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Stochastic Resonance Induced by over Damped Fractional-order Langevin Equation

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Abstract: Based on the existing theory of Fractional-order integral and derivative, this study discussed stochastic resonance induced by over-damped Fractional-order Langevin equation. Firstly, Fractional-order Langevin equation under over damped condition is derived and then its physical significance is discussed and internal mechanism of stochastic resonance phenomenon induced by over damped fractional-order Langevin equation is presented. The following corresponding numerical experiments state that it is easy to achieve to stochastic resonance by adjusting parameters of fractional-order Langevin equation such as fractional-order, noise intensity, or bi-stable system parameters and that it can be used to detect the frequency of the weak signal. The result of this study has positive practical value in weak periodic signal recovery.

Key words: Fractional-order Langevin equation, stochastic resonance, over damped

INTRODUCTION

During the past decades, stochastic resonance become one of the most active nonlinear phenomenon and attracted considerable attention. Stochastic resonance is firstly put forward by Benzi *et al.* (1981, 1983) has been extensively studied theoretically and experimentally around different power system and noise due to its vast applications in many fields. It describes a bi-stable system with nonlinear system, under the action of a small periodic modulation signal and inputting noise simultaneously, when the noise increases to certain intensity, the system output SNR will greatly boost and exist a best input noise intensity which can make the system produces a highest output SNR (Lingyun and Yuehua, 2013). Guerriero *et al.* (2009), Chen *et al.* (2008) and Applebaum (2009) have proved that stochastic resonance can increase weak signal detection performance within strong noise background. Generally, Langevin equation based on non-linear bi-stable system is used to the research model of stochastic resonance (Repperger *et al.*, 2005). Fractional-order calculus theory has attracted considerable attention for several decades in research on color noise, chaos and anomalous diffusion and so on (Ma and Hori, 2004). Goychuk and Hanggi (2011) found stochastic resonance in anomalous diffusion

process in their study in non-Markovian process in bi-stable renewal process model and had greater signal-to-noise improvement than traditional Markovian process. Recently, there are several researches on the phenomenon of stochastic resonance on fractional nonlinear systems (Xiao *et al.*, 2012). Based on the above-mentioned ideas, in this paper stochastic resonance induced by over damped fractional-order Langevin equation and application of which in weak signal detection are studied.

THEORY OF STOCHASTIC RESONANCE INDUCED BY OVER DAMPED FRACTIONAL-ORDER LANGEVIN EQUATION

Micro particle in medium got random collision from medium molecule due to the molecular thermal movement, resulting in a Brown movement. Langevin considered that Brownian particle with mass m in the mediums under the external forces including gradient potential field force $-U'(x)$, the damping force $-\eta\dot{x}$, random force $\xi(t)$ and the external signal power $F(t)$ (Calderon *et al.*, 2006). Due to Newton's second law, Langevin equation can be:

$$m\ddot{x} + \int_0^t \eta(t-t')\dot{x}(t')dt' = -U'(x) + F(t) + \xi(t) \quad (1)$$

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In the over-damped cases, the acceleration term $m\ddot{x}$ can be ignored, so the Langevin equation can be simplified as:

$$\int_0^t \eta(t-t') \dot{x}(t') dt' = -U'(x) + F(t) + \xi(t) \quad (2)$$

It is known that Brownian motion is characterized well by a bi-stable system. In this paper, we deal with the case that bi-stable system induced by periodic signals, its potential function is:

$$U(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4$$

And external power signal is $F(t) = A \cos(2\pi f_0 t)$, substituted to Langevin equation we have:

$$\int_0^t \eta(t-t') \dot{x}(t') dt' = ax - bx^3 + A \cos(2\pi f_0 t) + \xi(t) \quad (3)$$

where, $f(x, t) = -\partial U(x, t)/\partial x$ is a deterministic force; $\xi(t)$ denotes a zero mean, Gaussian white noise with autocorrelation function $\langle \xi(t) \rangle = 0$, $\langle \xi(t) \xi(t') \rangle = 2D\delta(t-t')$ and intensity D and $\eta(t)$ is the frictional memory kernel related to noise by the fluctuation-dissipation relation:

$$\langle \xi(t') \xi(t) \rangle = k_B T \eta(|t-t'|) \quad (4)$$

where, k_B is Boltzmann constant and T is medium temperature.

Damping force acting on the particles depends on the speed of the moments of the past time in different weights, which in the performance of damping kernel function with memory, setting the damping kernel function $\eta(t)$ is:

$$\eta(t) = \frac{1}{\Gamma(1-\alpha)} |t|^{-\alpha}, \quad 0 < \alpha < 1 \quad (5)$$

where, $\Gamma(\cdot)$ is Euler Gamma function.

