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Optimal Wavelet Packet Division Multiplexing Channel Estimator Using Tugnait Algorithm

¹Zhong Weizhi, ¹Jing Qingfeng and ²Guo Yan

¹College of Astronautics, Nanjing University of Aeronautics and Astronautics,
Nanjing, 210016, People's Republic of China

²Nanjing Institute of Electronic Technology, Nanjing, 210039, People's Republic of China

Abstract: Channel State Information (CSI) which can be obtained by channel estimator, is critical for the performance of Wavelet Packet Division Multiplexing (WPDM) transmission system. Considering the characteristics of WPDM and the multipath channel model, a channel estimation method based on Tugnait algorithm is proposed in this study. Without prior knowledge of the channel state, the proposed method improves the channel estimation precision especially in low Signal to Noise Ratio (SNR) condition. Theoretical analysis and simulation results show that compared with the second-order stationary statistics and Singular Value Decomposition (SVD) based channel estimator, proposed method has better performance especially in low SNR condition.

Key words: WPDM, tugnait algorithm, second-order stationary statistics, SVD

INTRODUCTION

Wireless digital communication systems using multicarrier schemes such as Orthogonal Frequency Division Multiplexing (OFDM) system are extensively studied. WPDM is a new kind of multicarrier transmission method (Lindsey, 1997; Lakshmanan and Nikookar, 2006), its good time-frequency localization and flexibility motivate a lot of current researches (Jamin and Petri, 2005; Linfoot, 2008; Gao *et al.*, 2004). In WPDM system, a packet structure can be divided not only in the frequency domain but also in the time domain which gives WPDM system the ability of Narrow Band Interference (NBI) rejection and reasonable channel resource allocation. Thus, the usage of WPDM to improve the performance of wireless digital communication system is an important research topic.

Since multicarrier schemes have been applied to the fourth generation mobile communication (4G) systems, the performance of WPDM technique in fading channel environments especially in frequency selective fading channel has gained a broad interest. In fading environment, to improve transmission performance, the Channel State Information (CSI) is needed to help the equalization at the receiver. Channel estimation is a technology of obtaining CSI by processing the received signal which is one of the key technologies of the WPDM system. There are many channel estimation algorithms

which can be divided into three categories: pilot based channel estimation, blind channel estimation and semi-blind channel estimation which is the combination of the two before. The first kind inserts training sequence in the transmitted signals, which can help the system recover the signal, but system resources are wasted. No need to send pilot tones or training symbols, blind channel estimation method can improve the spectrum using efficiency of the system and it gains more and more interests. In this study, the Tugnait algorithm, which is blind channel estimation method, is utilized to form the optimal WPDM channel estimator.

The 'classical' blind channel estimators normally used in WPDM receiver are second-order stationary statistics and Singular Value Decomposition (SVD) based estimation methods (Tugnait, 1993; Tanaka *et al.*, 2006; Liang *et al.*, 2006). Second-order stationary statistics based channel estimator use two order statistics to recover the channel amplitude information, however it can only recover the amplitude information but can't obtain the phase information. In addition, the algorithm is very sensitive to noise. Based on this situation, the kind of SVD channel estimation method is put forward. The SVD channel estimator can solve the problem of phase ambiguity and in the low SNR condition the performance of SVD is better than second-order stationary statistics algorithm. However, if the SNR is very low, SVD would have the flat bottom effect which lead to ambiguity

phenomenon of the characteristic value. To solve these problems, a WPDM channel estimator using Tugnait algorithm is proposed in this study. Compared with the autocorrelation based Finite Impulse Response (FIR) system identification, the method not only has the ability of the non minimum phase system identification, but also has strong ability of noise suppression. Theoretical analysis and simulation result show that, the proposed method with lower complexity can give better estimation performance than second-order stationary statistics and SVD based estimation methods, especially in low SNR condition.

WPDM TRANSMISSION SYSTEM

WPDM is a new kind of multicarrier transmission method; a typical block of basic WPDM system is given in Fig. 1. At transmitter, the binary information data are mapped into data symbols which depends on the modulation type. Then the serial data symbols are converted to parallel blocks and Inverse Discrete Wavelet Packet Transform (IDWPT) is applied to these parallel blocks. After the transform we obtain the WPDM signal which can be expressed as follow:

$$\begin{aligned}
 s(t) &= \sum_n x_{0,0}[n] \phi_{0,0}(t - nT_0) \\
 &= \sum_{l \in \Lambda, m \in M_l} \sum_k x_{l,m}[k] \phi_{l,m}(t - kT_1)
 \end{aligned}
 \tag{1}$$

where, $x_{l,m}[k]$ denotes the data symbol and modulates the wavelet packet function $\phi_{l,m}$. (l, m) denotes the terminal node of the l th level and $l = 1-L, \dots, 0, m = 0, \dots, 2^l-1$. L is the maximum decomposition level of the wavelet packet tree (WPT). Λ denotes the set of levels and M_l being the set of the terminals at the l th level. $T_1 = 2^{-1} T_0$ presents the symbol period of the l th level.

