http://ansinet.com/itj

ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 12 (6): 1239-1244, 2013
ISSN 1812-5638 / DOI: 10.3923/itj.2013.1239.1244

An Improved Cooperative Spectrum Sensing Algorithm Based on Random Matrix Theory

[1]Deng-Yin Zhang, [2]Xue-Feng Lin and [3]Xue-Mei Wang
[1]Science and Technology Park, [2]School of Telecommunication and Information Engineering,
[3]School of Computer Science and Technology,
Nanjing University of Posts and Telecommunications, Nanjing, China

Abstract: The combination of random matrix theory and cooperative spectrum sensing is one of the research hotspots of spectrum sensing. However, in multi-cognitive-user circumstance, the general combination algorithm has low detection performance under the case of few cognitive users. In this study, an improved CMME (Cooperation between the Maximum and Minimum Eigenvalue) based cooperative spectrum sensing algorithm was proposed. Through decomposing the signal achieve the goal of increase the number of logical sensed signals. The improved algorithm achieve the target of increasing signal related information. Moreover, the simulation results show that the proposed algorithm has a better perceived performance than classical CMME algorithm.

Key words: Cognitive radio, spectrum sensing, random matrix theory, signal decomposition, eigenvalue

INTRODUCTION

In order to solve the contradiction between low utilization of wireless spectrum and the so-called spectrum scarcity, Cognitive Radio (CR) proposes secondary access to already-licensed spectrum in an opportunistic fashion. And the fundamental issue in CR is cognitive users should detect and occupy the spectrum hole effectively on a non-interfering basis to primary users. There are some common methods involved in spectrum sensing, such as matched filter detection, energy detection, cyclostationary feature detection, relay detection (Zhu *et al*., 2010) etc. However, the detection performance of these methods are limited due to noise uncertainty. In recent years, some scholars have found that large-dimensional Random Matrix Theory (RMT) can be applied in spectrum sensing. Zeng and Liang (2007) presented the algorithm based on the maximum and minimum eigenvalue of large-dimensional RMT (MME) (Zeng and Liang, 2007) which enhanced the detection performance in case of a small number of samples. Large-dimensional RMT-based cooperative spectrum sensing (LSC) algorithms is proposed (Cardolo *et al*., 2008) which increases the probability of signal detection except for small amount of samples. And then, the Difference between the Maximum eigenvalue and the Minimum eigenvalue algorithm (DMM) (Wang and Lu, 2010), CMME (Lei *et al*., 2009), Double Eigenvalue Threshold algorithm (DET) (Kai-Tian and Zhen, 2010) algorithms emerged in succession, in which multiple users are deployed to perform sensing task and reporting their signal samples to fusion center. It has been found that the detection ability of them is weak when the amount of participating cognitive users is not large enough. In fact, in practical CR system, the capacity of the system is not a stable value (Mei-Ling *et al*., 2011), the total number of cognitive users is limited by the limited bandwidth (Sun *et al*., 2007). Besides, considering such cases as device running out of energy and physical damage etc, the actual number of collaborative user is changing at any time (Letaief and Zhang, 2009). Then how to ensure the detection performance of spectrum sensing in the case of the small number of collaborative users? After studying the above algorithms thoroughly, this paper puts forward an improved spectrum sensing algorithm on the basis of CMME. In light of the correlation detection theory, this algorithm effectively increases the signal correlation information in the signal covariance matrix by the operation of splitting and reconstructing on the sensed signals. As a result, the proposed algorithm solving the problem of degrading performance CMME encounters in case of few cognitive users.

SENSING MODEL

In the study, the cooperative sensing scene is shown in Fig. 1. Here, exist multiple Secondary Users (SU)

Corresponding Author: Deng-Yin Zhang, Science and Technology Park, Nanjing University of Posts and Telecommunications, Nanjing, China

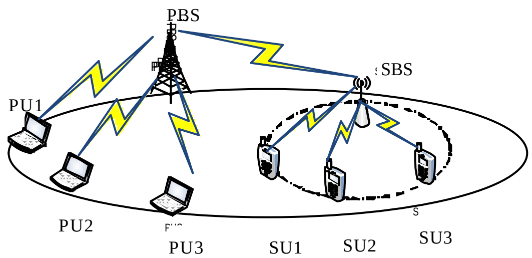

Fig. 1: Cooperative spectrum sensing scenarios

each of which sends signal samples receiving from Primary User (PU) to the Secondary Base Station (SBS) separately. And SBS processes the information collected and makes the final decision on whether some spectrum hole exists.