Let formula (5) substituted to left end of Eq. 3, due to definitions of Riemann-Liouville integral and Caputo fractional derivative (Podlubny, 1999), we have:

$$\int_0^t \frac{1}{\Gamma(1-\alpha)} (t-t')^{\alpha-1} \dot{x}(t') dt' = D^\alpha x(t) \quad (6)$$

Therefore, fractional-order Langevin equation of over-damped Brownian motion is as follows:

$$D^\alpha x(t) = ax - bx^3 + A \cos(2\pi f_0 t) + \xi(t), \quad 0 < \alpha < 1 \quad (7)$$

From the definition of Caputo derivative we can see that, fractional derivative of displacement $x(t)$ in bi-stable potential well of Brownian particle is equivalent to a weighted integral of its speed $\dot{x}(t)$, closer to the present moment, greater of the weight of $\dot{x}(t)$ and vice versa. Also, decreased ratio of weight is related to the order α of fractional derivative. Particularly, if $\alpha \rightarrow 1$, damping kernel function $\eta(t)$ is degraded to unit impulse function, loss of characteristic of memory, fractional-order derivative is becoming to integer derivative, the result of derivative is the speed $\dot{x}(t)$ of particle when initial value is 0. If $\alpha \rightarrow 0$, damping kernel function $\eta(t)$ is degraded to constant 1, meaning that having the same speed memory of every moment, called having idea memory, which derivative is displacement $x(t)$.

EXPERIMENTS AND RESULTS

In this study, weak signal in heavy noise background is detected by the system as shown in Fig.1 and the signal-to-noise ratio (SNR) is used as the index of stochastic resonance, which is defined as follows:

$$SNR = \frac{2[\lim_{\Delta\omega \rightarrow 0} \int_{\Omega-\Delta\omega}^{\Omega+\Delta\omega} S(\omega) d\omega]}{S_N(\Omega)} \quad (8)$$

where, $S(\omega)$ is the signal power and $S_N(\Omega)$ denote the noise energy which is corresponding to the frequency range of the input signal.

Figure 2 shows that the system can achieve stochastic resonance by modulating system parameters a , b , α . Firstly, let parameters $a = b = 1$ be fixed. Figure 2a shows that the SNR is a non-monotonic function of fractional-order α and here exits a maximum. Combined concerned with Fig. 2b, Brownian particles transmit the potential barrier excited from noise intensity and with increase of noise intensity, the SNR is increased, until noise intensity reaches a certain value, the SNR is increased to a maximum. After that, along with the noise intensity further increase, the SNR began to decrease gradually. Otherwise, due to without need noise intensity for Brownian particles to transmit to another potential well, so stochastic resonance will not happen, SNR is a monotonic function of noise intensity, as shown in Fig. 2b and c.

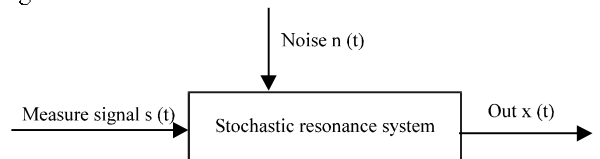


Fig. 1: Numerical simulation scheme of weak signal detection via stochastic resonance induced by over damped fractional-order Langevin equation

