2]$ in both sides of Eq. 11, which turns out to be the following expression:

$$1 = \frac{|R_{l,1}(2DN)|^2}{E[|\hat{y}_l(m')|^2]} + \frac{\sum_{m \neq m' - 2D} |R_{l,1}(m'N - mN)|^2}{E[|\hat{y}_l(m')|^2]} + \frac{\sum_{k \neq l} \sum_m |R_{k,1}(m'N - mN)|^2}{E[|\hat{y}_l(m')|^2]} \quad (12)$$

Generally, Eq. 12 is used for comparing performance of different filters.

SIMULATION AND ANALYSIS

This section exploits three kinds of prototype filters, namely RRC, Nyq2, Remez (equiripple filter), where the roll-off factor $\alpha = 0.22$, the oversampling factor $N = 18$, the subcarrier number $M = 16$ and the filter delay $2D = 12$ as those in conventional GMC systems. Here the Nyq2 applies a simple iterative algorithm to transform the low-pass filter into quasi-RRC structure, while it can control the passband and stopband ripple, which is different from the standard RRC filter. For the Remez filter, this study uses the function 'remez' in matlab toolbox. Since $\alpha = 0.22$ leads to the optimal delay $(2D) 22$ in the Nyq2 filter, $D \in \{4, 5, 6, 10, 11, 12\}$ are chosen as examples.

Aside from the infinite precision result, this study also presents the fix-point result, where $R_{k,i}(n)$ in Eq. 11 should be calculated with the finite word length, i.e., using the fix-point multiplication/addition/subtraction. This study exploits three quantization word lengths for the case $N/M = 18/16$: 14, 16 and 24. While the quantization word length belongs to $\{12, 14\}$ for the case $N/M = 6/4$. All numerical computations are collected at below.

Figure 2 presents the inference result with the infinite precision, where 'Nyq' and 'Rcosine' mean the Nyq2 filter and the RRC filter. From Fig. 2, we clearly see that the ISI is the main interference if the remez filter or the RRC filter is employed. Moreover, the ICI of each prototype filter produces small different. Additionally, the increase of D

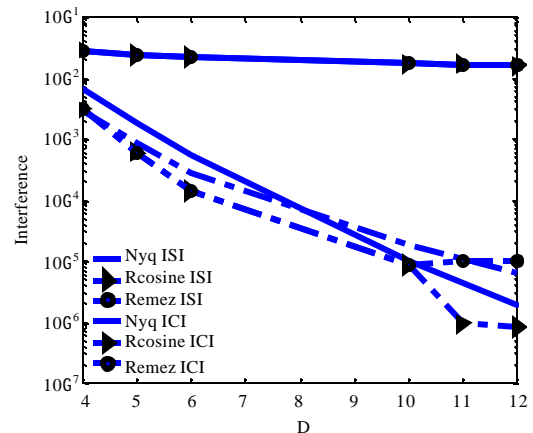


Fig. 2: ISI and ICI for three prototype filters

Table 1: Infinite precision results of interference analysis in time domain

D	Prototype filter	Desired power	ISI	ICI
4	Nyq2	0.99046	0.00665	0.00290
	Remez	0.96928	0.02766	0.00307
	Rcosine	0.96931	0.02762	0.00307
5	Nyq2	0.99729	0.00186	8.4173e-04
	Remez	0.97510	0.02429	6.11361e-04
	Rcosine	0.97502	0.02436	0.00062
6	Nyq2	0.99917	0.00054	2.8519e-04
	Remez	0.97767	0.02219	1.42375e-04
	Rcosine	0.97764	0.02222	1.35107e-04
10	Nyq2	0.99997	9.6493e-06	1.8881e-05
	Remez	0.98263	0.01736	8.5455e-06
	Rcosine	0.98269	0.01731	1.41919e-06
11	Nyq2	0.99998	4.3742e-06	1.0746e-05
	Remez	0.98355	0.01644	9.81452e-06
	Rcosine	0.98355	0.01645	9.37097e-07
12	Nyq2	0.99999	1.8578e-06	6.6016e-06
	Remez	0.98427	0.01572	9.62363e-06
	Rcosine	0.98430	0.01570	8.42259e-07

ICI: Inter-channel interference, ISI: Inter-symbol interference

usually degrades the interference. For deep insight, the data of Fig. 2 is presented in Table 1.

