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# Power Allocation Algorithm of Opportunities Cooperative Relay System Over Nakagami-m Fading Channels

[1]Xiaoyou Yu, [1,2]Ling Yin, [1]Tiejun Chen and [1]Yubin Bai
[1]School of Information Science and Engineering, Hunan University, 410082 Changsha, China
[2]SiHe Yuan Lab, School of Information Science and Engineering, Hunan University,
Changsha, 410080 Hunan, People's Republic of China

**Abstract:** In order to improve the performance of the cooperative relay communication system, this study proposed power allocation algorithm aiming at minimizing the system BER (Bit Error Ratio). Firstly, the relay model of system is constructed, which is proved to be convex optimization problem. Then, BER performance expression and optimal power allocation algorithm are presented. And ξ is defined as power allocation factor. Furthermore, sub-optimal power allocation algorithm of a simple and effective optimal power allocation algorithm is developed, which only depends on the average channel gains of the relays. Therefore, each relay nodes don't need to update channel status information in real-time and the closed form of ξ can be obtained. Simulation and numerical results are shown that power allocation factor is very close to the theoretical value. Compared with equal power allocation algorithm, the BER performance gains can be achieved by optimal and sub-optimal power allocation algorithms.

**Key words:** Power allocation algorithm, opportunistic relay selection, bit error ratio, Nakagami-m fading channel

## INTRODUCTION

Users through share each antenna to form a virtual antenna array in the cooperative communication network and space diversity can be achieved effectively to resist fading, therefore, it has become a hot research in recent years (Gomez-Cuba *et al*., 2011), while the relay selection and power allocation is an important research direction in cooperative communication (Maham and Hjorungnes, 2009a; Al-Qahtani *et al*., 2011). The author first proposed opportunistic selection based on the instantaneous channel state information and each time only select the largest SNR (Signal Noise Ratio) of the relay nodes to transmit information (Bletsas *et al*., 2006). Authors have studied error rate or the outage probability performance in AF (Amplify-and-Forward) opportunistic relaying collaborative network over different channel model (Chu, 2011; Zhang *et al*., 2009; Maham and Hjorungnes, 2009b) and draw the corresponding closed-form solution. Directed at the issue of whether to adopt cooperative transmission in a communication system, authors proposed opportunistic selection based on maximum ratio combining and sub-collection methods (Zou *et al*., 2009). Allocating the limit total power of the cooperative relay communication system rationally can improve the performance of the system. For the complexity of channel coefficient, McLaurin expand method was used and power allocation algorithm proposed, which based on minimizing the outage probability, the algorithm does not obtain the closed-form solution of optimal allocation factor. Yuan *et al*. (2010) and Li *et al*. (2008) analyzed allocation algorithms the over Rayleigh fading channel, the optimal and sub-optimal power allocation algorithm have been proposed to minimize outage probability, compared with equal power allocation algorithm, the proposed algorithm can enhance the performance of the system 1 to 2 dB. The author introduced greedy algorithm to analyze the optimal power allocation of orthogonal frequency sub-channels (Bakanoglu *et al*., 2011). Unlike Li *et al*. (2008), this study used Nakagami fading channel, which is a more realistic channel mode but it is more complexity.

The whole work is organized as follows: the relay model of system is constructed. Then, BER (bit error ratio) performance is presented, optimal and sub-optimal power allocation algorithms are introduced. Since the power allocation factor can be obtained by sub-optimal power allocation and also depends on average channel gains. Equal power allocation algorithm is utilized to compare with the two proposed algorithms in this study. Nakagami channel model is adopted in our simulation experiment and the accuracy of the three algorithms is compared with respect to different average channel gains and noise

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**Corresponding Author:** Xiaoyou Yu, School of Information Science and Engineering, Hunan University, Changsha, 410082, China

levels. The numerical simulation results show that the sub-optimal power allocation algorithm performs low complexity and efficiency, while the optimal power allocation algorithm archives best performance.

