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## Performance of Physical-layer Network Coding in Asymmetric Rayleigh Fading Two-way Relay Communication Systems

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**Abstract:** Physical-layer Network Coding (PNC) has drawn so much attention of wireless communication researchers. However, most of the studies focused on symmetric Two-Way Relay Communication (TWRC) systems. In order to analyze the performance of PNC in asymmetric scenario, in this study, we classify TWRC systems with Rayleigh fading channels into five asymmetric cases according to different poor channel allocation. In each symmetric and asymmetric case, we provide the detailed signal transmission and processing progress. Then we simulate the Bit Error Rate (BER) performance of PNC either in symmetric or asymmetric cases. From the results, we know that the Phase Asymmetry-Uplink (PA-U) case is the worst and Downlink Channel Asymmetry (DCA) case is the best among the five asymmetric cases. We also conclude that poor uplink channels will provide more severe effect to the performance degradation caused by system asymmetry than poor downlink ones.

**Key words:** Physical-layer network coding, two-way relay communications, rayleigh fading, bit error rate

### INTRODUCTION

Network Coding (NC) (Ahlsvede *et al.*, 2000), a revolutionary technology of communication networks, has become a very hot research area and so many network researchers focused on it such as (Wang and Zhang, 2012; Zhou *et al.*, 2010; Tao *et al.*, 2012; Wei *et al.*, 2011). A typical form of network coding is Physical-layer Network Coding (PNC) (Zhang *et al.*, 2006), which was proposed in 2006 and became a very attractive technology rapidly because it can improve about 100 and 50% system throughput than traditional multi-hop scheme and NC scheme in Two-Way Relay Communication (TWRC) systems (Shannon, 1961), respectively. Then, so many researchers focused on PNC and obtained lots of important achievements. Popovski and Yomo (2007) and Koike-Akino *et al.* (2009), the authors studied the optimal constellations for PNC in TWRC. Lu *et al.* (2011) and Lu and Liew (2012) proposed optimized decoding algorithm for asynchronous PNC. The authors of (Yang *et al.*, 2013) studied PNC in cooperative communication systems and proposed a new decode-and-forward scheme.

We observed that almost all of the studies of PNC in TWRC assumed the channels are symmetric, i.e., all channels have the same channel conditions. Only (Liu *et al.*, 2009) studied the asymmetric downlink Additive White Gaussian Noise (AWGN) channels with PNC. To the best our knowledge, there are no other

studies focused on PNC with asymmetric TWRC, especially no papers studied the asymmetric TWRC with Rayleigh flat fading channels. In fact, affected by multiple factors, the systems are not always symmetric. Then, the study of PNC in asymmetric TWRC is valuable.

In this study, we classify two-way relay communication systems into five asymmetric categories according to different poor channel allocation and apply PNC in these five cases. We provide the decision Log-Likelihood Ratio (LLR) for the relay node in each case and simulate the Bit Error Rate (BER) performance of PNC for symmetric and asymmetric cases. Then we compare the simulation results and get some impotent conclusions.

### TWO-WAY RELAY COMMUNICATION SYSTEM MODEL

A two-way relay communication system is shown in Fig. 1, in which the two end nodes, 1 and 2, want to exchange information with the help of the relay node R,

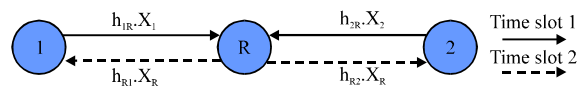


Fig. 1: Two-way relay communication system model (h is fading coefficient and X is packet)

due to there are no direct links between node 1 and 2. We don't consider channel coding and decoding and assume all nodes work in half-duplex mode, which means a node cannot transmit and receive simultaneously. All channels in this system are modeled as Rayleigh flat fading. The complex fading coefficient,  $h = h_{re} + jh_{im}$  is with magnitude,  $|h|$ , following a probability distribution:

$$p(|h|) = \frac{|h|}{\sigma^2} \exp\left(-\frac{|h|^2}{2\sigma^2}\right) \quad (1)$$