The significant property of the WPDM is the orthogonality of the wavelet packets and this feature ensure that the subcarriers have the orthogonality. The orthogonality of the wavelet packets described as follow:

$$\begin{cases}
 \langle \phi_{n,j}, \phi_{n,k} \rangle = \delta(j-k) \\
 \langle \phi_{2n,j}, \phi_{2n+1,k} \rangle = 0
 \end{cases}$$

where, (\cdot, \cdot) denotes a convolution operation.

Three levels tree is described in Fig. 2 and the corresponding sub band structure is displayed in Fig. 3. From the figure we can find that different tree structures represent different sub channel allocation, this feature of WPDM leads to reasonable channel resource allocation.

At the receiver, after the A/D transform, the discrete wavelet packet transform (DWPT) is applied to recover the original data $\hat{x}_{l,m}[k]$.

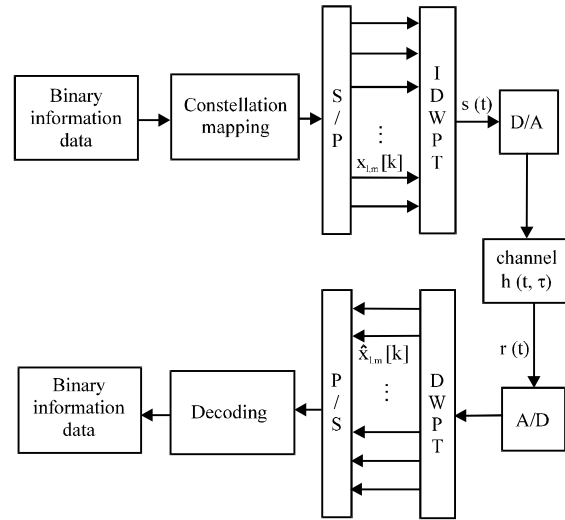


Fig. 1: Transmitter and receiver of WPDM system

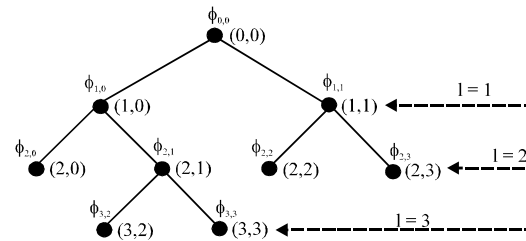


Fig. 2: Level L = 3 decomposition of the wavelet packet tree

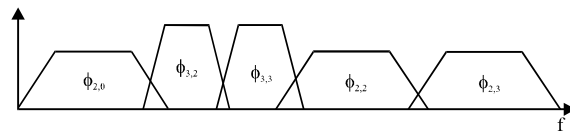


Fig. 3: Sub band structure of the tree

CHANNEL ESTIMATION METHOD

Simplified channel model: At the receiver, the simplified baseband model for the received signals can be formulated as follow:

$$\begin{aligned}
 r(t) &= s(t) * h(t, \tau) + w(t) \\
 &= \int_0^{T_{max}} s(t - \tau) h(t, \tau) d\tau + w(t)
 \end{aligned}
 \tag{2}$$

where, $w(t)$ is the the Additive White Gaussian Noise (AWGN) with zero-mean and variance of σ_w^2 . $s(t)$ is information sequence. $h(t, \tau)$ is the impulse response of the multipath channel and it can be modeled as:

$$h(t, \tau) = \sum_{n=0}^{N-1} \alpha_n(t) \exp[-j\theta_n(t)] \delta(\tau - \tau_n(t)) \quad (3)$$

where, N is the total number of propagation paths. $\{\alpha_n(t)\}_{n=1}^N$ and $\{\theta_n(t)\}_{n=1}^N$ are the random channel amplitude and phase of the n -th propagation path and $\tau_n(t)$ is the propagation delay of the n th path.