## SYSTEM MODEL

The cooperative relay communication system consists of N relay node, a source node S and a destination node D (Fig. 1), all the relay nodes are included in the set $S_{relay} = \{1, 2, \ldots, N\}$. Each node is equipped with only a pair of omni-directional antenna and can't send and receive information at the same time. $h_{si}$ and $h_{di}$ are channel coefficients of the source node S to the relay node $R_i$ and the relay node $R_i$ to the destination node D. Assuming that all the channels are conformed to independent Nakagami-m distribution, wherein m is fading parameters, the smaller m, the channel fading is more serious. The PDF (probability density function) of the power strength gain $\beta = |h|^2$ can be expressed as (Yuan *et al*., 2010):

$$p_\beta(x) = \frac{m^m}{\Gamma(m)\Omega^m} x^{m-1} \exp(-\frac{m}{\Omega}x) \tag{1}$$

The one-way cooperative relay model was shown in Fig. 1. Assuming there is no direct transmission link between the source node and the destination node. Channel noises $n_0$ is AWGN (Additive White Gaussian Noise) with zero mean and unit variance $N_0$ and $N_0 = 1$. In the whole system, the total emission power of source node and the nodes participated in cooperative relay was limited to P, set $\xi$ to power distribution factor, therefore, the source node transmission power $P_s = \xi P$, opportunistic relaying node transmitting power is $P_r = (1-\xi)P$, $\xi \in (0,1)$. The transmission process is divided into two stages. Firstly, the source node transmits information to all relay nodes. Then, the relay node forwards it to the destination node:

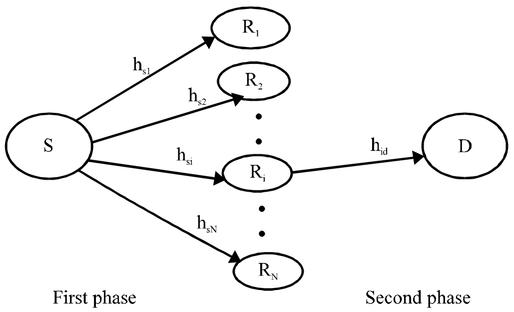

Fig. 1: Channel models of cooperative relay system

**First stage:** The source node broadcast message to all the relay nodes, all the relay nodes are eavesdropping all the time, so the received signal and SNR of the selected relay node can be expressed as:

$$\gamma_{si} = \frac{P_s |h_{si}|^2}{N_0} \tag{2}$$

$$y_{si} = h_{si}x + n_0 \quad i \in S_{relay} \tag{3}$$

**Second stage:** Selecting the appropriate relay node by opportunistic relay selection algorithm, the selected relay node use the AF model to forward information, so the received signal and SNR of the destination node can be expressed as:

$$\gamma_{id} = \frac{P_{ri} |h_{id}|^2}{N_0} \tag{4}$$

$$y_{id} = \partial_i h_{id} y_{si} + n_0 \quad i \in S_{relay} \tag{5}$$

Amplification factor can be expressed as:

$$\partial_i = \sqrt{P_{ri}} \Big/ \sqrt{P_s |h_{si}|^2 + N_0}$$

where, $P_{ri}$ is the transmission power of the ith relay node. Through arbitrary relay link S→$R_i$→D, the output SNR can be expressed as:

$$\gamma_i = \frac{\partial_i^2 |h_{si}|^2 |h_{id}|^2 P_s}{\partial_i^2 |h_{id}|^2 N_0 + N_0} = \frac{\gamma_{si}\gamma_{id}}{\gamma_{si} + \gamma_{id} + 1} \tag{6}$$

## OPPORTUNISTIC RELAY SELECTION

Based on opportunistic relay selection algorithm (Bletsas *et al*., 2006), the relay node was selected, which has a maximum output SNR from all candidate relay node, namely $\gamma_{b^*} = \max_{i \in S_{relay}}(\gamma_i)$ :

$$\gamma_{b^*} = \max_{i \in S_{relay}}(\frac{\gamma_{si}\gamma_{id}}{\gamma_{si} + \gamma_{id} + 1}) = \max_{i \in S_{relay}}(\frac{\frac{\zeta P |h_{si}|^2}{N_0} \frac{(1-\zeta) P |h_{id}|^2}{N_0}}{\frac{\zeta P |h_{si}|^2}{N_0} + \frac{(1-\zeta) P |h_{id}|^2}{N_0} + 1}) \tag{7}$$