When using PNC scheme, the TWRC system needs two time slots to complete an information exchanging cycle. In time slot 1, node 1 and 2 transmit their binary phase shift keying (BPSK) modulated information,  $X_1$  and  $X_2$ , simultaneously. The signal from the two nodes mixed in the air after multiplied complex fading coefficients,  $h_{1R}$  and  $h_{2R}$  and then affected by the additive noise,  $n_R$ . We define this transmission as uplink phase. Upon receiving the superimposed signal, relay R makes a LLR decision and obtain the XOR combined information,  $S_R = S_1 \oplus S_2$ . Then, it broadcasts the modulated  $X_R$  to destination node 1 and 2. The broadcast signal also will be multiplied fading

factors,  $h_{R1}$  and  $h_{R2}$  and added noises,  $n_1$  and  $n_2$ , before it is received by node 1 and 2, respectively. When receiving the broadcast signal, node 1 and 2 can decode and recover their desired information with the help of their self-information. The decoding and recovering expressed as  $\hat{S}_2 = S_1 \oplus \hat{S}_R$  and  $\hat{S}_1 = S_2 \oplus \hat{S}_R$  at node 1 and node 2, respectively. The detailed block diagrams of the uplink and downlink phase are shown in Fig. 2. We assume that all nodes transmit signal with power 1.

### DETAILED SIGNAL TRANSMISSION AND PROCESSING PROGRESS IN SYMMETRIC TWRC SYSTEMS

For symmetric TWRC systems shown in Fig. 1, in uplink phase, the BPSK modulated signals in a symbol period at node 1 and 2 are expressed as:

$$\begin{cases} x_1 = 1 - 2s_1 \\ x_2 = 1 - 2s_2 \end{cases} \quad (2)$$

The two signals mixed in the air after multiplied fading coefficients and then added noise. The received