From Table 1 the infinite precision result demonstrates again that the desired power of Nyq2 filter is always larger than those of RRC filter and Remez filter, then its ICI and ISI is always the smallest one. Therefore, we can conclude that the performance of Nyq2 filter is the best in the sense of infinite precision. Moreover, it is obviously that the performance of the GMC filter bank is improved with D increasing.

As for the finite word length, we can also present figures analogous to Fig. 2. However, since there are too many curves, such a figure is difficult to distinguish. Hence, only Table 2 and 3 are given for the finite word length case.

From Table 2, the numerical result demonstrates that the quantization bit-width requires at least sixteen bits to conserve an accurate result of $R_{k_i}(n)$. However, the larger bit-width and small filter delay result in small performance loss. When the bit-width is 14 and D equals 10, 11 or 12, all the filters perform worse, viz., no desired powers exceed 0.9. Moreover, when the bit-width is 16 and D equals to 4, 5 or 6, the performance loss is much smaller. Hence, the larger D should be equipped with the larger quantization word length and our examples should choose small D. Moreover, when D belongs to {4, 5, 6}, it can be easily find that the ISI keeps the same level while the ICI enlarges almost 10 times after quantization. According to the table, the quantization impacts the Remez filter mostly and the Rcosine filter least.

Table 2: Fixed-point results of interference analysis in time domain I

D	Prototype filter	Word-length	Desired power	ISI	ICI	
4	Nyq2	14	0.84556	0.01037	0.14407	
		16	0.97949	0.00641	0.01410	
		24	0.99045	0.00665	0.00290	
	Remez	14	0.78994	0.04248	0.16757	
		16	0.95317	0.03166	0.01517	
		24	0.96927	0.02766	0.00307	
	Rcosine	14	0.83556	0.00918	0.15526	
		16	0.98520	0.00129	0.01351	
		24	0.99655	0.00057	0.00288	
5	Nyq2	14	0.67962	0.01544	0.30494	
		16	0.96910	0.02246	0.02844	
		24	0.99729	0.00186	0.00084	
	Remez	14	0.67527	0.03882	0.28591	
		16	0.94586	0.02884	0.02530	
		24	0.97502	0.02437	0.00062	
	Rcosine	14	0.72659	0.01501	0.25840	
		16	0.97619	0.00131	0.02251	
		24	0.99833	0.00054	0.00166	
6	Nyq2	14	0.60599	0.02047	0.37353	
		16	0.96067	0.00201	0.03731	
		24	0.99917	0.00054	0.00028	
		Remez	14	0.58126	0.03679	0.38195
			16	0.93445	0.02668	0.03880
			24	0.97760	0.02226	0.00014
	Rcosine	14	0.60810	0.02337	0.36853	
		16	0.96062	0.00260	0.03678	
		24	0.99860	0.00013	0.00127	
		10 Nyq2	14	0.27468	0.03938	0.68594
			16	0.84105	0.00845	0.15050
			24	0.99997	8.5776e-06	2317e-05
	Remez	14	0.29839	0.04238	0.65923	
		16	0.83263	0.02238	0.14500	
		24	0.98266	0.01734	3.4851e-06	
	Rcosine	14	0.28601	0.04331	0.67068	
		16	0.84637	0.00934	0.14429	
		24	0.99860	7.0696e-06	0.00118	
11 Nyq2		14	0.23441	0.04297	0.72262	
		16	0.80185	0.01109	0.18705	
		24	0.99998	3.6564e-06	1.604e-05	
Remez	14	0.25887	0.04483	0.69630		
	16	0.80531	0.02243	0.17226		
	24	0.98355	0.01734	3.4851e-06		
Rcosine	14	0.24117	0.04696	0.71187		
	16	0.80568	0.01199	0.18233		
	24	0.99881	1.9216e-05	0.00117		
	12 Nyq2	14	0.20710	0.04531	0.74759	
		16	0.76545	0.01358	0.22097	
		24	0.99999	1.5829e-06	1.2587e-05	
Remez	14	0.22459	0.04814	0.72727		
	16	0.77835	0.02270	0.19895		
	24	0.98429	0.01571	4.7357e-06		
Rcosine	14	0.20733	0.05022	0.74245		
	16	0.76600	0.01474	0.21926		
	24	0.99880	1.5015e-05	0.00118		