Let:

$$\overline{\gamma} = \frac{P}{N_0},\ \beta_{si} = |h_{si}|^2, \beta_{id} = |h_{id}|^2$$

The Eq. 7 can be simplified to:

$$\gamma_{b^*} = \max_{i\in S_{relay}}\left(\frac{\zeta\bar{\gamma}\beta_{si}(1-\zeta)\bar{\gamma}\beta_{id}}{\zeta\bar{\gamma}\beta_{si}+(1-\zeta)\bar{\gamma}\beta_{id}+1}\right) = \max_{i\in S_{relay}}\left(\bar{\gamma}\frac{\beta_{si}'\beta_{id}'}{\beta_{si}'+\beta_{id}'+1/\bar{\gamma}}\right) \qquad (8)$$

The PDF of $\beta_{si}'$ and $\beta_{id}'$ can be expressed as:

$$p_{\beta_{si}'}(x) = \frac{m_{si}^{m_{si}}}{\Gamma(m_{si})\Omega_{si}'^{m_{si}}}x^{m_{si}-1}\exp\left(-\frac{m_{si}}{\Omega_{si}'}x\right) \qquad (9)$$

$$p_{\beta_{id}'}(x) = \frac{m_{id}^{m_{id}}}{\Gamma(m_{id})\Omega_{id}'^{m_{id}}}x^{m_{id}-1}\exp\left(-\frac{m_{id}}{\Omega_{id}'}x\right) \qquad (10)$$

where, $\Omega_{si}' = E[\beta_{si}'] = \zeta E[\beta_{si}] = \zeta\Omega_{si}, \Omega_{id}' = E[\beta_{id}'] = (1-\zeta)E[\beta_{id}] = (1-\zeta)\Omega_{id}$.

The PDF of $\gamma_i$ can be seen from Eq. 6 (Zhang *et al.*, 2009):

$$p_{\gamma_i}(x) = a_i\bar{\gamma}^{-m_i}x^{m_i-1}+o(x^{m_i}+\varepsilon) \qquad (11)$$

From the probability theory knowledge that the CDF (cumulative distribution function) of $\gamma_i$ is presented:

$$F_{\gamma_i}(x) = \frac{a_i}{m_i}\bar{\gamma}^{-m_i}x^{m_i} \qquad (12)$$

Where:

$$m_i = \min\{m_{si}, m_{id}\},\ a_i = \begin{cases} \dfrac{m_{si}^{m_{si}}}{\Gamma(m_{si})\Omega_{si}'^{m_{si}}} & m_{si} < m_{id} \\ \dfrac{m_{si}^{m_{si}}}{\Gamma(m_{si})\Omega_{si}'^{m_{si}}} + \dfrac{m_{id}^{m_{id}}}{\Gamma(m_{id})\Omega_{id}'^{m_{id}}} & m_{si} = m_{id} \\ \dfrac{m_{id}^{m_{id}}}{\Gamma(m_{id})\Omega_{id}'^{m_{id}}} & m_{si} > m_{id} \end{cases}$$

$$\gamma_{b^*} = \max\{\gamma_i\}$$

so the CDF and PDF of $\gamma_{b^*}$ are expressed, respectively:

$$F_{\gamma_{b^*}}(x) = P(\gamma_{b^*} < x) = P(\max\{\gamma_i\} < x) = \prod_{i=1}^{N}F_{\gamma_i}(x) \qquad (13)$$

$$\begin{aligned} P_{\gamma_{b^*}}(x) &= \frac{\partial\prod_{i=1}^{N}F_{\gamma_i}(x)}{\partial x} = \left(\sum_{i=1}^{K}P_{\gamma_i}(x)\right)\prod_{j=1,j\neq i}^{K}F_{\gamma_j}(x) \\ &= \prod_{i=1}^{N}\left(\frac{a_i}{m_i}\bar{\gamma}^{-m_i}\right)\left(\sum_{i=1}^{N}m_i\right)x^{\sum_{i=1}^{N}m_i-1} \end{aligned} \qquad (14)$$