Suppose that the channel is frequency selective fading. According to Eq. 3, the channel model can be given by:

$$h(t, \tau) = \sum_{n=0}^{N-1} h_n(t) \delta(\tau - \tau_n(t)) \quad (4)$$

where, $h_n(t)$ is a complex Gaussian distribution and $\tau_1 = 0$. In this case, the received signal $r(t)$ can be described as:

$$r(t) = \sum_{n=0}^{N-1} h_n(t) s(t - \tau_n(t)) + w(t) \quad (5)$$

To introduce the Tugnait algorithm into channel estimator, the channel model must be described as a typical linear model.

Take two paths channel model for example, the received signal can be given as:

$$r(t) = h_0(t)s(t) + h_1(t)s(t - \tau) + w(t) \quad (6)$$

After matched filtering (the correlating receiver is equivalent to a receiver with a matched filter $\varphi_{0,0}(-t)$) and sampling and according to the Eq. 1, the received signal can be expressed as follow:

$$\begin{aligned} r_f[n] &= \int r(t) \varphi_{0,0}(t - nT_0) dt \\ &= h_0[n] x_{0,0}[n] + h_1[n] \sum_k x_{0,0}[k] R_\varphi(nT_0 - kT_0 - \tau) + w_{0,0}[n] \end{aligned} \quad (7)$$

where, $R_\varphi(\cdot)$ denotes the autocorrelation function of the wavelet packet function $\varphi_{0,0}(t)$. The Eq. 7 can be transformed into the following form:

$$\begin{aligned} r_f[n] &= (h_0[n] + h_1[n] R_\varphi(-\tau)) x_{0,0}[n] + \\ &\left\{ h_1[n] \sum_{k=n-kT_0} x_{0,0}[n-k] R_\varphi(kT_0 - \tau) + w_{0,0}[n] \right\} \\ &= h_f[n] x_{0,0}[n] + w_f[n] \end{aligned} \quad (8)$$

In Eq. 8, the second part includes the noise $w_{0,0}[n]$ and the interference value.

Under frequency selective fading channel, as $\tau \gg T_0$, it can assume that $R_\varphi(-\tau) \approx 0$. Assuming that the channel is slowly fading channels, therefore the channel coefficients are constant during a frame. In this case, the Eq. 8 can be given by:

$$r_f[n] = h_0 x_{0,0}[n] + h_1 \left[n - \frac{\tau}{T_0} \right] + w_{0,0}[n] \quad (9)$$

Similarly, when there are L paths and the time delay of the i th path meet the condition that $\tau_i = iT_0$. In the case, Eq. 9 can be formulated as follows:

$$r_f[n] = \sum_{i=0}^{L-1} h[i] x_{0,0}[n-i] + w_{0,0}[n] = y[n] + w[n] \quad (10)$$

where, $h[i]$ presents the channel parameters which need to be estimated. The model described in Eq. 10 is a typical ARMA linear model and thus the estimation of $h[i]$ belongs to the FIR system identification. Using high cumulant to identify the FIR system is a hot issue (Tugnait, 1991), thus, the model established in Eq. 10 is the foundation of the proposed channel estimation method.

Channel estimation performance evaluation: Channel estimation is a technology of obtaining CSI by processing the received signal, which is one of the key technologies of the WPDM system. WPDM channel estimation system is described in Fig. 4. In the receiver end, the received signal $r(t)$ passes through matched filter firstly and then be sampled. After these actions, channel estimation techniques are used to obtain the CSI to assist the late equalization.

Channel estimation accuracy has greatly influence on system performance. Two paths channel model, which given by Eq. 7, is taken as an example to describe the estimation error and the noise impact on the system performance.

According to Eq. 8, the channel is compensated by the channel estimate:

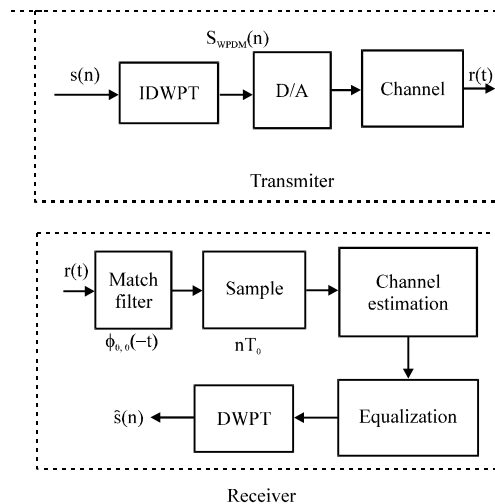


Fig. 4: WPDM channel estimation system

$$\hat{r}_f[n] = \frac{r_f[n]}{\hat{h}_f[n]} = x_{0,0}[n] + \left(\frac{e_f[n]x_{0,0}[n] + w_f[n]}{\hat{h}_f[n]} \right) \quad (11)$$