ICI: Inter-channel interference, ISI: Inter-symbol interference

In order to show the influence of N/M, we further study the fixed-point performance for N/M = 6/4, where the word length is 12 or 14. The result of Table 3 is similar to that of Table 2. Summarized, the performance of GMC modulation and demodulation filter bank is limited by the value of D, the quantization word length and the filter type.

Table 3: Fixed-point results of interference analysis in time domain II

D	Prototype filter	Word-length	Desired power	ISI	ICI
4	Nyq2	12	0.91680	0.027791	0.05541
		14	0.98660	0.00946	0.00394
	Remez	12	0.91564	0.03602	0.04834
		14	0.96876	0.02801	0.00324
	Rcosine	12	0.94279	0.01322	0.04398
		14	0.99615	0.00128	0.00258
5	Nyq2	12	0.82733	0.04012	0.13255
		14	0.98498	0.00450	0.01051
	Remez	12	0.86561	0.03877	0.09562
		14	0.96975	0.02391	0.00634
	Rcosine	12	0.89082	0.02462	0.08456
		14	0.99294	0.01470	0.00559
6	Nyq2	12	0.78287	0.05371	0.16342
		14	0.98287	0.00428	0.01285
	Remez	12	0.82293	0.04417	0.13290
		14	0.96875	0.02150	0.00975
	Rcosine	12	0.82704	0.04123	0.13173
		14	0.98775	0.00300	0.00926
10	Nyq2	12	0.49863	0.12961	0.37177
		14	0.92584	0.01859	0.05557
	Remez	12	0.61689	0.09290	0.61689
		14	0.98120	0.01602	0.00278
	Rcosine	12	0.58749	0.10171	0.31080
		14	0.95056	0.01193	0.03751
11	Nyq2	12	0.44080	0.14428	0.41492
		14	0.90875	0.02285	0.06840
	Remez	12	0.56078	0.10517	0.56078
		14	0.98130	0.01511	0.00359
	Rcosine	12	0.54041	0.11769	0.34190
		14	0.93591	0.01572	0.04837
12	Nyq2	12	0.41051	0.15374	0.43575
		14	0.89024	0.02805	0.08171
	Remez	12	0.52311	0.11863	0.52311
		14	0.98140	0.01426	0.00434
	Rcosine	12	0.50021	0.13130	0.36849
		14	0.91995	0.01987	0.06018

ICI: Inter-channel interference, ISI: Inter-symbol interference

CONCLUSION

In this study, we derive the closing expression of interference and signal of GMC modulation and demodulation filter bank in time domain and use Rcosine, Nyq2 and Remez filters as the prototype filter to perform the comparison, which including both the floating-point and fixed-point cases. The result comes out that the performance of GMC modulation and demodulation filter bank is limited by the value of D, the quantization word length and the filter type. In detail, the RRC filter performs the best with the bit-width is lower than 16 and the Nyq2 filter yields the best performance when the bit-width is larger than 24. These results and conclusions, especially fixed-point ones, provide a reference for the GMC application and are useful for the engineer.

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