Through Laplasse transform, moment generating function can be obtained as follows:

$$M_{\gamma_{b^*}}(s) = \left(\sum_{i=1}^{N}m_i\right)\frac{\Gamma\left(\sum_{i=1}^{N}m_i\right)}{S^{\sum_{i=1}^{N}m_i}}\prod_{i=1}^{N}\left(\frac{a_i}{m_i}\bar{\gamma}^{-m_i}\right) = bs^{-d}+o(s^{-d}) \qquad (15)$$

Let:

$$b = \bar{\gamma}^{-\sum_{i=1}^{N}m_i}\left(\sum_{i=1}^{N}m_i\right)\Gamma\left(\sum_{i=1}^{N}m_i\right)\prod_{i=1}^{N}\frac{a_i}{m_i},\ d = \sum_{i=1}^{N}m_i$$

According to Wang and Giannakis (2003), the receiver using MPSK (Multi-phase Shift Keying) modulation, BER expression can be expressed as:

$$P_e = \frac{b}{\pi}\int_0^{\frac{M-1}{M}\pi}\left(\frac{\sin^2\theta}{\sin^2(\pi/M)}\right)^d d\theta \qquad (16)$$

## POWER ALLOCATION ALGORITHM

Now the total power is limited to P, minimizing the BER can achieve the best power allocation factor. Therefore, the optimization problem can be converted to:

$$\begin{cases} \min\{P_e\} \\ 0<\zeta<1 \end{cases} \qquad (17)$$

**Optimal allocation algorithm:** When $m_{si} = m_{id} = m_i$, the BER expression can be modified to:

$$P_e = \frac{1}{\pi}\bar{\gamma}^{-\sum_{i=1}^{N}m_i}\left(\sum_{i=1}^{N}m_i\right)\Gamma\left(\sum_{i=1}^{N}m_i\right)\prod_{i=1}^{N}\frac{a_i}{m_i}\int_0^{\frac{M-1}{M}\pi}\left(\frac{\sin^2\theta}{\sin^2(\pi/M)}\right)^d d\theta \qquad (18)$$

BER is related with power allocation factor $\xi$. Let:

$$f(\zeta) = \prod_{i=1}^{N}\left(\frac{1}{(\zeta\Omega_{si})^{m_i}}+\frac{1}{((1-\zeta)\Omega_{id})^{m_i}}\right)$$

obviously $0<\zeta<1$ is a convex set and $f(\zeta)$ is a strictly convex function. Minimizing $P_e$ is equivalent to get a minimum value of $f(\zeta)$, so the optimal power allocation factor can be expressed as:

$$\zeta^* = \arg\min_{0<\zeta<1}\prod_{i=1}^{N}\left(\frac{1}{(\zeta\Omega_{si})^{m_i}}+\frac{1}{((1-\zeta)\Omega_{id})^{m_i}}\right) \qquad (19)$$

Namely optimization problems can be transformed into:

$$\begin{cases} \min\{f(\zeta)\} \\ \text{s.t.}\ \ 0<\zeta<1 \end{cases}$$

this optimization problem is a constrained nonlinear issue. The derivative function of $f(\zeta)$ is expressed as:

$$f'(\zeta) = \sum_{i=1}^{N}\left(\frac{-m_i}{\zeta^{m_i+1}\Omega_{si}^{m_i}}+\frac{m_i}{(1-\zeta)^{m_i+1}\Omega_{id}^{m_i}}\right)\prod_{j=1,j\neq i}^{N}\left(\frac{1}{(\zeta\Omega_{sj})^{m_i}}+\frac{1}{((1-\zeta)\Omega_{jd})^{m_i}}\right) \qquad (20)$$

It can get the optimal power allocation factor by setting $f'(\zeta)$, then, the best $\zeta^*$ can be obtained by iterative algorithm.