where, $\hat{h}_f[n]$ is the estimation value of $h_f[n]$ and $e_f[n]$ is the estimation error. Passing the compensated sequence $\hat{r}_f[n]$ through wavelet packet decomposition filter, it can be denoted as:

$$\begin{aligned} \hat{x}_{l,m}[k] &= \sum_n \hat{r}_f[n] f_{l,m}[n-2^l k] \\ &= x_{l,m}[k] + \sum_n \left\{ \frac{e_f[n]x_{0,0}[n] + w_f[n]}{\hat{h}_f[n]} \right\} f_{l,m}[n-2^l k] \end{aligned} \quad (12)$$

where, $f_{l,m}[n]$ is the equivalent filter of the (l,m) terminal node. It can be found that only the real part of the second term in Eq. 12 is of interest. For the notation simplicity, it can be denoted as:

$$I_{r,l,m}[k] \approx \text{Re} \left\{ \sum_n \frac{\{e_f[n]x_{0,0}[n] + w_f[n]\} f_{l,m}[n-2^l k]}{\hat{h}_f[2^l k]} \right\} = \frac{\text{Re} \{e_{r,l,m}[k]\}}{|\hat{h}_f[2^l k]} \quad (13)$$

where:

$$e_{r,l,m}[k] = \text{Re} \left\{ \sum_n \{e_f[n]x_{0,0}[n] + w_f[n]\} f_{l,m}[n-2^l k] \right\} \quad (14)$$

$e_f[n]$ is a sequence of independent random variables. Thus, $e_{r,l,m}[k]$ is the sum of N correlated random variables, where N is the length of filter $f_{l,m}[k]$. However, $e_{r,l,m}[k]$ can still be approximated by Gaussian distribution. Therefore, the mean and variance of $e_{r,l,m}[k]$ are:

$$E\{e_{r,l,m}[k]\} = 0 \quad (15)$$

and:

$$\begin{aligned} \sigma_{f,l,m}^2 &= E\{e_{r,l,m}^2[k]\} \\ &= \frac{1}{2} (\sigma_e^2 \sigma_x^2 + \sigma_w^2) \sum_n f_{l,m}^2[n-2^l k] + \\ &\frac{1}{2} \sigma_x^2 \sigma_w^2 \sum_n \sum_j R_{h_x}((n-j)T_0) \sum_{i=0}^{\infty} R_{\varphi}(i-\tau) R_{\varphi}(j-n+i-\tau) f_{l,m}[n-2^l k] f_{l,m}[j-2^l k] \end{aligned} \quad (16)$$

where, σ_e^2 is the power of estimation error, σ_x^2 denote the power of the signal $x_{0,0}[n]$ and σ_w^2 is the power of AWGN $w_{0,0}[n]$. Thus, the probability density function of $I_{r,l,m}[k]$ can be obtained as:

$$P_{h_m}(z) = \frac{1}{2^{3/2} \sigma_{f,l,m} \left(\frac{z^2}{2\sigma_{f,l,m}^2} + \frac{1}{\sigma_{h_f}^2} \right)^{3/2}} \sigma_{h_f}^2 \quad (17)$$

If binary transmission is assumed for each terminal, the probability of error of $x_{l,m}[k]$ is:

$$\begin{aligned} P_{l,m}(\epsilon) &= \Pr(\hat{x}_{l,m}[k] > 0 | x_{l,m}[k] = -1) \\ &= \Pr(I_{r,l,m}[k] > 1) = \int_1^{\infty} P_{h_m}(z) dz \\ &= \frac{1}{2} \left(1 - \frac{\sigma_{h_f}^2}{\sqrt{\sigma_{h_f}^2 + 2\sigma_{f,l,m}^2}} \right) \end{aligned} \quad (18)$$

It can be seen that the probability of error at terminal (l, m) is a function of $\sigma_{f,l,m}^2$. For $\sigma_{f,l,m}^2$ contain the noise value, and from Eq. 18 it can be found that the bit error rate would increase with the noise power increasing. Therefore, it can be found that noise is the main factors affecting the estimation precision.