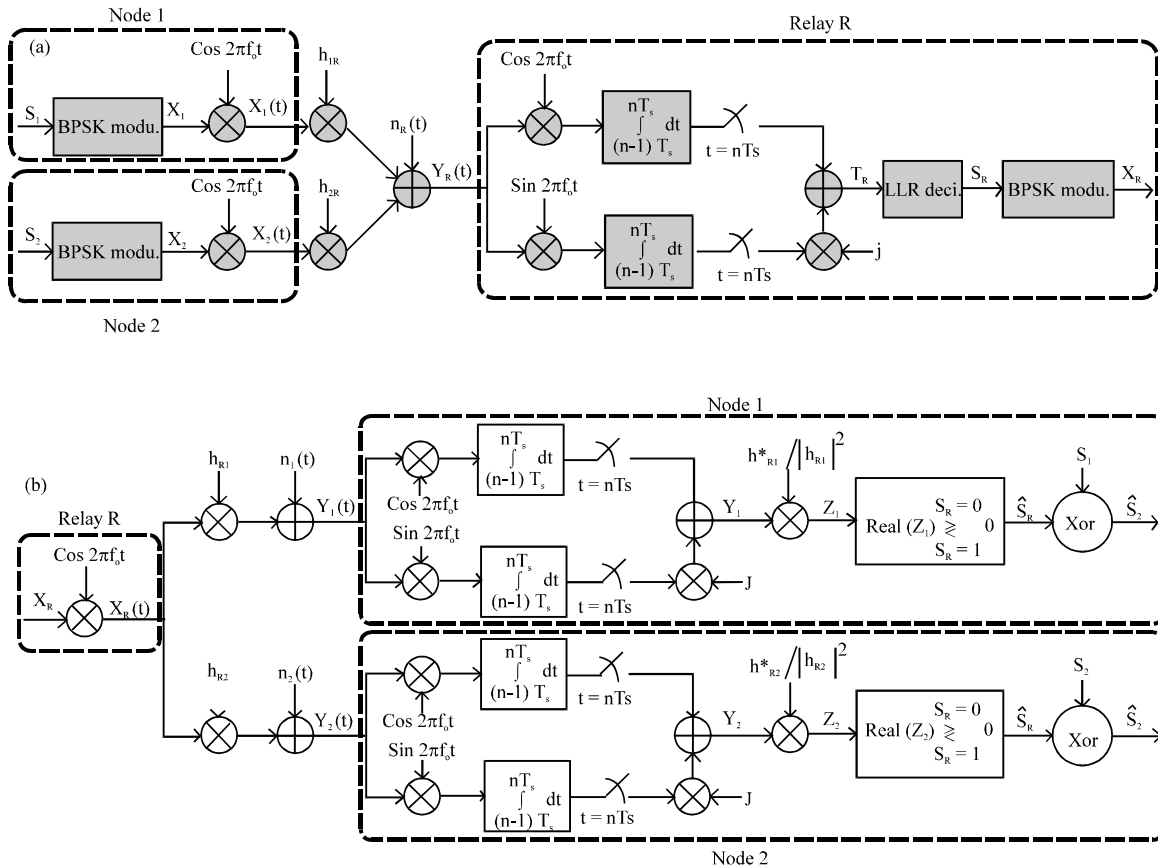


Fig. 2(a-b): Block diagrams of the TWRC system with PNC, (a) Uplink phase and (b) Downlink phase

baseband signal in a symbol period at relay node R after down conversion from the carrier frequency and low-pass filtering is expressed as:

$$y_R = h_{1R}x_1 + h_{2R}x_2 + n_R \quad (3)$$

where,  $n_R = n_{Rc} + jn_{Ri}$  is the additive white Gaussian noise with variance  $N_0/2$  per dimension. In order to get the XOR combined bit  $s_R$ , it calculates the LLR and makes a decision. The LLR is calculated as:

$$\begin{aligned} \Lambda &= \log \left( \frac{\Pr(s_R = 0/y_R)}{\Pr(s_R = 1/y_R)} \right) \\ &= \log \left( \frac{\sum_{c=\pm(h_{1R}+h_{2R})} \Pr((x_1 + x_2 = c)/y_R)}{\sum_{c=\pm(h_{1R}-h_{2R})} \Pr((x_1 + x_2 = c)/y_R)} \right) \\ &= \log \left( \frac{e^{-\frac{|y_R - (h_{1R} + h_{2R})|^2}{N_0}} + e^{-\frac{|y_R + (h_{1R} + h_{2R})|^2}{N_0}}}{e^{-\frac{|y_R - (h_{1R} - h_{2R})|^2}{N_0}} + e^{-\frac{|y_R + (h_{1R} - h_{2R})|^2}{N_0}}} \right) \end{aligned} \quad (4)$$

and the decision is:

$$\Lambda \underset{s_R=1}{\overset{s_R=0}{\leq}} 0 \quad (5)$$

In the downlink phase, the combined signal  $S_R$  is BPSK modulated and broadcasted to the two destination nodes. The baseband signal in a symbol period received by node 1 and 2 can be expressed as:

$$\begin{cases} y_1 = h_{R1}x_R + n_1 \\ y_2 = h_{R2}x_R + n_2 \end{cases} \quad (6)$$

Then, the baseband signal will be multiplied  $h_{Ri}^*/|h_{Ri}|^2$  for Maximum Ratio Transmission (MRT) (Lo, 1999). After making a hard decision shown in Fig. 2, we can obtain the estimated XOR bit  $\hat{s}_R$ . Then, with the help of self-information, node 1 and 2 can decode and recover their desired information as:

$$\begin{cases} \hat{s}_2 = \hat{s}_R \oplus s_1 \\ \hat{s}_1 = \hat{s}_R \oplus s_2 \end{cases} \quad (7)$$

The systems' BER for this symmetric case will be given by computer simulation shown in the later section.

### FIVE ASYMMETRIC CASES OF TWRC SYSTEMS

In practice, wireless channels are not always ideal and will be affected by multiple factors such as obstacles; distance so on and so forth. As a result of that, channels in TWRC systems are not always symmetric. So, it is valuable to study PNC in asymmetric TWRC. In order to describe the asymmetric scenarios, we introduce an asymmetric factor,  $\mu$ , which is belong to interval (0, 1).  $\mu = 1$  implies the channels are symmetric. The asymmetry depth becomes more severe along with  $\mu$  decreases. The asymmetric factor will be multiplied to the transmitting signals within a poor channel. TWRC systems can be classified into five asymmetric cases according to different poor channel allocation as follows.