**Suboptimal allocation algorithm:** Equation 20 is more complex, this study proposed a simple and effective suboptimal power allocation algorithm, since all channel fading parameters are the same $m_i = m$. In Eq. 20, for all the relay nodes:

$$\prod_{j=1, j\neq i}^{N}\left(\frac{1}{(\zeta\Omega_{sj})^{m_i}}+\frac{1}{((1-\zeta)\Omega_{jd})^{m_i}}\right)$$

is different and not equal to zero. Therefore, $f'(\zeta) = 0$ approximate to:

$$\sum_{i=1}^{K}\left(\frac{-1}{\zeta^{m+1}\Omega_{si}{}^{m}}+\frac{1}{(1-\zeta)^{m+1}\Omega_{id}{}^{m}}\right)= 0$$

this study achieves suboptimal solution as follows:

$$\zeta^{*\prime} = \frac{\sqrt[m+1]{\sum_{i=1}^{N}\Omega_{si}{}^{-m}}}{\sqrt[m+1]{\sum_{i=1}^{N}\Omega_{si}{}^{-m}}+\sqrt[m+1]{\sum_{i=1}^{N}\Omega_{id}{}^{-m}}} \qquad (21)$$

As the Eq. 21 shows, the suboptimal algorithm depends on average channels gains, the number of relay node and Nakagami fading parameter m, respectively.

## SIMULATION RESULTS AND ANALYSIS

Two proposed power allocation algorithms, i.e., OPA (Optimal Power Allocation) algorithm and SPA (Sub-optimal Power Allocation) algorithm, are applied to cooperative relay communication system. In order to measure the quality of the two algorithms, the EPA (Equal Power Allocation) algorithm in (Li *et al*., 2008) is adopted. The SPA algorithm is simple and effective and only depends on the average channel gain, while OPA achieves the best performance.

There are three relay nodes in the cooperative relay system, namely N = 3. The distance between source node and destination node is normalized to 1 and $d_{sd}$:$d_{sr}$:$d_{rd}$ = 1:1:1, the channel fading coefficients is expressed as $m_{si} = m_{id} = 0.7$. As shown in the simulation results, power allocation factor under three SNRs is presented and BER performance of three power allocation algorithms is compared under different average channel gains.

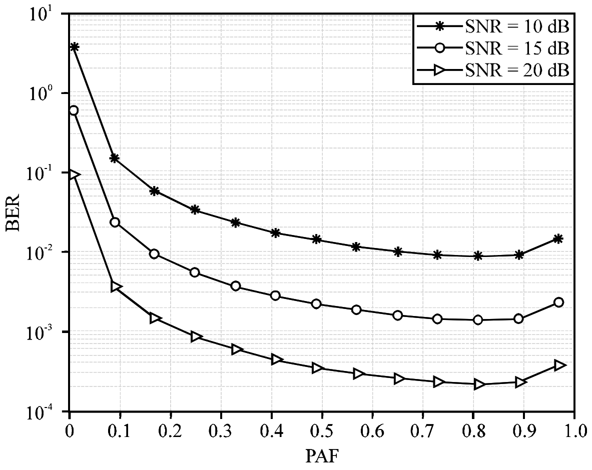

Fig. 2: PAF affects the BER performance, $\Omega_{si} = 2$, $\Omega_{id} = 2$, i = 1, 2, 3, PAF: Power allocation factor, BER: Bit error ratio

In Fig. 2 and 3, the average gains $\Omega_{si} = 2$, $\Omega_{id} = 200$, i = 1, 2, 3 for each channel and $\Omega_{id}$ is bigger than $\Omega_{si}$. As shown in Fig. 3, OPA and SPA algorithms are consistent with the theoretical result and which are about 2 dB better than the EPA algorithm. What is more important, the two proposed algorithms only depend on average channel gains. The OPA algorithm can be calculated by iteration method and the SPA algorithm as shown in Eq. 21, has the characteristic of low complexity. In order to obtain optimal BER performance, the power allocation factor is about 0.86. When the link quality between source node and relay node is poor, the source node should achieve more transmission power.

As shown in Fig. 4 and 5, the channel average gains $\Omega_{si} = 200$, $\Omega_{id} = 2$, i = 1, 2, 3 and $\Omega_{id}$ is smaller than $\Omega_{si}$. The analysis of the simulation results is similar to the above two figures. But the value of power allocation factor has a great difference. As is shown in Fig. 4, the optimum power allocation factor is about 0.13. Therefore, when the link quality between relay node and the destination node is poor, the selected relay node should be allocated more transmission power to guarantee the communication quality.