Tugnait algorithm based channel estimation method:

Here, according to the simplified channel ARMA model and the conclusion of channel estimation performance evaluation, a Tugnait algorithm based WPDM channel method is proposed which belongs to the system identification. The proposed algorithm use third-order cumulant which can effectively inhibit the noise impact on the estimated results (Noh *et al.*, 2006) and increase the estimation precision.

Before the discussion of the proposed method, some restrictive conditions must be considered:

- The signal sequence $\{x_{0,0}[n]\}$ is independent and identically distributed and non-Gaussian with $E\{x_{0,0}^2[n]\} = \sigma_x^2$, $E\{x_{0,0}^3[n]\} = \gamma_{3x} \neq 0$ and $E\{x_{0,0}^6[n]\} < \infty$
- The noise sequence $\{w_{0,0}[n]\}$ is assumed to be zero mean, i.i.d and independent of $\{x_{0,0}[n]\}$ with $E\{w_{0,0}^2[n]\} = \sigma_w^2$ and $E\{w_{0,0}^3[n]\} = \gamma_{3w} = 0$

The objective is to estimate the coefficients $h[i]$ from the cumulat statistics of the observations $\{r_f[n], 1 \leq n \leq N\}$. In order to reduce the noise effect on the estimation performance, the third-order cumulant operation has been done on both sides of the Eq. 10 and it can be expressed as:

$$\begin{aligned} c_{3r}(m,m) &= c_{3y}(m,m) + c_{3w}(m,m) \\ &= c_{3y}(m,m) = E\{y[n]y[n+m]y[n+m]\} \\ &= \gamma_{3x} \sum_{i=0}^{L-1} h[i]h^2[i+m] \end{aligned} \quad (19)$$

From the Eq. 19 it can be found that, the noise cumulant value $c_{3w}(m, m)$ (Noh *et al.*, 2006) which greatly reducing the noise effect on the channel parameter

estimation. In order to estimate the channel parameter $h[i]$, let $C(z)$ denote the z-transform of the sequence $c_{3x}(m, m)$, then:

$$C(z) = \gamma_{3x} \sum_{i=0}^{L-1} h[i] z^i \sum_{k=0}^{L-1} h^2[i] z^{-k} \quad (20)$$

$$= \gamma_{3x} H(z^{-1}) H_2(z)$$

The autocorrelation function of $y[n]$ is denoted as:

$$R_y(m) = E\{y(n)y(n+m)\} \quad (21)$$

$$= \sigma_x^2 \sum_{i=0}^{L-1} h[i] h[i+m]$$

And its z transform is $P(z) = \sigma_x^2 H(z) H(z^{-1})$. Eliminate $H(z^{-1})$ from Eq. 20 and $P(z)$ to get:

$$H_2(z)P(z) = \frac{\sigma_x^2}{\gamma_{3x}} H(z)C(z) \quad (22)$$

With the time domain expressions of Eq. 22, the Eq. 23 and 24 can be obtain and by solving the equations, the channel parameters $h[i]$ can be obtained.

$$\sum_{i=0}^{L-1} h[i] [c_{3x}(m-i, m-i)] - \sum_{i=0}^{L-1} \{ \epsilon h^2[i] \} R_r(m-i) = -c_{3x}(m, m) \quad (23)$$

Where $-(L-1) \leq m \leq -1$ and $(L-1)+1 \leq m \leq 2(L-1)$

$$\sum_{i=0}^{L-1} h[i] [c_{3x}(i-m, L-1)] - \{ \epsilon h[L-1] \} R_r(m) = -c_{3x}(-m, L-1) \quad (24)$$

where, $1 \leq |m| \leq (L-1)$

where, $\epsilon = \gamma_{3x} / \sigma_x^2$ and in the above equations $R_r(m)$ presents the autocorrelation function of $r_r[n]$ which can be expressed as:

$$R_r(m) = E\{r_r(n)r_r(n+m)\} = \sigma_w^2 \sum_{i=0}^{L-1} h[i] h[i+m] + \sigma_w^2 \delta(m) \quad (25)$$

where, $\sigma_w^2 \delta(m)$ is the noise autocorrelation function. Equation 23 and 24 are parameterized with $2(L-1)+2$ unknowns and $4(L-1)$ equations and by solving those equations, the channel parameter $h[i]$ can be successively obtained. Equation 23 and 24 constituted the Tugnait based channel estimation algorithm. Take two path channel model as an example, in the example $L = 2$ and Eq. 23 and 24 can be described as:

$$h[1]c_{3x}(m-1, m-1) - \sum_{i=0}^1 \{ \epsilon h^2[i] \} R_r(m-i) = -c_{3x}(m, m) \quad (26)$$

where, $m = -1$ and $m = 2$

$$h[1]c_{3x}(1-m, 1) - \{ \epsilon h[1] \} R_r(m) = -c_{3x}(-m, 1) \quad (27)$$

where, $|m| = 1$

From the Eq. 26 and 27, it can be found can find that, the number of equations is four and the channel parameters $h[0]$ and $h[1]$ can be easily obtained by solving these four equations.