**Phase asymmetry-uplink (PA-U):** When the two channels in uplink phase,  $C_{1R}$  and  $C_{2R}$  are poor and the signals transmitted within these two channels should be multiplied the asymmetric factor  $\mu$  as shown in Fig. 3. This case is defined as phase asymmetry-uplink (PA-U). During the uplink phase, the baseband signal received by relay node R is:

$$y_R = \mu h_{1R} x_1 + \mu h_{2R} x_2 + n_R \quad (8)$$

And the LLR used to make decision at relay R is calculated as:

$$\Lambda = \log \left( \frac{e^{-\frac{|y_R - \mu(h_{1R} + h_{2R})|^2}{N_0}} + e^{-\frac{|y_R + \mu(h_{1R} + h_{2R})|^2}{N_0}}}{e^{-\frac{|y_R - \mu(h_{1R} - h_{2R})|^2}{N_0}} + e^{-\frac{|y_R + \mu(h_{1R} - h_{2R})|^2}{N_0}}} \right) \quad (9)$$

The decision rule is (5). During the downlink phase, the signal transmission, signal processing and decoding operation are same as symmetric case expressed as (6) and (7).

**Phase asymmetry-downlink (PA-D):** When the two channels in downlink phase,  $C_{R1}$  and  $C_{R2}$  are poor as shown in Fig. 4. This situation is defined as phase

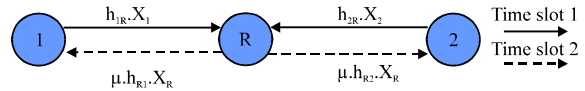


Fig. 3: Phase asymmetry-uplink (PA-U)

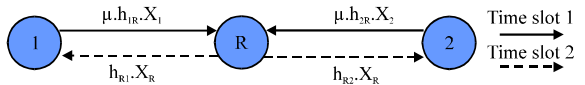


Fig. 4: Phase asymmetry-downlink (PA-D)

asymmetry-downlink (PA-D). During the uplink phase, the signal transmission and processing at relay R is same as symmetric case expressed as (3), (4) and (5). During the downlink phase, the baseband signals received by node 1 and 2 are expressed as:

$$\begin{cases} y_1 = \mu h_{R1} X_R + n_1 \\ y_2 = \mu h_{R2} X_R + n_2 \end{cases} \quad (10)$$

The following signal processing is similar as symmetric case: MRT and make decision. The decoding is expressed as (7).

**Uplink channel asymmetry (UCA):** Just one channel of the uplink phase,  $C_{1R}$  or  $C_{2R}$ , is poor as shown in Fig. 5. This case is defined as uplink channel asymmetry (UCA). Without loss of generality, we assume  $C_{1R}$  is poor as shown in Fig. 5a. As a result, during the uplink phase, the baseband signal received by relay R is expressed as:

$$y_R = \mu h_{1R} X_1 + h_{2R} X_2 + n_R \quad (11)$$

Then, the LLR is calculated as:

$$\Lambda = \log \left( \frac{e^{-\frac{|y_1 - (\mu h_{1R} + h_{2R})|^2}{N_0}} + e^{-\frac{|y_1 + (\mu h_{1R} + h_{2R})|^2}{N_0}}}{e^{-\frac{|y_1 - (\mu h_{1R} - h_{2R})|^2}{N_0}} + e^{-\frac{|y_1 + (\mu h_{1R} - h_{2R})|^2}{N_0}}} \right) \quad (12)$$

and the decision is (5). During the downlink phase, the progress is same as symmetric case expressed as (6) and (7).

**Downlink channel asymmetry (DCA):** One of the channel in downlink phase,  $C_{R1}$  or  $C_{R2}$ , is poor as shown in Fig. 6. This situation is defined as Downlink Channel Asymmetry (DCA). In the uplink phase, the signal transmission and processing at relay R are same as symmetric case expressed as (3), (4) and (5). During the downlink phase, without loss of generality, we assume  $C_{R1}$  is poor as shown in Fig. 6a. Then the received baseband signals at node 1 and 2 are:

$$\begin{cases} y_1 = \mu h_{R1} X_R + n_1 \\ y_2 = h_{R2} X_R + n_2 \end{cases} \quad (13)$$

Then, the MRT operation and decision are same as symmetric case and the decoding is also (7).