Seen from the above two cases, when the average gains of $S \to R_i$ and $R_i \to D$ have a big difference, compared to EPA algorithm, the SPA algorithm is closer to OPA algorithm. And the SPA algorithm proposed in this study, not only reduces complexity but also is easy to realize in the actual system.

In Fig. 6 and 7, average gain $\{\Omega_{s1}, \Omega_{s2}, \Omega_{s3}\} = \{4, 4.5, 4\}$, $\{\Omega_{1d}, \Omega_{2d}, \Omega_{3d}\} = \{4.5, 4.5, 4.5\}$ and $\Omega_{id}$ substantially

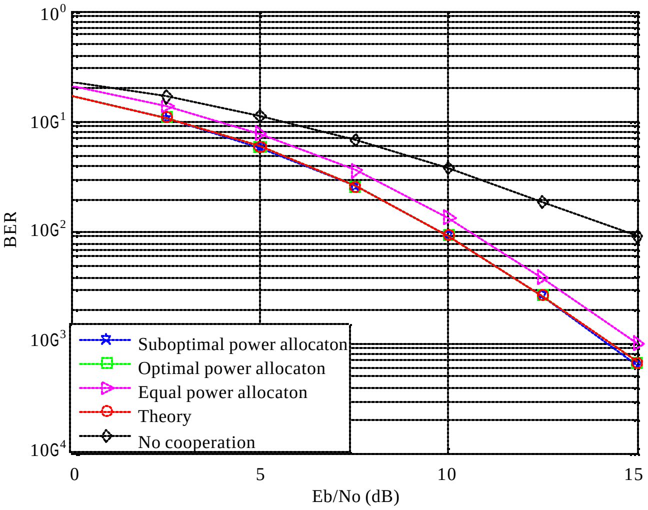

Fig. 3: Compared the BER performance, $\Omega_{si} = 2$, $\Omega_{id} = 200$, i = 1, 2, 3, BER: Bit error ratio

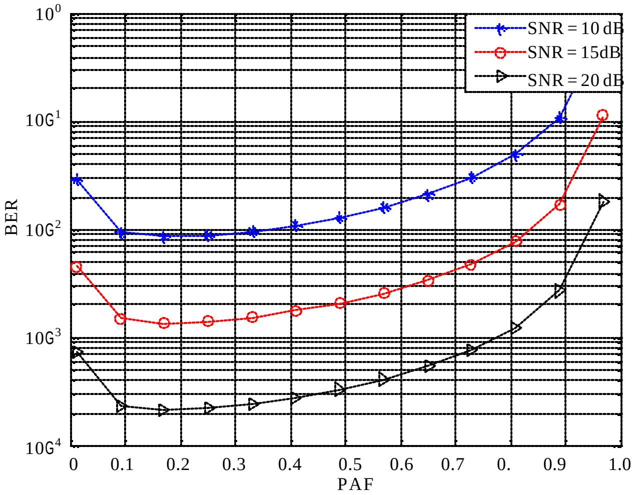

Fig. 4: PAF affects the BER performance, $\Omega_{si} = 200$, $\Omega_{id} = 2$, i = 1, 2, 3, PAF: Power allocation factor, BER: Bit error ratio

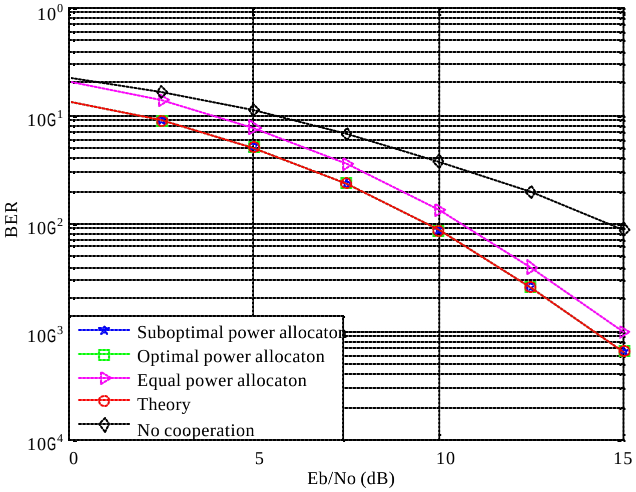

Fig. 5: Compared the BER performance, $\Omega_{si} = 200$, $\Omega_{id} = 2$, i = 1, 2, 3, BER: Bit error ratio