For any SU, the hypothetical model for spectrum sensing at the ith time instant is formulated as Eq. 1:

$$x(i)=\begin{cases} n(i), & H_0 \\ h(i)s(i)+n(i), & H_1 \end{cases} \quad (1)$$

where, x(i) and s(i) denote the signal sample received by SU and the signal transmitted by PU, respectively. n(i) represents the noise with variance σ^2 and h(i) is channel gain coefficient. In H_0, primary signal is absent while only noise is received in H_1.

The following three matrixes: X, n and S which denote the received signal matrix, noise matrix and transmitted primary signal matrix, respectively:

$$X=[X_1^T \quad X_2^T \quad \dots \quad X_M^T] \quad (2)$$

$$n=[n_1^T \quad n_2^T \quad \dots \quad n_M^T] \quad (3)$$

$$S=[S_1^T \quad S_2^T \quad \dots \quad S_M^T] \quad (4)$$

Assume there are M cognitive users and they operate independently. The collected samples are organized in SBS in form of Eq. 5 which is a M×N dimensional matrix:

$$X=\begin{pmatrix} X_1^T \\ X_2^T \\ \vdots \\ X_M^T \end{pmatrix}=\begin{pmatrix} x_1(1) & x_1(2) & \cdots x_1(i)\cdots & x_1(N) \\ x_2(1) & x_2(2) & \cdots x_2(i)\cdots & x_2(N) \\ \vdots & \vdots & \ddots & \vdots \\ x_M(1) & x_M(2) & \cdots x_M(i)\cdots & x_M(N) \end{pmatrix} \quad (5)$$

In practical cognitive environment, the number of cognitive users is changing at any time. When fewer SUs participate in sensing, the dimensions of X decreases accordingly. And it means the correlation degree of covariance matrix in CMME is reduced which results in the decline of detection performance. In order to solve this problem, it is essential to enhance the correlation in covariance matrix. Hence, the matrix X would be transferred as follows: each vector in Eq. 5 is divided into q parts with the same length, each of which includes L = N/q samples. After this splitting operation on the original signal vector from jth user, the according q parts could be described as:

$$X_{j1}(i')=h_{j1}(i')S_{j1}(i')+n_{j1}(i'),i'=1,\dots,\frac{N}{q}$$

$$X_{j2}(i')=h_{j2}(i')S_{j2}(i')+n_{j2}(i'),i'=\frac{N}{q}+1,\dots,\frac{N}{q}\times 2$$

$$\dots,X_{jm}(i')=h_{jm}(i')S_{jm}(i')+n_{jm}(i'),i'=\frac{N}{q}\times(m-1)+1,\dots,\frac{N}{q}\times m,\dots$$

$$X_{jq}(i')=h_{jq}(i')S_{jq}(i')+n_{jq}(i'),i'=\frac{N}{q}\times(q-1)+1,\dots,\frac{N}{q}\times q$$

Hence, M sensing signals are total decomposed to K = M×q sub-signal vectors and a new matrix with the dimension of K×L was constructed as Eq. 6:

$$Y=\begin{Bmatrix} Y_{11} \\ Y_{12} \\ \vdots \\ Y_{jm} \\ \vdots \\ Y_{M(q-1)} \\ Y_{Mq} \end{Bmatrix}=\begin{Bmatrix} X_{11}(1) & X_{11}(2) & \cdots & X_{11}(L) \\ \vdots & \vdots & \cdots & \vdots \\ X_{1q}(1) & X_{1q}(2) & \ddots & X_{1q}(L) \\ \vdots & \vdots & \cdots & \vdots \\ X_{jm}(1) & \ddots & X_{jm}(i) & X_{jm}(L) \\ \vdots & \ddots & \ddots & \vdots \\ X_{Mq}(1) & X_{Mq}(2) & \cdots & X_{Mq}(L) \end{Bmatrix} \quad (6)$$

where, Y_{jm} (j = 1,···,M; m = 1,···,q) denotes the mth part of the jth signal vector in X. And the matrix Y can be represented in the same form with Eq. 1 in 7:

$$Y = H'S'+n' \quad (7)$$

In which, the vector H', S', n' are as follows:

$$H' = (h_{11}, h_{12}, \cdots, h_{jm}, \cdots, h_{Mq})$$
$$S' = (S_{11}, \cdots, S_{1q}, \cdots, S_{jm}, \cdots, S_{Mq})_T$$
$$n' = (n_{11}, \cdots, n_{1q}, \cdots, n_{jm}, \cdots, n_{Mq})^T$$