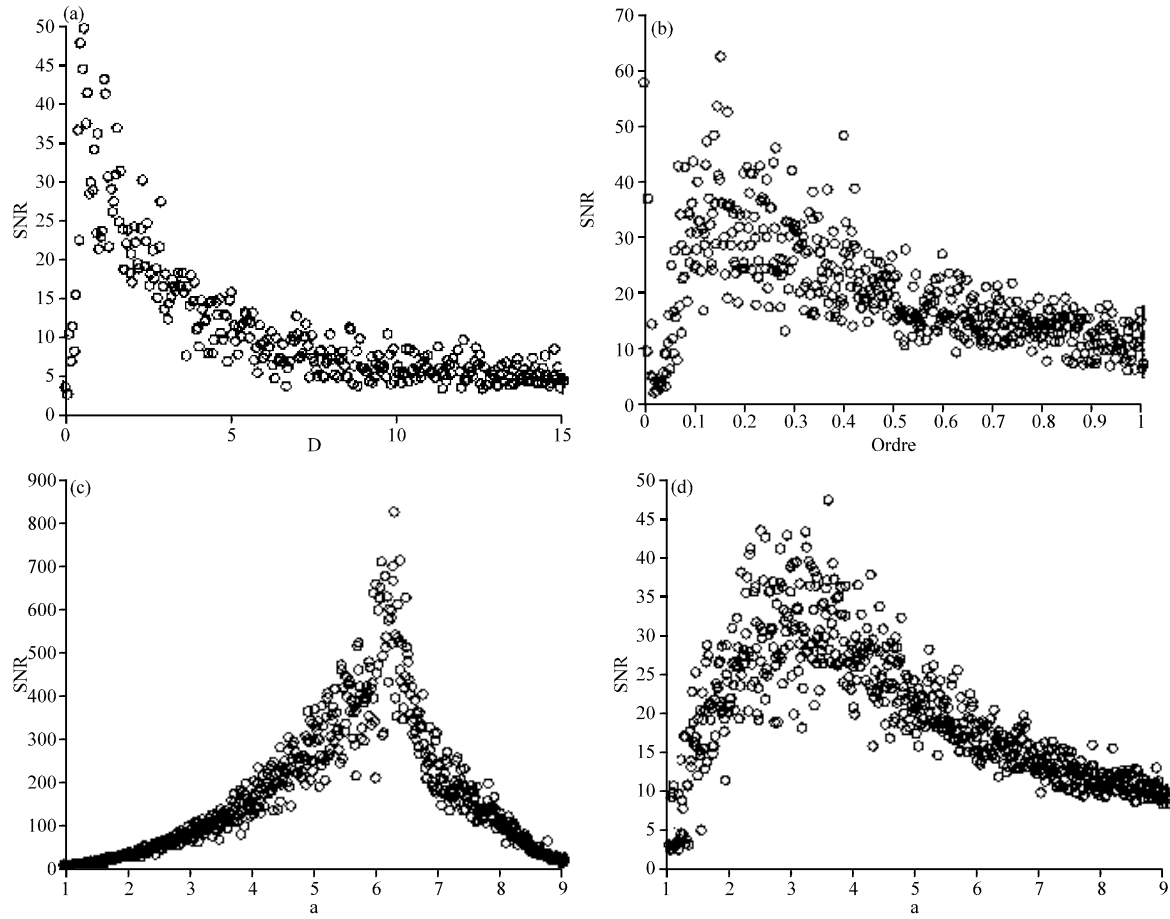


Fig. 2(a-d): Graph of SNR versus (a) D, (b) Fractional-order α , (c) a and (d) b

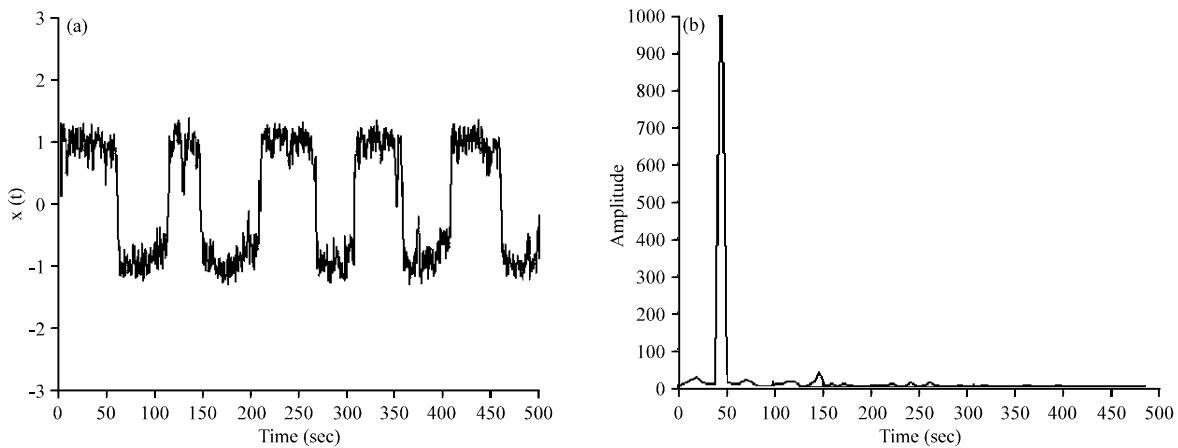


Fig. 3(a-b): (a) Output signal $x(t)$ of Eq. 7 and (b) Power spectrum of $x(t)$ of Eq. 7

Thus for non-linear system of induced by over damped Langevin equation, stochastic resonance occurs by modulating the fractional-order α . Without loss of

generality, fractional-order $\alpha = 0.2$ is taken, the non-linear system display stochastic resonance, which is shown in Fig. 3. Figure 3a and b depict the output signal $x(t)$ and its