**Node asymmetry (NA):** A pair of uplink and downlink channels between an end node (1 or 2) and the relay R,

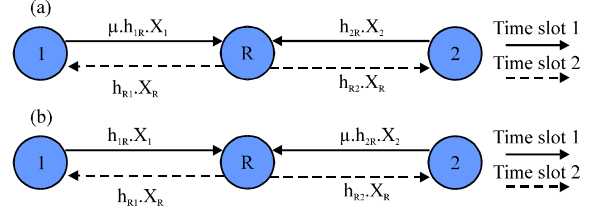


Fig. 5(a-b): Uplink channel asymmetry (UCA)

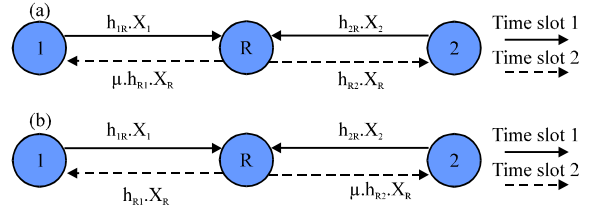


Fig. 6(a-b): Downlink channel asymmetry (DCA)

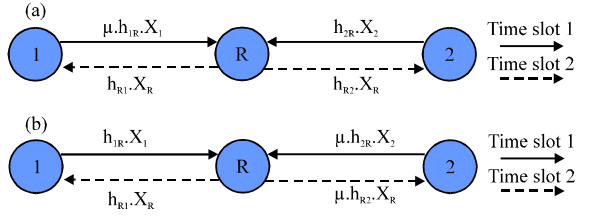


Fig. 7(a-b): Node asymmetry (NA)

( $C_{1R}$ ,  $C_{R1}$ ) or ( $C_{2R}$ ,  $C_{R2}$ ), are poor, as shown in Fig. 7. This situation is defined as node asymmetry (NA). Without of generality, we assume channels between node 1 and relay R, ( $C_{1R}$ ,  $C_{R1}$ ), are poor as shown in Fig. 7a. During the uplink phase, the signal transmission and processing progress is same as case UAC expressed as (11), (12) and (5). Similarly, the progress in the downlink phase is same as case DAC expressed as (13) and (7). Then node 1 and 2 can obtain their desired information.

## BER SIMULATION RESULTS AND ANALYSIS

We simulate the BER performance of PNC for symmetric and asymmetric cases in these two scenarios: 1) Signal to Noise Ratio (SNR) of all channels is variable from 0 to 40 dB and  $\mu = 0.7079$ , i.e.,  $\mu^2 = -3$  dB. That means the poor channels are weaker than other channels and the SNR difference is -3 dB, as shown in Fig. 8 b is the partly enlarged version of Fig. 8a for clear illustration); 2) SNR of all channels is fixed at 40 dB and  $\mu^2$  is variable from -40 to 0 dB, i.e.,  $\mu^2$  is variable from 0 to 40dB, as shown in Fig. 9.

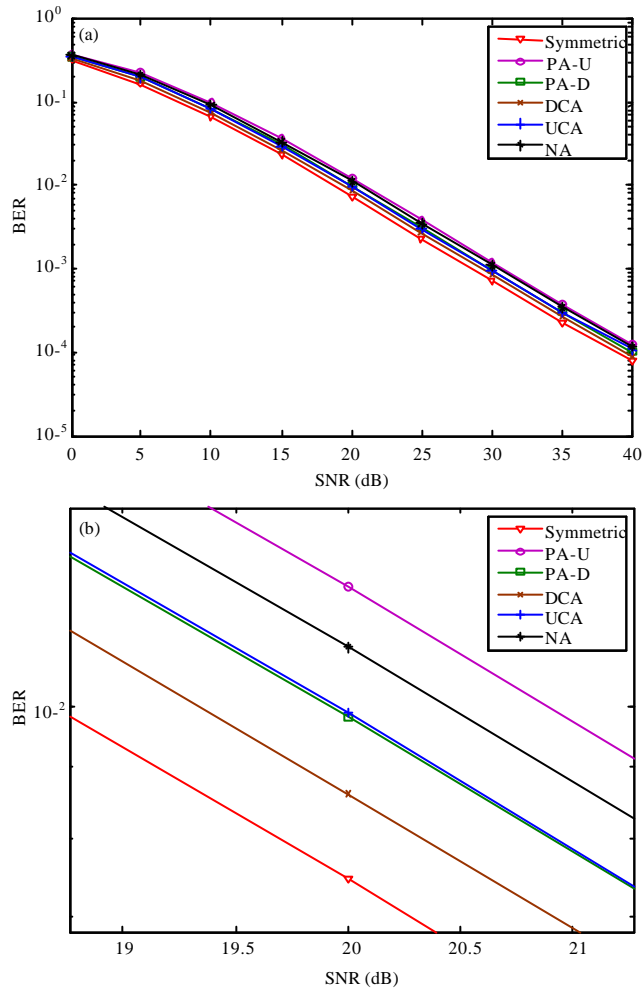


Fig. 8(a-b): BER of symmetric and asymmetric cases when SNR is variable (a) BER of symmetric and asymmetric cases when  $\mu^2 = -3\text{dB}$  and SNR is variable from 0 to 40 dB and (b) Partly enlarged version of Fig. 8(a)