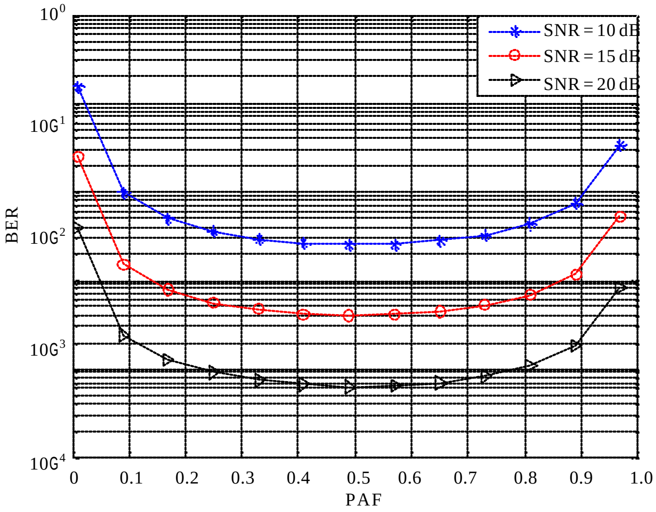

Fig. 6: PAF affects the BER performance, $\{\Omega_{s1}, \Omega_{s2}, \Omega_{s3}\} = \{4, 4.5, 4\}$, $\{\Omega_{1d}, \Omega_{2d}, \Omega_{3d}\} = \{4.5, 4.5, 4.5\}$, PAF: Power allocation factor, BER: Bit error ratio

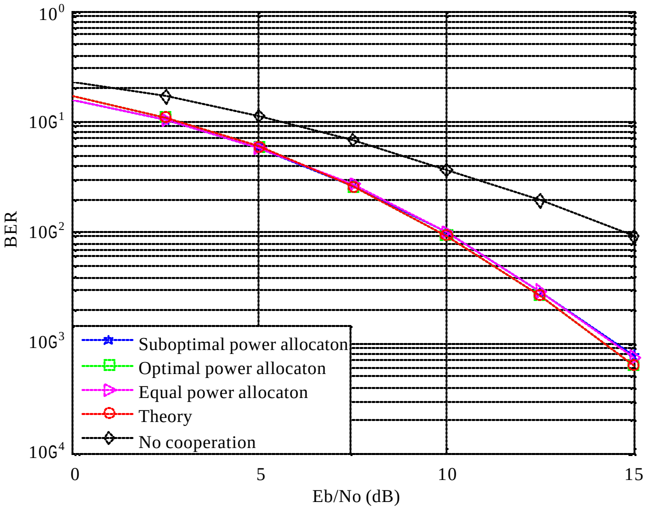

Fig. 7: Compared the BER performance, $\{\Omega_{s1}, \Omega_{s2}, \Omega_{s3}\} = \{4, 4.5, 4\}$, $\{\Omega_{1d}, \Omega_{2d}, \Omega_{3d}\} = \{4.5, 4.5, 4.5\}$, BER: Bit error ratio

equals to $\Omega_{si}$. Simulation results of OPA, SPA and EPA algorithms are consistent with the theoretical result. As Eq. 21 shows, to obtain optimal BER performance, the optimal power allocation factor is about 0.5. Therefore, SPA algorithm is equal to EPA algorithm.

## CONCLUSION

Transmission power can affect the BER in cooperative system. In this study, based on opportunistic relay selection, two power allocation algorithms are presented over Nakagami channel model. Nonlinear optimization model was given to minimize BER performance of the system. SPA algorithm of a simple and effective OPA algorithm is developed, which only depends on the average channel gains of the relays. Therefore, each node is not required to update the channel state information in real time. This study presents the performance of SPA algorithm is very close to OPA algorithm. The low complexity and better performance of the SPA algorithm can help the realization of cooperative relay technology.

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