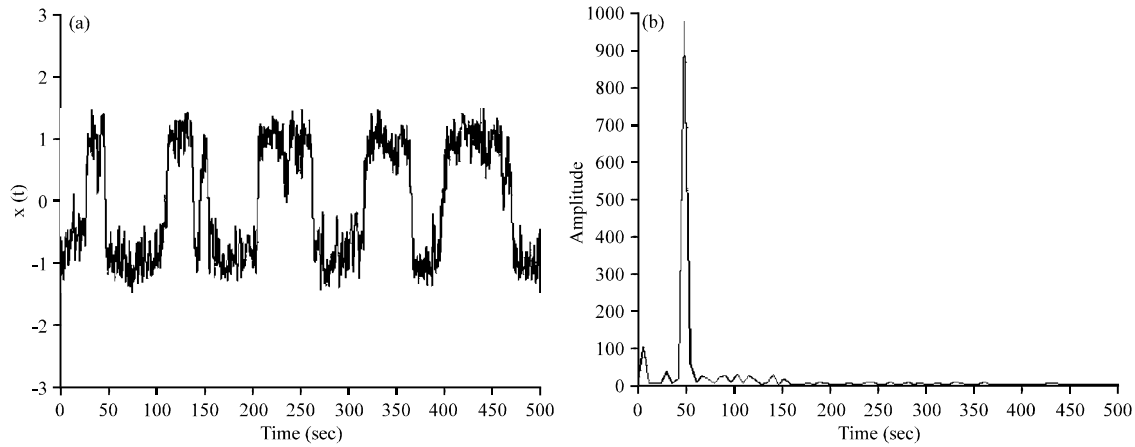


Fig. 4(a-b): (a) Output signal $x(t)$ of Eq. 7 and (b) Power spectrum of $x(t)$ of Eq. 7 when $a = 6$

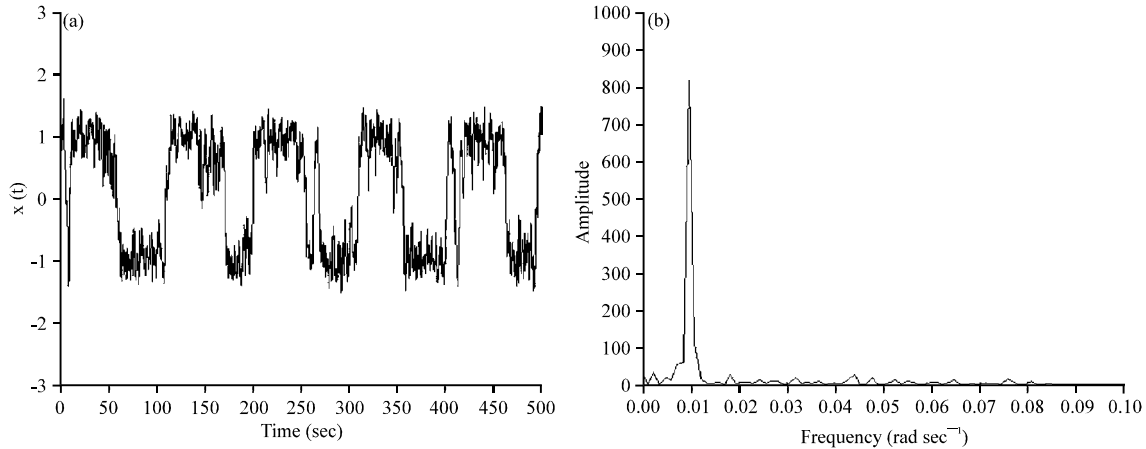


Fig. 5(a-b): (a) Output signal $x(t)$ of Eq. 7 and (b) Power spectrum of $x(t)$ of Eq. 7 when $a = 1$

power spectrum, respectively. It is clearly can be seen that the frequency of the weak signal can be detected.

Similarly, let parameters $b = 1$, $\alpha = 0.2$ be fixed. Figure 2c shows that the SNR is a non-monotonic function of the parameter α and here exits a maximum. Thus stochastic resonance can occurs easily by modulating the parameter α . Without loss of generality, fractional-order $a = 6$ is taken, which is shown in Fig. 4. Figure 4a and b depict the output signal $x(t)$ for $a = 6$ and its power spectrum, respectively. It is clearly can be seen that the frequency of the weak signal can be detected.

Also, let parameters $a = 1$, $\alpha = 0.2$ be fixed, stochastic resonance would also occur by modulating the parameter b , which is shown in Fig. 2d and Fig. 5.

CONCLUSION

In this study, stochastic resonance induced by over-damped Fractional-order Langevin equation is discussed. Firstly, we derived fractional-order Langevin equation under over damped conditions and studied stochastic resonance induced by it, including the internal mechanism of the phenomenon of which. Then experimental results show that stochastic resonance induced by over damped fractional-order Langevin equation can occur by modulating fractional-order α , the noise intensity D , or parameters a , b . One of the characteristic of stochastic resonance in fractional-order Langevin equation is that it can enhance stochastic resonance by modulating the fractional-order α . At last, corresponding experimental

results state that by adjusting the above-mentioned parameters, the weak signal can be detected with power spectra.

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