But there is one point to note, the boundaries of m lead to the noise autocorrelation function $\sigma_w^2 \delta(m) = 0$ and because of this, the $R_r(m)$ can not be effected by noise which leads to the Eq. 23 and 24 immune from the noise. Therefore, the Tugnait based channel estimation algorithm has better performance than the others.

SIMULATION RESULTS

Here, the estimation performance of the proposed channel estimator by simulation verification is evaluated, and the simulation results with the typical second-order stationary statistics (Tanaka *et al.*, 2006) and SVD (Bertrand *et al.*, 2002) based channel estimator are compared.

In simulation experiment, the transmission channel is assumed to be five-ray fading channel which is slow fading and coherent to the linear model described in Eq. 10, the channel parameter is empirically chosen as $h = [0.9355, 0.4057, 0.1763, 0.7917, 0.6154]$ and the time delay is $T = [0, T_0, 2T_0, 3T_0, 4T_0]$. Moreover, the channel is corrupted by AWGN.

The comparison results of the real value and the estimation value of channel parameter h is described in Fig. 5. In the simulating operation, the estimation value is gained by the Tugnait algorithm based estimation method when SNR is 20 dB and the parameter h is expressed as a function of frequency.

From the simulation results, it can be found that the real value and the estimation value nearly the same. To further investigate the performance of the algorithm, the performance of different channel estimation methods are evaluated by simulation and the results are plotted in Fig. 6. From the simulation results, it can be observed that comparing with the typical second-order stationary statistics and SVD based channel estimator, the proposed estimation method has better performance, especially in low SNR condition. This is because the proposed method identifies the system parameters by solving equation group and coupled with third-order

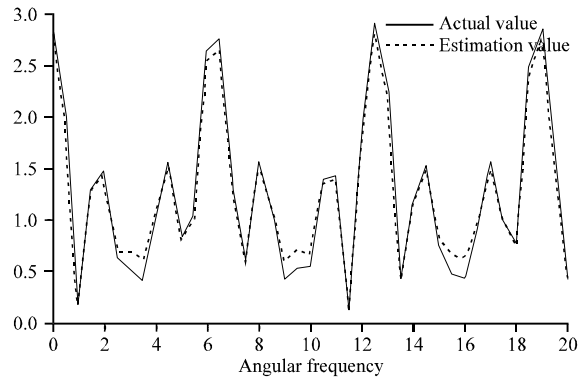


Fig. 5: Comparison of actual value and the estimation value

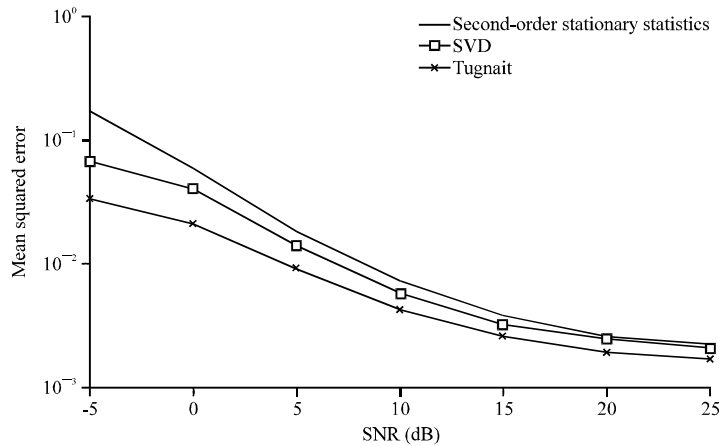


Fig. 6: Performance of different estimation methods

cumulants which can effectively suppress the AWGN and improve the estimation precision.

CONCLUSION

Based on simplified channel model and analysis of the factors affect channel estimation performance, a Tugnait algorithm based WPDM channel estimation method is presented in this study. Theoretical analysis proves the feasibility to decrease the channel estimation error with noise suppression by Tugnait algorithm. Simulation result show that the channel parameters of proposed method approach the real value well and the mean squared error of proposed method is lower than SVD and second-order stationary statistics based channel estimators, especially in low SNR condition.

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