ALGORITHMS AND ANALYSIS

Given below is the improved CMME spectrum sensing algorithm which based on large dimensional random matrix, it is assumed that the channel noise is white, the transmitted symbols and noise are uncorrelated. General steps of the improved algorithm are as follows:

- SBS collects signal samples from different cognitive users and forms the matrix X as shown in Eq. 5
- For each row vector in X, decomposing operation is performed to form q pieces of subsignal vectors with the same length. And then all the sub-signal vectors will be arranged to construct a new matrix Y as shown in Eq. 6
- According to equation $R_Y = \frac{1}{L} YY^T$, compute the covariance matrix R_Y. And obtain the maximum eigenvalue λ_{max} and the minimum eigenvalue λ_{min} of the matrix R_Y
- Make the final decision based on the rule:

$$\lambda_{max} > \gamma\ \lambda_{min} m\ H_1$$

$$\lambda_{max} > \gamma\ \lambda_{min} m\ H_0$$

where judgment threshold γ is defined as:

$$\gamma = \frac{(\sqrt{N}-\sqrt{M^2q})^2}{(\sqrt{N}+\sqrt{M^2q})^2}\left(1+\frac{(\sqrt{N}+q\sqrt{M})^{-2/3}\cdot q^{-1/3}}{(MN)^{1/6}}F_1^{-1}(1-P_{fa})\right)$$

Then, the following will give an analysis on the algorithm and decision threshold. For any two vectors of the matrix Eq. 7, the correlation between them is described as Eq. 8 which indicates the cross-correlation and can simplified as $R_{jm\times xy}(i) = E[h_{jm}(i)s_{jm}(i)s_{xy}(i)^T]$ except when j = x, m = y:

$$\begin{aligned} R_{jm\times xy}(i) &= E\,(X_{jm}\,(i)X_{sy}\,(I)^T) \\ &= E[(h_{jm}\,(I)s_{jm}\,(i)+n_{jm}(i))\ h_{sy}\,(i)+n_{sy}(i)^T \\ &= E[h_{jm}\,(i)s_{jm}(i)s_{xy}(i)_T+n_{jm}(i)n_{xy}(i)^T] \end{aligned} \quad (8)$$

Since the cross-correlation detection eliminates the effect of noise self-correlation on the signal, the cross-correlation detection in weak signal detection is superior to the self-correlation detection. Let R_Y be the covariance matrix of Y:

$$R_Y = E(YY^T) = E[(H's')(H's')^T]+E[n'(n')^T] = R_{s'}+R_{n'} = R_{s'}+\sigma^2 I_K \quad (9)$$

where, σ^2 is the variance of the noise and I_K is the unit matrix with order K. In R_Y, all the elements are cross-correlation values except for the diagonal ones. For CMME algorithm, covariance matrix R_X is from matrix 5. From the analysis between matrix 5 and 7, it can be found that the proposed algorithm increases the number of collaboration users logically. Moreover, R_Y expands in dimension by q times then R_X and comprises more correlation information. Accordingly, the decision threshold in improved algorithm is closer to the ideal one which will result in the increase of detection probability.

Actually, the number of signal sample in processing is limited, so the actual covariance matrix are as follows:

$$\hat{R}_Y(L) = \frac{1}{L} YY^T \quad (10)$$

$$\hat{R}_{S'}(L) = \frac{1}{L}(H's')(H's')^T \quad (11)$$

$$\hat{R}_{n'}(L) = \frac{1}{L} n'(n')^T \quad (12)$$

If signal and noise are considered as random process, the following relationship is established:

$$R_Y \approx \hat{R}_Y(L) = \hat{R}_{S'}(L) + \hat{R}_{n'}(L) \quad (13)$$

Let, λ_{max} and λ_{min} be the maximum and minimum eigenvalue of $\hat{R}_Y$ and ρ_{max} and ρ_{min} be the maximum and minimum eigenvalue of $\hat{R}_{S'}$. When H_1, according to Eq. 13, there is $\lambda_{max} = \rho_{max}+\sigma^2$, $\lambda_{min} = \rho_{min}+\sigma^2$ and $\lambda_{max}/\lambda_{min}>1$. In H_0, $\lambda_{max} = \lambda_{min} = \sigma^2$ and $\lambda_{max}/\lambda_{min} = 1$. In practice, since covariance matrix is based on limited number of sampled data, it is impossible for N to be infinite, so is L. Therefore, the decision rule should be adjusted to:

(a) If $\lambda_{max} > \gamma\lambda_{min}$, signal exists
(b) Otherwise, signal does not exist

As a special Wishart random matrix, the joint Probability Density Function (PDF) about eigenvalues of $\hat{R}_Y(L)$ is so complicated that its exact expression has not been found till now. However, according to the recent researches of I.M.Johnstone and K. Johansson on the distribution of the maximum eigenvalue of $\hat{R}_Y(L)$, the following theorems (Tracy and Widom, 1996) are established:

- **Theorem 1:** Assume that the noise is real. Let $A(L) = (L/\sigma^2)\cdot R(L)$, $\mu = (\sqrt{L-1}+\sqrt{K})^2$ and $\nu = (\sqrt{L-1}+\sqrt{K})^2\cdot(1/\sqrt{L-1}+1/\sqrt{K})^{1/3}$. Assume that $\lim_{L\to\infty} K/L = c(0<c<1)$. Then λ_{max} (A(L))-μ/ν converges (with probability one) to the Tracy-Widom distribution of order 1
- **Theorem 2:** Assume that the noise is complex. Let $A(L) = L/\sigma^2 \hat{R}_Y(L)$, $\mu' = (\sqrt{L}+\sqrt{K})^2$ and $\nu' = (\sqrt{L}+\sqrt{K})(1/\sqrt{L}+1/\sqrt{K})^{1/3}$. Assume that $\lim_{L\to\infty} K/L = c(0<c<1)$. Then λ_{max} (A(L))-μ'/ν converges (with probability one) to the Tracy-Widom distribution of order 2

Let F_1 and F_2 be the cumulative distribution of the Tracy-Widom distribution with order 1 and 2, respectively. And the inverse function can be used to compute the point value, for example, $F_1^{-1}(0.95) = 0.98$, $F_1^{-1}(0.9) = 0.45$. Let P_d be the probability of detection which refers the probability of the algorithm having detected the signal. Let P_{fa} be the probability of false alarm which refers to the probability of detecting signal in absence of any data transmission from PU. Since, there is no information about the signal (actually we even do not know if there is signal or not), it is difficult to set the threshold based on the P_d. Hence, here choose the threshold based on the P_{fa}:

$$\begin{aligned} P_{fa} &= P\left(\frac{\lambda_{max}}{\lambda_{min}} > \gamma\right) = P\left(\frac{\sigma^2}{N}\lambda_{max}(\hat{R}(L)) > \gamma\lambda_{min}\right) \\ &\approx P(\lambda_{max}(\hat{R}(L)) > \gamma(\sqrt{L}-\sqrt{K})^2) \\ &= P\left(\frac{\lambda_{max}(\hat{R}(L)) - \mu}{\nu} > \frac{\gamma(\sqrt{L}-\sqrt{K})^2 - \mu}{\nu}\right) \\ &= 1 - F_1\left(\frac{\gamma(\sqrt{L}-\sqrt{K})^2 - \mu}{\nu}\right) \end{aligned} \tag{14}$$

Given μ and ν, the decision threshold γ is derived as:

$$\gamma = \frac{\eta_2}{\eta_1}\left(1 + \frac{(\sqrt{L}+\sqrt{K})^{-2/3}}{(LK)^{\frac{1}{6}}} F_1^{-1}(1-P_{fa})\right) \tag{15}$$

where, η_1, η_2 is the minimum and maximum convergence eigenvalue value of M-P law (Mehta, 2006), respectively, shows as $\eta_1 = \sigma^2(1-\sqrt{c})^2, \eta_2 = \sigma^2(1+\sqrt{c})^2$. Given L = N/q, K = M*q, the decision threshold γ turns to:

$$\gamma = \frac{(\sqrt{N}-\sqrt{M^2q})^2}{(\sqrt{N}+\sqrt{M^2q})^2}\left(1 + \frac{(\sqrt{N}+q\sqrt{M})^{-2/3} \cdot q^{-1/3}}{(MN)^{1/6}} F_1^{-1}(1-P_{fa})\right) \tag{16}$$

Compared with CMME detection threshold, the threshold γ of this algorithm is more accurate. For complex noise, the only difference is the function F_1 will be replaced by F_2, the Cumulative Distribution Function (CDF) of the Tracy-Widom distribution of order 2.