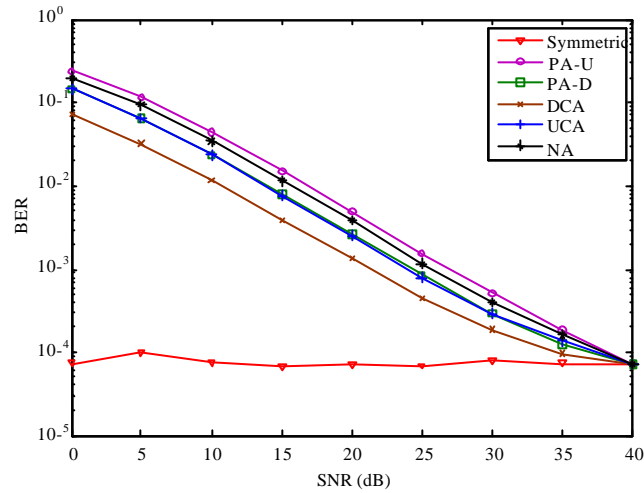


Fig. 9: BER of symmetric and asymmetric cases when SNR = 40dB and  $\mu^2$  is variable from -40 to 0 dB

From Fig. 8, we can see that the BER performances of all asymmetric cases have degradation compared with symmetric case, as expected. The worst case is PA-U, which has about 2.3 dB degradation than symmetric case. Other asymmetric case are slightly better than PA-U. Case DCA has the best BER performance among the asymmetric cases. However, there is still 0.8 dB loss than symmetric case.

For scenario (2), as shown in Fig. 9, BER of symmetric case is almost a constant due to the fixed SNR. Similar to the results of Fig. 8, case PA-U is also the worst and case DCA is the best among the asymmetric cases. The BER differences between symmetric and asymmetric cases decrease along with the value of  $\mu$  increase, as expected. When  $\mu=1$ , i.e.,  $\mu^2 = 0$  dB, all the cases has the same BER performance which is corresponding to the definition of asymmetric factor  $\mu$ .

Seen from Fig. 8 and 9, we can get an important conclusion: the top three worst asymmetric cases, PA-U, NA and UCA, all contain 1 or 2 poor uplink channels. That means the effect of poor uplink channels is more severe to the performance degradation than poor downlink channels. The reasons can be explained as follows. At relay R, the XOR combined bit,  $s_R = s_1 \oplus s_2$ , will be in error if any one of  $s_1$  and  $s_2$  is incorrect. And as a result, upon receiving the error broadcast XOR bit, both of the two destination nodes will decode incorrect bit. That means one error occurred during the uplink phase will affect the two decoded bits of node 1 and 2, but one error occurred in downlink phase just affect one decoded bit at node 1 or node 2. Based on this conclusion, we should pay more attentions to poor uplink channels if we want to improve the BER performance of PNC in asymmetric TWRC systems.

### CONCLUSION

In this study, the performance of physical-layer network coding in asymmetric two-way relay communication systems with Rayleigh flat fading channels is analyzed and simulated. The asymmetric TWRC systems are classified into five cases according to different poor channel allocation. And the detailed signal transmission and processing progresses in symmetric and asymmetric cases are described. The BER performances for these cases are simulated. From the results, we can see that all asymmetric cases are worse than symmetric one. Among the asymmetric cases, phase asymmetry-uplink (PA-U) case is the worst and downlink channels asymmetry (DCA) case is the best.

We can also get another important conclusion: poor uplink channels provide more severe effect than poor downlink channels in the performance degradation caused by asymmetric channel conditions. This conclusion is very valuable when we design some schemes to improve the performance of PNC in asymmetric TWRC systems.

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