SIMULATION

According to the theoretical analysis, due to the decomposition of the sensing signal increase the number of cooperative logic signals, the improved algorithm has better detection performance.

Assume that the primary user signal experiences Rayleigh fading and the noise is a Gaussian White one. The transmitter signal is a narrow-band FM signal and its bandwidth is limited in 10 kHZ, with the sampling frequency five times than the bandwidth. For comparison, the CMME algorithm is also simulated.

Figure 2 shows how the ratio of the maximum eighenvalue and minimum eighenvalue and the decision threshold change with sample number N. Here, the Signal-to-Noise Ratio (SNR) was set to -10dB, cooperative cognitive users M = 4, each sensing signal was decomposed to q = 2 sections, the probability of false alarm $P_{fa} = 0.05$. It can be seen that the decision threshold of the algorithm varies with sampling number and it tends to be nearer to the ideal value when N gets larger. On the other hand, due to the presence of the false alarm ratio, it seldom happens that the minimum and maximum eighenvalue value ratio exceeds the threshold value. Figure 2 illustrates the effectiveness of the decision threshold.

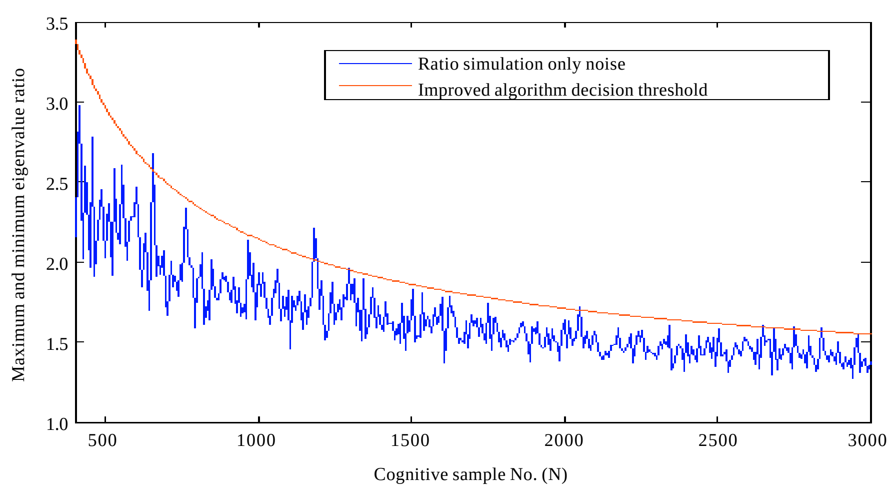

Fig. 2: Relationship between the theoretical and the actual threshold value

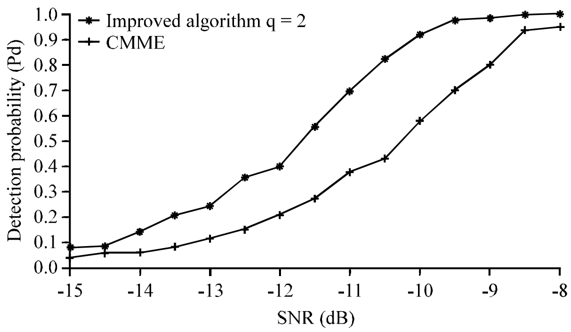

Fig. 3: Detection performance comparison of the two algorithms

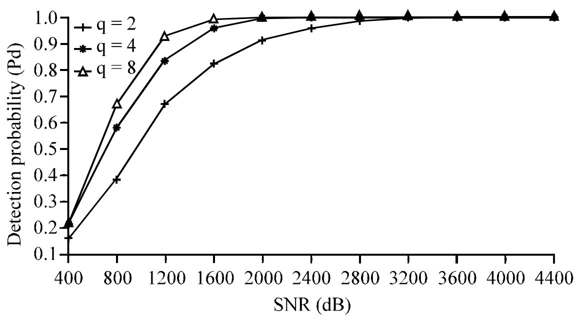

Fig. 4: Number of sub-signals affect detection performance of improved algorithm

Figure 3 compares the detection performance between the improved algorithm and CMME with SNR ranging from -15 to -8 dB. Here, let M = 4, q = 2, N = 2000. As shown in Fig. 3, the detection probability of both algorithms both increase accordingly as SNR increases. And it is obvious that the detection probability of proposed algorithm is greater than the CMME algorithm. For example, when SNR is -11 dB, the detected probability of improved algorithm is close to 70% while CMME it is close to 40%. This simulation result illustrate the detection performance of the improved algorithm is better than CMME algorithm.

Figure 4 illustrates the relationship between detection probability and sample number N in proposed algorithm when q = 2, q = 4, q = 8, respectively. Here, M = 4. Obviously, the larger N is, the finely signal is decomposed. And with the same sample number, the probability of detection increases when q is greater. In addition, as q increases, the speed of increment in detection probability gets lower. So, if M is fixed, by choosing an appropriate q can we achieve the required increase in detection performance.

The simulation results show that, through splitting and reconstructing on the sensed signal, the improved algorithm can get some performance gain in detection. However, in the calculation of the maximum and minimum eighenvalue values, the time complexity $o(K^3)$ is related to K(K is the number of dimension of the covariance matrix). So, compared with CMME, it is unavoidable for the improved algorithm to introduce higher computational overhead.

CONCLUSION

Based on the correlation detection theory, this study presents an improved algorithm based on the CMME to solve the problem of degrading detection performance of CMME in case of low signal -to-noise ratio and less collaborative users. In spite of the additional complexity introduced by the splitting and reconstructing operations on sensed signals, simulation results show that, compared with CMME, the proposed algorithm achieves better detection performance in deed.

ACKNOWLEDGMENTS

The study was partially supported by Swedish Research Links [No. 348-2008-6212], the National Natural Science Foundation of China [6107193], China's Project 863 [2010AA701202] and SRF for ROCS, SEM [NJ209002]. The assistance of LI Kuankuan and Zhang Hui in the preparation of the figures is greatly appreciated.

REFERENCES

Cardolo, L.S., M. Debbah and P. Bianchi, 2008. Cooperative spectrum sensing using random matrix theory. Proceedings of the International Symposium on Wireless Pervasive Computing, May 7-9, 2008, Santorini, Greece, pp: 334-338.

Kai-Tian, C. and Y. Zhen, 2010. DET cooperative spectrum sensing algorithm based on random matrix theory. J. Electron. Inform. Technol., 32: 129-134.

Lei, W., Z. Bao-Yu and L. Lei, 2009. Cooperative spectrum sensing based on random matrix theory. J. Electron. Inform. Technol., 31: 1925-1929.

Letaief, K.B. and W. Zhang, 2009. Cooperative communications for cognitive radio networks. Proc. IEEE, 97: 878-893.

Mehta, M.L., 2006. Random Matrices. 3rd Edn., Academic Press, Lonton, UK., pp: 394-438.

Mei-Ling, L., Y. Chao-Wei, L. Lin and Y. Rui-Zhe, 2011. Performance analysis and optimization of cooperative spectrum sensing for maximizing secondary throughput. J. Commun., 32: 53-60.

Sun, C., W. Zhang and K.B. Letaief, 2007. Cooperative spectrum sensing for cognitive radios under bandwidth constraints. Proceedings of the IEEE International Conference on Wireless Communications and Networking, March 11-15, 2007, Kowloon, China, pp: 1-5.

Tracy, C.A. and H. Widom, 1996. On orthogonal and symplectic matrix ensembles. Commun. Math. Phys., 177: 727-754.

Wang, Y.X. and G.Y. Lu, 2010. DMM based spectrum sensing method for cognitive radio systems. J. Electron. Inform. Technol., 32: 2571-2575.

Zeng, Y. and Y.C. Liang, 2007. Maximum-minimum eigenvalue detection for cognitive radio. Proceedings of the 18th Annual IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, September 3-7, 2007, Athens, Greece, pp: 1-5.

Zhu, J., B.Y. Zheng and Y. Zhou, 2010. Cooperative spectrum sensing in multiser cognitive radio networks with best relay selection. Acta Electronica Sinica, 38: 92-98.